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Britta Kohlbrecher
University of Erlangen-Nürnberg

Christian Merkl
University of Erlangen-Nürnberg

Daniela Nordmeier
Deutsche Bundesbank

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Revisiting the Matching Function*

Britta Kohlbrecher[a], Christian Merkl[a] and Daniela Nordmeier[b]

[a] Friedrich-Alexander-Universität (FAU) Erlangen-Nürnberg,

[b] Deutsche Bundesbank.

There is strong empirical evidence for Cobb-Douglas matching functions. We show in this paper that this widely found relation between matches on the one hand and unemployment and vacancies on the other hand can be the result of different underlying mechanisms. Obviously, it can be generated by assuming a Cobb-Douglas matching function. Less obvious, the same relationship results from a vacancy free entry condition and idiosyncratic productivity shocks. A positive aggregate productivity shock leads to more vacancy posting, a shift of the idiosyncratic selection cutoff and thereby more hiring. We calibrate a model with both mechanisms to administrative German labor market data and show that idiosyncratic productivity for new contacts is an important driver of the elasticity of the job-finding rate with respect to market tightness. Accounting for idiosyncratic productivity can explain the observed negative time trend in estimated matching efficiency and asymmetric business cycle responses to large aggregate shocks.

JEL Classification: E24, E32, E20, E30

Key words: matching function, idiosyncratic productivity, job creation, vacancies, time trend, asymmetries

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1. Introduction

We show in this paper that a labor market model with a constant contact rate for unemployed workers, a free-entry condition for vacancies, and idiosyncratic productivity shocks for new contacts generates the same equilibrium comovement between matches, unemployment, and vacancies as a model with a standard Cobb-Douglas contact function.[1] We use German administrative wage data to calibrate our model and show that a large part of the elasticity of the job-finding rate with respect to market tightness - the parameter estimated in matching functions - is driven by idiosyncratic productivity shocks.

There is widespread empirical evidence for a Cobb-Douglas constant returns matching function across countries, occupations, or other disaggregation levels (see Blanchard and Diamond (1990) for an early work and Petrongolo and Pissarides (2001) for a survey). The coefficients from these matching function estimations are often used to parametrize Cobb-Douglas contact functions in theoretical models. Thus, the job-creation mechanism in search and matching models is usually exclusively driven by a theoretical contact function.[2] In reality, job creation consists of more than one margin. After workers and firms get in contact (e.g. in an interview), only a certain fraction of workers is selected. Not all workers are suitable for an employer and thus only those with the best characteristics (e.g. idiosyncratic shocks) are selected. This second mechanism is also well established in the literature.[3] However, most existing macro-labor business cycle papers use a degenerate selection mechanism, i.e. idiosyncratic shocks play no meaningful role for job creation.[4] There are some works that combine a contact function, a vacancy free-entry condition, and idiosyncratic productivity (e.g. Krause and Lubik, 2007; Merz, 1999; Thomas and Zanetti, 2009; Zanetti, 2011). However, idiosyncratic productivity shocks are mainly used to model the behavior of separations. Depending on the timing, matches may also be affected. In contrast to our paper, the interaction of idiosyncratic shocks and the job-finding rate is not the focus of the analysis.

[1] In what follows, "contact function" refers to the theoretical function that establishes contacts between workers and firms. Due to idiosyncratic shocks, not all of the contacts may become matches. "Matching function" refers to the empirical connection between matches on the one hand and vacancies and unemployment on the other hand.

[2] See e.g. Hall (2005); Hagedorn and Manovskii (2008), and Shimer (2005) or Christiano et al. (2015) for an estimated medium-scale model.

[3] For a seminal contribution with idiosyncratic productivity see Jovanovic (1979). Traditional search models (e.g. McCall, 1970; Mortensen, 1987) rely on exogenous wage distributions. If they are interpreted as the result of some underlying idiosyncratic productivity heterogeneity, they fall into the same category of models. The stochastic job-matching model (Pissarides, 2000, chapter 6) combines a traditional Cobb-Douglas contact function and permanent idiosyncratic productivity shocks.

[4] See the upper panel in Figure 6 in the Appendix for an illustration.

In this paper, we focus on the potential role of idiosyncratic productivity for job creation. Other than the existing literature, we show how idiosyncratic shocks affect the shape of the estimated matching function, i.e. the elasticity of the job-finding rate with respect to market tightness. Imagine that every worker gets in contact with a firm with a constant probability. This is a special case of a Cobb-Douglas contact function in which the overall number of contacts does not respond to vacancies. We denote this as "degenerate" contact function henceforth. Due to different idiosyncratic productivity, firms only select those workers with large enough realizations.[5] Even though the aggregate number of contacts would not respond to vacancies in such a case, both vacancies and the job-finding rate are procyclical. A positive aggregate productivity shock would still lead to a rise of the ex-ante expected profits in firms' vacancy free-entry condition and thereby stimulate vacancy creation. In addition, larger aggregate productivity makes it profitable for firms to hire workers with less favorable characteristics (i.e. lower idiosyncratic productivity). This increases the job-finding rate.

We show analytically and numerically that the degenerate contact function with idiosyncratic productivity shocks generates an equilibrium comovement between matches, unemployment, and vacancies that is observationally equivalent to a Cobb-Douglas constant returns contact function up to a first-order Taylor approximation. One of our contributions is to show that dynamic labor market models with two standard modeling ingredients (vacancy free entry and idiosyncratic productivity) generate a simulated time-series behavior that is in line with the results from matching function estimations. Obviously, many theories generate procyclical employment/hours. However, the combination of a vacancy free-entry condition and idiosyncratic productivity generates a Cobb-Douglas and close to constant returns comovement between matches, unemployment, and vacancies.[6]

We prove that the shape of the idiosyncratic productivity distribution at the hiring cutoff determines the precise nature of the comovement, i.e. the coefficients in an estimated matching function. One of the key contributions of this paper is the link between high quality German administrative labor market data, the idiosyncratic shock distribution for new contacts, and the aggregate matching function. We use administrative wage data to impose discipline on the shape and dispersion of the idiosyncratic shock distribution. This allows us to run meaningful counterfactual exercises.

To assess the relative importance and implications of idiosyncratic productivity, we

[5] Brown et al. (2015) and Lechthaler et al. (2010) use similar mechanisms in the context of more complex models.

[6] We are not aware of any other model (except for the matching function) that generates such a time-series behavior.

combine a traditional Cobb-Douglas contact function and idiosyncratic productivity in a dynamic model calibrated to German data. We calibrate the idiosyncratic shock distribution by using residual wages for new employment spells and we target the elasticity of the job-finding rate with respect to market tightness in the data obtained from a matching function estimation. Due to idiosyncratic productivity shocks, the required weight on vacancies in the calibrated contact function is much smaller than the coefficient from the matching function estimation would suggest. Idiosyncratic productivity drives a large part of the observed elasticity, namely about three quarters in our baseline calibration. Thus, our paper reveals that the conventional practice to use matching function estimations in order to parametrize contact functions has caveats. More precisely: Assume that the weight on vacancies in the traditional contact function is parametrized with the values obtained from a matching function estimation. Then, in a model with idiosyncratic shocks, the model based comovement between matches, unemployment, and vacancies will not be in line with the data any more. In this scenario, a matching function estimation based on simulated data generates a larger weight on vacancies than in the empirical data. This is also relevant from a normative perspective. According to Hosios' (1990) rule, an economy is constrained efficient when the bargaining power is equal to the elasticity of the contact function with respect to vacancies.

For small business cycle shocks or up to a first-order Taylor approximation, the relative importance of the contact function and idiosyncratic shocks does not matter much from a positive perspective. However, for the nonlinear dynamics of the labor market it is very important to understand the driving forces of match formation. We give two examples where idiosyncratic shocks for new contacts provide a rationale for puzzling empirical phenomena. First, we explain the negative time trend in matching function estimations (Petrongolo and Pissarides, 2001; Poeschel, 2012) by a decline in vacancy posting costs (e.g. due to new technologies). Second, we show that large aggregate shocks lead to asymmetric responses on the labor market (see also Kohlbrecher and Merkl, 2016). The response of unemployment and the job-finding rate to a large negative productivity shock is more than twice as large as to a positive shock of equal size. The reason is that for large aggregate shocks the curvature of the idiosyncratic shock distribution near the cutoff point matters a lot. Idiosyncratic productivity shocks are thus a potential explanation why business cycle fluctuations generate labor market asymmetries (see McKay and Reis (2008) and Abbritti and Fahr (2013) for empirical evidence).

Our argument that a Cobb-Douglas constant returns matching function may be a reasonable assumption for small shocks, but inappropriate for large shocks, has not been made in the labor market context so far. However, there are other streams of the literature which provide a similar insight. This is for example the case for production functions (Christensen et al., 1973). Similarly, different models of price rigidities may appear very similar in terms of the first-order Taylor approximation, but yield very different outcomes on a microeconomic level (Klenow and Kryvtsov, 2008). In addition, when performing nonlinear exercises such as a disinflation, these observationally equivalent models generate very different results both compared to their linear benchmark (e.g. Ascari and Merkl, 2009) and in comparison to one another (e.g. Ascari and Rossi, 2012).

The rest of the paper proceeds as follows. Section 2 derives a simple model with contact function, free entry of vacancies, and idiosyncratic productivity shocks for new contacts. Section 3 provides an analytical expression for the equilibrium comovement of the job-finding rate and the market tightness in this framework. Section 4 first calibrates the model with a degenerate contact function. Second, it combines a non-degenerate contact function and idiosyncratic productivity shocks to match the estimated matching function. Section 5 shows why it is important to account for idiosyncratic productivity in labor market models. Although a combined model with contact function and idiosyncratic productivity shocks is observationally equivalent to a standard matching function up to a first-order Taylor approximation, we provide two examples where results are very different. Section 6 concludes.

2. A Simple Model

2.1. Model Environment

Our economy is populated by a continuum of workers who can either be employed or unemployed. Employed workers are separated with an exogenous probability σ.[7] Unemployed workers search for a job. We assume that they get in contact with a firm with probability $f_t \leq 1$.[8]

When unemployed workers get in contact with a firm, they draw an idiosyncratic productivity realization ε_{it}, i.e. some workers are more productive than others. This nests the case of search and matching models in which endogenous separations hit before

[7] In their cross-country comparison, Elsby et al. (2013) show that separations are a more important driver of unemployment in Germany than in the United States. However, the assumption of exogenous separations is chosen for illustration purposes and it is without loss of generality. In Appendix B.2 we show that our key results also hold with endogenous separations.

[8] The contact probability may either be driven by a standard contact function as in Mortensen and Pissarides (1994) and Pissarides (2000) or it may be degenerate in the sense that workers have a constant contact probability, as standard in search models (e.g. McCall, 1970; Mortensen, 1987) or as assumed in selection models (Brown et al., 2015; Lechthaler et al., 2010).

production takes place (e.g. Krause and Lubik, 2007) or the stochastic job-matching model (Pissarides, 2000, chapter 6). Firms will only hire workers when the productivity realization ε_{it} is at least as large as the cutoff productivity $\tilde{\varepsilon}_t$ that makes a firm indifferent between hiring and not hiring.[9] We use a model in which idiosyncratic productivity shocks are only drawn in the first period of employment. However, as shown in the Appendix, this assumption is without loss of generality.

Firms have to post vacancies to obtain a share of the economy wide applicants (namely, the firm's vacancy divided by the overall number of vacancies, which is determined by a free-entry condition).

2.2. Contacts

Contacts are assumed to follow a Cobb-Douglas function with constant returns to scale (CRS)

$$C_t = \vartheta v_t^{\xi} u_t^{1-\xi}, \tag{1}$$

where C_t denotes the overall number of contacts, ϑ is the contact efficiency, and u_t and v_t are beginning of period unemployment and vacancies respectively. The contact probability for a worker is thus $f_t = \vartheta \theta_t^{\xi}$ and the contact probability for a firm $q_t = \vartheta \theta_t^{\xi-1} = f_t/\theta_t$, where $\theta_t = v_t/u_t$ denotes market tightness. In the special case with $\xi = 0$ the contact function is degenerate in the sense that more vacancies do not lead to more contacts in the aggregate.

2.3. The Selection Decision

Once a contact between a searching worker and a firm has been established, firms decide whether to hire/select a particular worker or not. There is a random worker-firm pair specific idiosyncratic productivity shock ε_{it} which is *iid* across workers and time, with density function $f_{\varepsilon}(\varepsilon)$ and the cumulative distribution $F_{\varepsilon}(\varepsilon)$. Due to the *iid* assumption, we abstract from the worker-firm pair specific index i from here onward. The idiosyncratic productivity realization ε_t is observed by the worker and the firm. Thus, the expected discounted profit of hiring an unemployed worker, $\pi_t^E(\varepsilon_t)$, is equal to the current aggregate productivity a_t, plus the idiosyncratic productivity shock ε_t, minus the current wage $w_t^E(\varepsilon_t)$, plus the expected discounted future profits for incumbent worker-firm pairs π_{t+1}^I:

[9] Hochmuth et al. (2016) show with a new survey dataset for Germany that the selection rate is very procyclical over the business cycle. This is in line with our proposed model mechanism.

$$\pi_t^E(\varepsilon_t) = a_t + \varepsilon_t - w_t^E(\varepsilon_t) + \beta(1-\sigma)E_t\left[\pi_{t+1}^I\right], \tag{2}$$

with

$$\pi_t^I = a_t - w_t^I + \beta(1-\sigma)E_t\left[\pi_{t+1}^I\right], \tag{3}$$

where β is the discount factor and σ is the exogenous separation probability. In the baseline scenario, incumbent worker-firm pairs are not subject to idiosyncratic productivity shocks. Thus, the profit for incumbent workers π_t^I does not depend on ε_t.

The firm selects an unemployed worker whenever there is an expected positive surplus:

$$\tilde{\varepsilon}_t = w_t^E(\tilde{\varepsilon}_t) - a_t - \beta(1-\sigma)E_t\left[\pi_{t+1}^I\right]. \tag{4}$$

Thus, the selection rate is given by:

$$\eta_t = \int_{\tilde{\varepsilon}_t}^{\infty} f_\varepsilon(\varepsilon)\,d\varepsilon. \tag{5}$$

2.4. Vacancies

As in Pissarides (2000, chapter 1), we assume that each vacancy corresponds to one firm. For entering the market, firms have to pay a fixed vacancy posting cost κ. The value of a vacancy, Ψ_t, is

$$\Psi_t = -\kappa + q_t\eta_t E_t\left[\pi_t^E|\varepsilon_t \geq \tilde{\varepsilon}_t\right] + (1-q_t\eta_t)E_t[\Psi_{t+1}], \tag{6}$$

where $q_t = C_t/v_t$ is the probability that a vacancy leads to a contact (i.e. overall contacts divided by overall vacancies). Thus:

$$\begin{aligned}\Psi_t = -\kappa + q_t\eta_t&\left(a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty}\left(\varepsilon - w_t^E(\varepsilon)\right)f_\varepsilon(\varepsilon)\,d\varepsilon}{\eta_t} + \beta(1-\sigma)E_t\left[\pi_{t+1}^I\right]\right)\\ &+ (1-q_t\eta_t)E_t[\Psi_{t+1}].\end{aligned} \tag{7}$$

Firms will post vacancies up to the point where the value is driven to zero (free-entry condition). Thus, in equilibrium:

$$\frac{\kappa}{q_t\eta_t} = E_t\left[\pi_t^E|\varepsilon_t \geq \tilde{\varepsilon}_t\right], \tag{8}$$

or

$$\frac{\kappa}{q_t\eta_t} = a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty}\left(\varepsilon - w_t^E(\varepsilon)\right) f_\varepsilon(\varepsilon)\, d\varepsilon}{\eta_t} + \beta(1-\sigma) E_t\left[\pi_{t+1}^I\right]. \tag{9}$$

It is straightforward to see that the model nests the standard matching model where all workers are selected (i.e. with no role for idiosyncratic shocks) by setting $\eta_t = 1$ and $\varepsilon_t = 0$ and dropping superscripts E and I. In this case, the right hand side is $a_t - w_t + \beta(1-\sigma) E_t[\pi_{t+1}] = a_t - w_t + \beta(1-\sigma) E_t\left[\frac{\kappa}{q_{t+1}}\right]$.

Note that even in the special case of a degenerate contact function, it is perfectly rational for firms to enter the market. Under a positive aggregate productivity shock, the expected returns of hiring a worker increase. Thus, more firms will enter the market to compete for these profits until the free-entry condition holds again. This makes vacancies procyclical in response to aggregate productivity shocks.[10]

2.5. Wages

We assume standard Nash bargaining for both new and existing matches over the joint surplus of a match. Workers have linear utility over consumption. Let V_t^U, V_t^E, and V_t^I denote the value of unemployment, the value of a job for an entrant worker, and the value of a job for an incumbent worker.

$$V_t^U = b + \beta E_t\left[f_{t+1}\eta_{t+1}V_{t+1}^E(\varepsilon_{t+1}|\varepsilon_{t+1} \geq \tilde{\varepsilon}_{t+1}) + (1 - f_{t+1}\eta_{t+1})V_{t+1}^U\right], \tag{10}$$

$$V_t^E(\varepsilon_t) = w_t^E(\varepsilon_t) + \beta E_t\left[(1-\sigma)V_{t+1}^I + \sigma V_{t+1}^U\right], \tag{11}$$

$$V_t^I = w_t^I + \beta E_t\left[(1-\sigma)V_{t+1}^I + \sigma V_{t+1}^U\right]. \tag{12}$$

The wage for an entrant and the wage for an incumbent worker are thus determined by the following maximization problems, with γ denoting workers bargaining power:

$$w_t^E(\varepsilon_t) \in \arg\max \left(V_t^E(\varepsilon_t) - V_t^U\right)^\gamma \left(\pi_t^E(\varepsilon_t)\right)^{1-\gamma}, \tag{13}$$

$$w_t^I \in \arg\max \left(V_t^I - V_t^U\right)^\gamma \left(\pi_t^I\right)^{1-\gamma}. \tag{14}$$

[10] In principal, the model with degenerate contact function may be calibrated such that there are on average more contacts than vacancies in steady state ($q = C/v$). However, the probability of a firm to fill a vacancy is $q\eta$. Note that we exclude the scenario from our analysis where zero vacancies are posted. This would only be the case if expected profits are zero (i.e. where all firms are indifferent between producing and not producing).

The solutions to these problems are

$$w_t^I = \gamma \left(a_t + \beta E_t \left[f_{t+1}\eta_{t+1}\pi_{t+1}^I\right]\right) + (1-\gamma)\left(b + \gamma\beta E_t \left[(\varepsilon_{t+1}|\varepsilon_{t+1} \geq \tilde{\varepsilon}_{t+1}) f_{t+1}\eta_{t+1}\right]\right) \quad (15)$$

and

$$w_t^E = w_t^I + \gamma\varepsilon_t. \quad (16)$$

Two comments are in order: First, note that without selection, equation (15) boils down to the well known wage equation $w_t = \gamma(a_t + \kappa\theta_t) + (1-\gamma)b$. Second, in the Nash-bargained wage equation (16) there is a linear relation between idiosyncratic productivity and the entrant wage. We will use this theoretical connection to identify idiosyncratic productivity shocks via the wage distribution (for a homogeneous reference group).

Finally, it is worth emphasizing that our theoretical results do not hinge on Nash bargaining for wages. We show in the Appendix that our main results hold for nearly any wage formation that takes the general form of equation (16).

2.6. Employment

We assume an economy with a fixed labor force L, which is normalized to 1. The employment stock is thus equal to the employment rate n. The employment dynamics in this economy is determined by

$$n_{t+1} = (1 - \sigma - f_t\eta_t)\, n_t + f_t\eta_t. \quad (17)$$

The number of searching workers is equal to the number of unemployed workers at the beginning of period t, i.e.

$$u_t = 1 - n_t. \quad (18)$$

Given that all workers have the same job-finding rate in our model, there is no duration dependence of unemployment. The interaction of labor selection with duration dependence mechanisms such as human capital depreciation is left for future research.

2.7. Labor Market Equilibrium

The labor market equilibrium is a solution to the system consisting of the following equations: the contact function (1), the equation for firms' profit (3), the productivity cutoff point (4), the selection rate (5), the vacancy free-entry condition (9), the wage equations (15) and (16), the employment dynamics equation (17), and the definition of unemployment (18) as well as a law of motion for aggregate productivity, which is assumed to follow an AR(1) process.

3. Analytics

This section shows analytically how a vacancy free-entry condition and idiosyncratic shocks alone generate an equilibrium comovement of matches, unemployment, and vacancies that is observationally equivalent to a traditional Cobb-Douglas contact function. In order to isolate the effect of the selection mechanism, we start with a degenerate contact function. We prove that in this case the elasticity of the job-finding rate with respect to market tightness, which is equivalent to the weight on vacancies in an estimated matching function, is described by the first derivative of the expected idiosyncratic productivity shock. We illustrate the implications for different distributions and cutoff points. In a next step, we show how our results differ once we allow for a traditional non-degenerate contact function. Finally, we discuss the robustness of our results. To obtain analytical results, all derivations in this section are based on a steady state version of our model, i.e. we assume that there is no aggregate uncertainty and we analyze the reaction of the job-finding rate and vacancies with respect to permanent changes in aggregate productivity. These assumptions will be relaxed in Section 4.

3.1. Degenerate Contact Function

We start with a degenerate contact function ($\xi = 0$). This allows us to isolate the effect of the selection mechanism before putting both mechanisms, contact and selection, together.

The equilibrium comovement between the job-finding rate and market tightness can be described by the equations for the hiring cutoff point $\tilde{\varepsilon}$, the job-finding rate $f\eta$, the market tightness, defined as $\theta = v/u$, and the wage w^I. We have made use of the fact that $w^E(\varepsilon) = w^I + \gamma\varepsilon$ and substituted accordingly:

$$\tilde{\varepsilon} = \frac{w^I - a}{\left(1 - \beta\left(1 - \sigma\right)\right)\left(1 - \gamma\right)}, \tag{19}$$

$$f\eta = f \int_{\tilde{\varepsilon}}^{\infty} f_{\varepsilon}\left(\varepsilon\right) d\varepsilon, \tag{20}$$

and using equation (19):

$$\theta = (1-\gamma)\frac{f\eta}{\kappa}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)\,d\varepsilon}{\eta} - \tilde{\varepsilon}\right). \tag{21}$$

In standard empirical matching function estimations, the job-finding rate (jfr) is regressed on market tightness, where β_1 shows how strongly the job-finding rate and market tightness comove in percentage terms and ψ_t is an error term, namely:

$$\ln jfr_t = \ln f_t\eta_t = \beta_o + \beta_1 \ln\theta_t + \psi_t. \tag{22}$$

In a standard search and matching model with degenerate selection and a Cobb-Douglas CRS specification, β_1 is likewise the elasticity of matches with respect to vacancies. In the context of a model with degenerate contact function, vacancy free-entry condition, and idiosyncratic productivity, equation (22) does not correspond to the data driving process. However, the point of our methodological exercise is to analyze what an applied econometrician would find if she looks at the data through the lens of a reduced-form matching function estimation (as very often done in the empirical literature). We are able to provide analytical results for a constrained constant returns matching function estimation. In addition, we will show numerically in Section 4.3 that the results carry through when we estimate an unconstrained matching function.

The job-finding rate and market tightness are both dependent on aggregate productivity. By deriving the elasticity of the job-finding rate with respect to productivity and by deriving the elasticity of market tightness with respect to productivity,[11] we obtain an analytical expression for the empirical elasticity of the job-finding rate with respect to market tightness,[12] namely:

$$\frac{\partial\ln(f\eta)}{\partial\ln a} = \frac{-f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\eta}, \tag{23}$$

$$\frac{\partial\ln\theta}{\partial\ln a} = \frac{-\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta} - \tilde{\varepsilon}}. \tag{24}$$

Thus:

$$\frac{\partial\ln(f\eta)}{\partial\ln\theta} = \frac{f_{\varepsilon}(\tilde{\varepsilon})}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)\,d\varepsilon}{\eta} - \tilde{\varepsilon}\right). \tag{25}$$

[11]See Technical Appendix for details.

[12]Note that Merkl and van Rens (2012) show that the job-finding rate and its dynamics are isomorphic in a model with idiosyncratic training costs (under a Pareto distribution) and in the search and matching model. However, their model does not contain vacancies and is thus silent on the shape of the matching function.

Interestingly, neither the wage nor the first derivative of the cutoff with respect to productivity, $\frac{\partial \tilde{\varepsilon}}{\partial a}$, appears in equation (25). We explicitly take into account the first derivative of the wage with respect to productivity in our derivations in the Appendix (i.e. we consider that different wage formation regimes or parameters lead to different wage reactions). However, as long as the wage and aggregate and/or idiosyncratic productivity do not move one to one,[13] the terms drop out in the analytical derivations. Thus, our results hold for a very broad set of wage formation mechanisms.

It is important to emphasize that the relationship between the job-finding rate and market tightness in equation (25) is not causal. With a degenerate contact function, more vacancies do not lead to more contacts in aggregate. However, the model with idiosyncratic shocks generates a positive comovement between the job-finding rate and market tightness in equilibrium. Thus, equation (25) shows that it is not necessary to assume a standard contact function to obtain a positive comovement of these two variables.

To put it differently: If a model with degenerate contact function, idiosyncratic shocks, and free entry of vacancies is simulated and a matching function estimation is performed based on the simulated data, the estimation will generate a positive coefficient on vacancies, although the underlying contact function has a weight on vacancies of 0. We verify this numerically in Section 4.3.

What is the underlying economic mechanism and intuition? When aggregate productivity rises, firms have an incentive to hire workers with lower idiosyncratic productivity. Thus, the job-finding rate is procyclical. When productivity rises, this also increases the returns from posting a vacancy. Thus, firms compete for the larger pie of profits, more of them enter the market and thus increase the market tightness in the economy. These two mechanisms combined lead to the positive equilibrium comovement between the job-finding rate and the market tightness that we observe in equation (25).

Interestingly, the elasticity of the job-finding rate with respect to market tightness derived in equation (25) has an economic interpretation. It corresponds to the first derivative of the conditional expectation of idiosyncratic productivity with respect to the cutoff point, i.e.:

$$\frac{\partial \ln (f\eta)}{\partial \ln \theta} = \frac{f_\varepsilon(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_\varepsilon(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon} \right) = \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_\varepsilon(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}. \tag{26}$$

Thus, up to a first-order Taylor approximation, the comovement between the job-

[13] If the wage comoves one to one with productivity (in absolute terms, e.g. $w_t^E = a_t + \varepsilon_t$), the job-finding rate and vacancies would have a zero elasticity with respect to productivity. This trivial case is excluded from our analysis.

finding rate and the market tightness is determined by equation (26), and thus only depends on the distribution of idiosyncratic productivity and the position of the cutoff point. The quality of this approximation will be checked numerically in Section 4.3.

Figure 1 illustrates the prediction of our model for different idiosyncratic shock distributions. The upper panel plots the density functions of ε for normal, logistic, and lognormal distributions. The lower panel plots equation (26) evaluated at the corresponding cutoff point (on the abscissa). As shown above, this value corresponds to the elasticity of the job-finding rate with respect to market tightness, i.e. the implied weight on vacancies in an estimated matching function. Two observations are worth pointing out. First, for these standard distributions the weight on vacancies is always between 0 and 1. Second, for an elasticity of the job-finding rate with respect to market tightness smaller than 0.5, as commonly found in the empirical literature, the cutoff point needs to be to the left of the peak of the density function. This will be important later on when we compute the idiosyncratic shock distribution based on the empirical wage distribution.

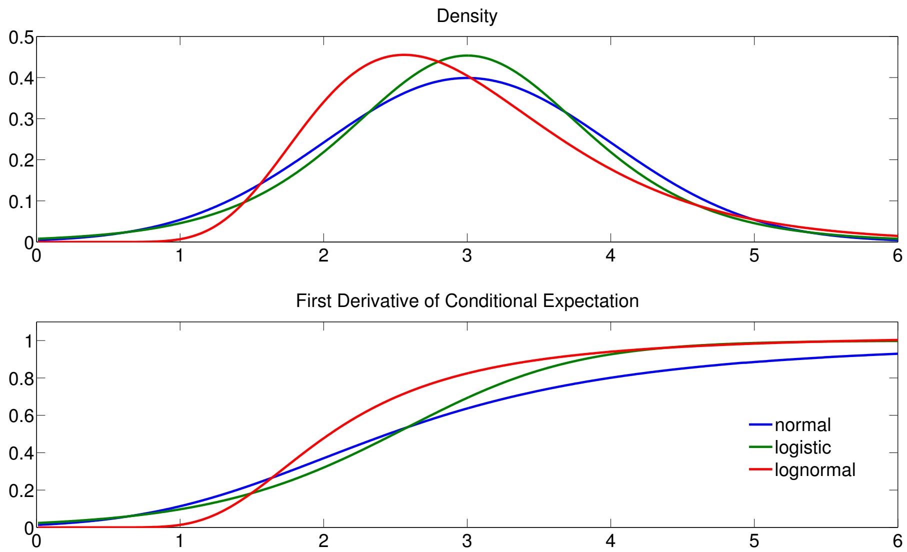

Figure 1: Predicted matching coefficients for standard distributions. Density function (upper panel) and first derivative of conditional expectation (lower panel) for different standard distributions (normal, logistic, and lognormal). For comparability reasons, the variance is normalized to 1 and the mean is set to 3.

Why is the elasticity of matches with respect to vacancies larger on the right hand side of the peak of the density function and smaller on the left hand side? The reason is that market tightness is driven by the free-entry condition of vacancies (see equation (21)).

When aggregate productivity increases, workers with lower idiosyncratic productivity are hired, i.e. the hiring cutoff moves to the left. On the left hand side of the peak of the density function, a small mass of additional workers with low idiosyncratic productivity will be hired. Thus, vacancies move by a lot because the additional hiring activity does not lower the average idiosyncratic productivity by much. Large vacancy movements relative to the job-finding rate lead to a small estimated coefficient in equation (22).

3.2. Traditional Contact Function

Now, let us assume a traditional contact function with $0 < \xi < 1$. In this case, the probability for a worker to make a contact ($f = C/u$) depends on aggregate productivity. In our real business cycle framework, we thus expect a procyclical movement of the contact rate ($\frac{\partial f}{\partial a} > 0$).

To analyze the implications of this modification, we recalculate the elasticities of the job-finding rate and market tightness with respect to productivity:

$$\frac{\partial \ln (f\eta)}{\partial \ln a} = -\frac{f_\varepsilon(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta} + \frac{\partial \ln f}{\partial \ln a}, \tag{27}$$

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_\varepsilon(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}} + \frac{\partial \ln f}{\partial \ln a}. \tag{28}$$

The elasticities of the job-finding rate and market tightness with respect to productivity are the elasticities with a fixed contact rate plus the elasticity of the contact rate with respect to productivity. Defining $\xi_{jfr/\theta} = \frac{\partial \ln jfr}{\partial \ln \theta}$, $\xi_{\eta/a} = -\frac{f_\varepsilon(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta}$, $\xi_{\theta/a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_\varepsilon(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}}$, and $\xi_{f/a} = \frac{\partial \ln f}{\partial \ln a}$, we can write the elasticity of the job-finding rate with respect to market tightness as:

$$\xi_{jfr/\theta} = \frac{\xi_{\eta/a} + \xi_{f/a}}{\xi_{\theta/a} + \xi_{f/a}}. \tag{29}$$

Taking first derivatives allows us to see how this elasticity changes with a procyclical contact rate:

$$\frac{\partial \xi_{jfr/\theta}}{\partial \xi_{f/a}} = \frac{\xi_{\theta/a} - \xi_{\eta/a}}{\left(\xi_{\theta/a} + \xi_{f/a}\right)^2}. \tag{30}$$

In the previous section, we have shown that for a variety of standard assumptions and cutoff points, the elasticity of the selection rate with respect to market tightness is smaller than 1 ($\frac{\xi_{\eta/a}}{\xi_{\theta/a}} < 1$). Thus, the numerator of (30) is positive, and $\frac{\partial \xi_{jfr/\theta}}{\partial \xi_{f/a}} > 0$, i.e. a stronger procyclicality of the contact rate increases the weight of vacancies in an estimated matching function. In different words: If both a traditional contact function and idiosyncratic shocks are important for match formation, both of them contribute to a positive weight on vacancies in an estimated matching function.

3.3. Robustness Checks

Our results hold for a broad set of models that contain idiosyncratic productivity shocks. The Technical Appendix shows that we obtain the same analytical results in a search and matching model with endogenous separations (where *iid* shocks hit every period) and in a model where idiosyncratic shocks are drawn for the entire span of employment. In all these cases, the elasticity of the job-finding rate with respect to market tightness is given by equation (26).[14]

4. Theory and Evidence

In the previous section we have shown that the selection mechanism has the potential to drive a substantial part of the observed comovement between the job-finding rate and market tightness. In this section we explore if this mechanism is quantitatively meaningful in a dynamic environment and how it interacts with a traditional contact margin.

We first establish a reference point by estimating an empirical matching function. Based on our analytical results, we use residual wages for new employment spells as a proxy for the idiosyncratic productivity distribution.[15] This determines the contribution of idiosyncratic productivity to the matching elasticity in a dynamic simulation of the model. Finally, we add a traditional contact function using the empirical matching elasticity as a target to pin down the contact elasticity.

For all these exercises, we use administrative labor market data for Germany (see e.g. Dustmann et al., 2009; Schmieder et al., 2012). The German administrative database has several advantages over commonly used U.S. data. First, it provides actual labor market transitions on a daily basis. This means that we do not have to construct labor

[14] For multiplicative idiosyncratic shocks, the results only change insofar as the equilibrium comovement is determined by the derivative of the logarithm of the conditional expectation of the shock with respect to the logarithm of the cutoff point.

[15] In a previous version of the paper (Kohlbrecher et al., 2014), we used the wage for a homogenous reference group (medium-skilled, middle aged men with certain characteristics) as an alternative to the Mincer residuals. The key quantitative results are very similar.

market flows from unemployment, employment, and duration data and we do not face a time aggregation bias (see, e.g., Shimer, 2005, 2012; Nordmeier, 2014). Second, we can use several control variables that might influence the search and matching process. Third, we can observe wages for new matches, which we use for our Mincer regression to obtain residual wages. These wages are from the same database that we construct our flow data from. A coherent definition is important because we use the wage distribution to approximate idiosyncratic shocks and hence to analyze their role for the matching function. Finally, we observe the number of posted vacancies. Therefore, we do not need to rely on a job-advertising index to construct a time series for vacancies.[16]

4.1. Empirical Matching Function

We estimate a standard Cobb-Douglas CRS matching function for the German labor market. Thus, we regress the job-finding rate, jfr, on labor market tightness θ, a linear time trend t, and a shift dummy d_{2005} that accounts for the redefinition of unemployment in the course of the so-called Hartz reforms:[17]

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \beta_2 t + \beta_3 d_{2005} + \psi_t, \tag{31}$$

where the job-finding rate at time t denotes all matches during month t over the beginning-of-month-t unemployment stock and market tightness refers to the beginning-of-month-t vacancy to unemployment ratio. The coefficient β_1 represents the matching elasticity with respect to vacancies and thus is the relevant reference point for our numerical exercises below. We further include observable control variables to account for the effects of a changing unemployment pool and different search intensities on the aggregate matching probability:[18]

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \beta_2 t + \beta_3 d_{2005} + \sum_{i=0}^{N} \gamma_i control_{it} + \psi_t. \tag{32}$$

Table 1 displays the estimation results of the matching function specification with and without control variables. Both estimations show a fairly good fit in terms of the adjusted R^2 measure. However, the Durbin-Watson statistic indicates that it is impor-

[16] See Appendix C for a detailed data description.

[17] In 2005, the official unemployment measure in Germany was extended to include recipients of former social assistance.

[18] It is well known that there is duration dependence of individual job-finding rates. Recent research by Hornstein (2012) and Barnichon and Figura (2015) suggests that this may be due to composition effects of the unemployment pool. Katz and Meyer (1990) find evidence for an influence of unemployment benefit receipt on workers' job acceptance behavior.

tant to control for the composition of the unemployment pool because this specification overcomes the positive autocorrelation in the error term. We also performed an IV estimation using lagged unemployment and vacancies to account for a potential endogeneity problem in specification (1), but the coefficients did not change notably.[19] The point estimate of β_1 in our preferred specification is 0.35 and the 95% confidence interval spans from 0.23 to 0.46. The matching elasticities of vacancies and unemployment are thus roughly one third and two thirds, respectively. These results are in line with the survey of matching function estimations by Petrongolo and Pissarides (2001). Moreover, the constant returns to scale assumption ($\beta_V = 1 - \beta_U$) cannot be rejected when we replace labor market tightness by the vacancy and unemployment stocks. We also find a significantly negative time trend in matching efficiency, which is a common result in matching function estimations (see e.g. Petrongolo and Pissarides, 2001; Poeschel, 2012).

log jfr	(1)	(2)
constant	-2.3498***	-4.3403**
log θ	0.2458***	0.3463***
t	-0.0003***	-0.0054**
d_{2005}	-0.0798***	-0.0766
controls	no	yes
adj. R^2	0.5134	0.6162
DW statistic	1.3664	1.8407
CRS t-statistic	1.0074	1.6141

Table 1: Matching function estimations. OLS estimation with Newey-West standard errors for monthly data from 1993-2007. ***, **, and * indicate significance at the 1%, 5%, and 10% levels. Control variables: *long, young, old, low-skilled, high-skilled, foreign, female, married, child, UB I.*

4.2. The Wage Distribution

How important is the selection mechanism in driving the elasticity of the job-finding rate in matching function estimations? To test for this, we use wages from German administrative data for new matches to infer the shape of the actual distribution of idiosyncratic productivity.[20]

We estimate a Mincer type regression which controls for observable characteristics of individual workers:

$$\ln w_{it}^E = \beta_1 X_{it} + \beta_2 \ln w_i^I + \mu_{it}, \tag{33}$$

[19]The results of the IV estimation are available on request.
[20]See Appendix C for a description of the wage data.

where w_{it}^E is the daily wage at the beginning of a new employment spell of person i in period t. X_{it} is a set of observables, such as experience and age (both in levels and squared). X_{it} also includes occupation fixed effects, dummies for education groups and for different size categories of the establishment and time dummies. A detailed variable list can be found in the Appendix. In addition, we control for the last wage of person i in the previous employment spell w_i^I.[21] The reason is that the last wage may contain person specific information that is not controlled for by the observable characteristics. It is meant to control for unobservable systematic heterogeneity.

The object of interest in our estimated regression is the residual wage μ_{it}. We use the residual wage for two purposes. First and most importantly, μ_{it} for all years is used as an input for our model calibration. Second, we employ the distribution of μ_{it} to calculate the residual wage dispersion over time.

We have assumed that wages are formed according to $w_t^E(\varepsilon_t) = w_t^I + \gamma\varepsilon_t$. We will use the proportionality between contemporaneous idiosyncratic productivity and the wage for our empirical analysis. Our proportionality assumption allows us to infer the shape of the distribution of the idiosyncratic productivity directly from the wage data. The histogram of residual wages is displayed in Figure 2.[22] When equation (16) holds, idiosyncratic productivity is just a scaled version of this distribution.

We assume that the cutoff point for hiring is located at the 10th percentile of the residual wages. It is quite standard in the literature to use the 10th percentile as a measure for the lowest wage to prevent that results are driven by outliers at lower percentiles. The 50th to 10th percentile ratio is, for instance, a conventionally used measure for wage dispersion (see e.g. Hornstein et al., 2011). The 10th percentile is therefore used for our benchmark calibration. For robustness, we also calibrate the model for the 5th percentile of the residual wages.

For our numerical analysis we fit the distribution in two different ways. First, we apply a kernel density estimation with a normal kernel. We can use this non-parametric fit to numerically calculate the derivative of the conditional expectation using equation (26). This gives us a first estimate of the matching elasticity in a pure selection model in steady state. Results for this non-parametric fit are shown in the Appendix. As we are ultimately interested in combing the selection and contact margins in a dynamic model, we require a functional form for the distribution. We therefore fit several standard distributions to the data by maximum likelihood. The logistic distribution gives us the

[21] Note that we do not assign a time subscript to the last employment wage because the timing of the last spell depends on the individual employment history.

[22] We use the exponential of the residual wage for the calibration because we estimated a log-log regression and the idiosyncratic shock enters as level in the model.

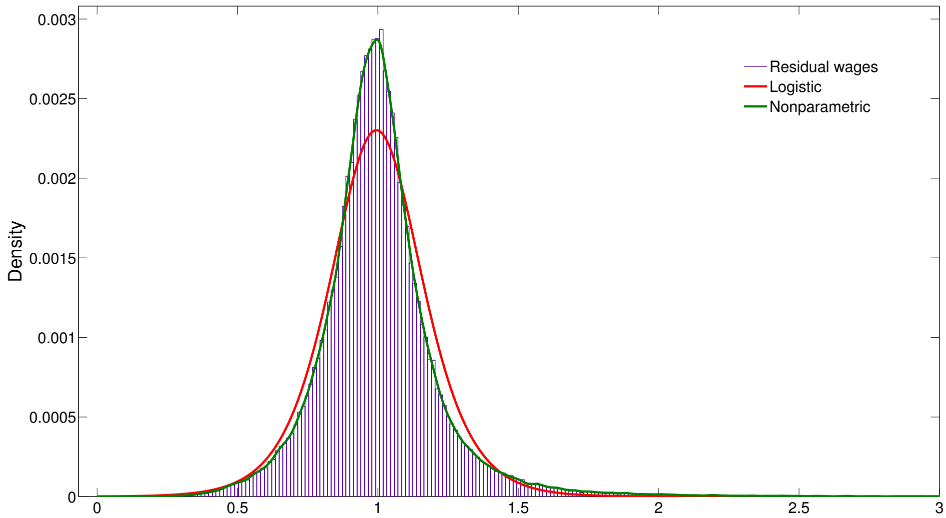

Figure 2: Distribution of the exponential of residual wages

best fit and therefore serves as our proxy for the idiosyncratic distribution in all following exercises. Both fits of the wage distribution are shown in Figure 2. The logistic fit is of course imperfect but reasonably good in the vicinity of the cutoff. In our calibration exercise, we assume that the distribution follows the shape of the logistic distribution around the cutoff. The shape of this distribution is particularly relevant when we analyze large business cycle shocks in a nonlinear framework because the cutoff point will move at the left corner of the distribution. Fortunately, the fit of the logistic distribution is particularly good at the left hand side of the distribution, which is the relevant part for the nonlinear exercises.

4.3. Numerical Results

We calibrate the model to the German labor market using the labor flow data and the logistic fit of the residual entry wage distribution. Details on the parametrization are given in Appendix E. We simulate the model 1000 times with aggregate productivity governed by a first-order autocorrelation process. The simulation is based on a second-order Taylor approximation.[23] Each time we use 180 periods corresponding to the time span used for our empirical matching function estimation and estimate a Cobb-Douglas CRS matching function:

[23]This explicitly allows for some nonlinearities not covered by our analytical steady state results.

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \psi_t. \tag{34}$$

Note that this specification does not correspond to the data driving process of our model. However, we follow the standard practice of the empirical literature to illustrate what this procedure would yield in our context.

In a first step, we simulate the model with a degenerate contact function, e.g. $\xi = 0$. This allows us to isolate the selection effect and to check whether our comparative static results hold in a dynamic setting and whether the constant returns to scale assumption is valid. Table 2 reports the results. The estimated elasticity of the job-finding rate with respect to market tightness is 0.18 and 0.29 for the calibration to the 5th and 10th percentile of the residual wage distribution respectively. These results are already remarkably close to the coefficient of 0.35 that we get from the real data in Section 4.1. The numbers from the simulation exercise and the comparative static exercise using equation (26) are literally the same. This shows that our comparative static results carry over to a dynamic environment.

	5th percentile	10th percentile
Simulation result: CRS constrained		
constant	-2.57	-2.30
log θ	0.18	0.29
Simulation result: unconstrained		
constant	-2.62	-2.30
log U	0.80	0.72
log V	0.18	0.29
Steady State prediction: Equation (26)		
constant	-	-
log θ	0.18	0.29

Table 2: Matching function based on simulation with degenerate contact function. Steady state results are calculated numerically from the logistic fit of the distribution using equation (26). The dynamic simulation results are OLS estimates from the simulated series using a logistic distribution. Reported coefficients are means over 1000 simulations.

We further analyze whether we have artificially imposed the CRS assumption in our comparative static exercise. We estimate the following unconstrained matching function: $\log m_t = \beta_0 + \beta_1 \log v_t + \beta_2 \log u_t + \psi_t$, where m_t denotes all matches in period t. Table 2 shows that the sum of estimated coefficients $(\beta_1 + \beta_2)$ is close to 1. The estimated coefficients are also statistically significant at the 1% level in every single run of the simulation. Interestingly, when we estimate other functional forms such as CES, the Cobb-Douglas specification is confirmed.[24]

Finally, the dynamic simulation puts us in a position to combine idiosyncratic productivity with a traditional contact function. Hence, we assume that the contact probability is defined by $f_t(v_t, u_t) = \vartheta\theta_t^{\xi}$ with $0 < \xi < 1$, i.e. the job-finding rate is not only driven by the movement of the cutoff point for idiosyncratic productivity but also by a procyclical contact rate.

We analyze how much of the empirical comovement between matches, unemployment, and vacancies is due to the contact function and how much is due to idiosyncratic productivity. For this purpose, we use the same calibration as before (see Table 2) and determine the contact elasticity ξ so as to get an overall elasticity of matches with respect to vacancies of 0.35 as found in the empirical matching function. The results are shown in Table 3.

	5th percentile	10th percentile
Coefficient on vacancies with $\xi = 0$		
$\log \theta$	0.18	0.29
Calibrated elasticity of the contact function (ξ)		
$\log \theta$	0.22	0.09
Coefficient on vacancies: Combined model		
$\log \theta$	0.35	0.35

Table 3: Estimated coefficient on vacancies in dynamic simulations with different contact function specifications. The dynamic simulation results are OLS estimates from the simulated series using a logistic distribution. Reported coefficients are means over 1000 simulations.

Our numerical results are in line with our theoretical results from Section 3.2. When a procyclical contact rate and idiosyncratic productivity are combined, this leads to a larger weight on vacancies in an estimated matching function. The first line in Table 3 shows the results for the model simulation with idiosyncratic productivity but with a degenerate contact function. The second line shows the elasticities of a traditional contact function that would correspond to the overall elasticity of matches with respect to vacancies if there was no idiosyncratic productivity. The third line shows the combination of the two mechanisms. Interestingly, the resulting estimated matching function has a coefficient on vacancies which is close to the sum of the coefficients on vacancies in the

[24] As a robustness check we have also performed IV estimations using the lagged value of market tightness as an instrument. This does not alter our results. Results are available from the authors on request.

two cases.

Our calibration suggest that one half or more of the observed elasticity of matches with respect to vacancies is actually driven by idiosyncratic productivity. When we use the 10th percentile of the wage distribution, about three quarters of the coefficient on vacancies are driven by idiosyncratic shocks. Thus, our exercise shows that there is a potentially large bias in standard matching function estimations if idiosyncratic productivity plays a role. Hence, in many model applications, the contact functions may be misspecified, assigning too large a role for vacancies in the process of match formation.

Another way of analyzing the relative contribution of the contact vis-à-vis the selection margin is do a variance decomposition. In logs, the job-finding rate is the sum of the contact and the selection rate: $\log jfr_t = \log \eta_t + \log f_t$. Thus, the relative contribution of the selection rate and the contact rate to the variance of the job-finding rate are $\chi_\eta = cov\left(\log \eta_t, \log jfr_t\right)/var\left(\log jfr_t\right)$ and $\chi_f = cov\left(\log f_t, \log jfr_t\right)/var\left(\log jfr_t\right)$, respectively. The relative contribution of the selection rate is 75% (40%) for the calibration to the 10th (5th) percentile. It has to be kept in mind that both the contact rate and the selection rate strongly comove with market tightness. Thus, the order of magnitude for the relative importance of these two margins is similar as conveyed by Table 3.

Interestingly, when we compare the combined model in Table 3 to a standard search and matching model with degenerate selection but with a weight on vacancies in the contact function of 0.35, it turns out that the two models also produce the same Beveridge curve. Thus, observational equivalence does not only hold for the matching function but also for the Beveridge curve.

Given this observational equivalence, it is worthwhile emphasizing that the elasticity of the contact function with respect to vacancies is very important in search and matching models. Hosios (1990) shows that matching models with a CRS matching function are constrained efficient when firms' bargaining power in Nash bargaining is equal to the weight on vacancies in the contact function.[25] We have shown that using the weight on vacancies from empirical estimations is inappropriate in the presence of idiosyncratic productivity shocks and leads to a misspecification. Against the background of Hosios' (1990) rule, welfare implications of policy interventions may thus be judged incorrectly.

In what follows, we define the combined model (contact and selection) calibrated to the 10th percentile of the wage distribution as our baseline, which will be used for all

[25]For an interesting application for the interaction of the matching function elasticity, the bargaining power, and the optimal cyclicality of unemployment benefit durations in Europe and in the United States, see Moyen and Stähler (2014).

quantitative exercises.[26]

4.4. Wage Dispersion and Cyclicality: Data vs. Model

Two statistics are interesting against the background of our model, namely the cyclical behavior of the residual wage dispersion and the cyclicality of wages in the model and in the data. Our model predicts that an increase of the selection rate leads to a larger dispersion of wages because firms are willing to hire workers with a lower idiosyncratic productivity realization in a boom. In our baseline model, the selection rate and the conditional standard deviation of entry wages are strongly positively correlated. The selection rate is not contained in the administrative data. The most direct way of checking whether this pattern is present in the data is to look at the correlation between the aggregate job-finding rate and the residual wage dispersion.[27] Therefore, we calculate the correlation between the job-finding rate and different dispersion measures for residual wages (namely, the 50-10 percentile, the 90-10 percentile and the variance). For this purpose, we calculate the distribution of μ_{it} on an annual basis.[28]

While the correlation between the dispersion measures and the job-finding rate are only moderately positive for the entire sample, the correlation becomes larger when excluding the first year. There is an unexpected movement at the beginning of the 1990s, which may be due to the aftermath of German unification. In short: the comovement between the job-finding rate and the residual wage dispersion appears to be roughly in line with our model prediction (at least after 1994). Due to the length of our sample, we can only provide preliminary evidence for Germany. However, Morin (2013) shows for a 40 year time span for the United States that the residual wage dispersion is procyclical, i.e. positively correlated with GDP and negatively correlated with unemployment.

For the wage cyclicality, we take the results by Stüber (2016) as a reference point who also uses administrative data for Germany. He estimates a wage regression for about 30 years in a first step. In a second step, he regresses the coefficients of the time dummies from the first-stage regression on log GDP and the unemployment rate. According to his results, the estimated coefficient is 0.36 for log real GDP and -1.26 for the unemployment rate. This is a useful benchmark for our model.

[26] Note, however, that our quantitative results would be fairly similar for the calibration to the 5th percentile.

[27] Our model predicts a strong positive comovement between the contact and the selection rate in response to aggregate productivity shocks.

[28] We do not choose a higher frequency due to the nature of the administrative data. When a new job is created at the beginning of the year and when it lasts until the end of the year, the average daily wage is calculated based on the total annual wage and the number of working days (see Appendix for details).

Correlation with JFR	50-10 perc.	90-10 perc.	Variance
1993-2007:	0.05	0.06	0.13
1994-2007:	0.24	0.24	0.35

Table 4: Correlation of residual wage dispersion and the job-finding rate.

In our model simulation, the elasticity of the aggregate wage with respect to output is 0.47 and the elasticity with respect to unemployment ist -1.86. Thus, the cyclicality of aggregate wages in our model is well in line with the cyclicality of aggregate wages in the data.[29]

5. Why It Matters to Account for Idiosyncratic Productivity Shocks

The theoretical section has shown that idiosyncratic productivity shocks and a vacancy free-entry condition alone generate an equilibrium comovement between matches, vacancies, and unemployment. We have used wage data to show that this may generate a large part of the observed comovement between these variables in the German data. Given that there are multiple ways of obtaining the observed comovement between matches, vacancies, and unemployment, does it matter whether the labor market is modeled with a contact function only (i.e. degenerate selection) or with a combined model (with both a standard contact function and idiosyncratic productivity shocks)? In the previous section, we have briefly argued that this matters from a normative perspective (Hosios' (1990) rule). This section shows that it also matters from a positive perspective. First, we provide a novel explanation for the puzzling fact that many empirical matching function estimations document a decline of the matching efficiency over time. Second, a model with idiosyncratic shocks and vacancy free entry generates highly asymmetric labor market reactions to business cycles.

5.1. Vacancy Posting Costs and the Time Trend in the Matching Function

There is a lot of anecdotal evidence that new technologies such as databases or the dissemination of the internet have made vacancy posting cheaper (see for example Kuhn

[29]Note, however, that our baseline calibration would generate a countercyclical wage for the average entrant due to the very cyclical movements of the cutoff point. This is not a general result in our model and depends on the curvature of the idiosyncratic wage distribution at the cutoff point and the parametrization (such as the bargaining power). If we calibrate to other percentiles of the wage distribution, the entrant wage is procyclical in some cases.

and Mansour, 2014).[30] For example, the newspaper based job-advertising index has collapsed because firms have started using internet based advertising (e.g. Barnichon, 2010). Obviously, firms have started substituting because there was a cheaper technology available. However, it is difficult to quantify this reduction of vacancy posting costs.

In a standard search and matching model with degenerate selection, a decline in vacancy posting costs would lead to a drop in unemployment but leave the estimated matching function unaffected. To give an example: For our calibration, in a model with contact function only, a drop in vacancy posting costs of 20% would lead to a reduction in steady state unemployment of 1.2 percentage points. In our baseline version (with both mechanisms at work), the effect is reduced by a factor of 5.

The effects of a long run downward trend in vacancy posting costs are also very different in a model with labor selection. To illustrate our point, we simulate a hypothetical situation in the combined model where vacancy posting costs drop by 50% over 420 periods (35 years). More specifically, we simulate the nonlinear trajectory of the economy to a new steady state when vacancy posting costs decline linearly over time. In addition, the economy is subject to aggregate productivity shocks during the entire time span.[31] We take 180 periods (15 years) in the middle of this process and estimate a matching function based on the simulated data. This corresponds to the observation period in our empirical data and a 20% drop in vacancy posting costs during those 180 periods. We choose to estimate a subperiod of a longer time series because we consider the decline of vacancy posting costs as a long lasting process. In the estimations, we now also include a time trend, which turns out to be statistically significant and negative (see Table 5).

$\log jfr$	Coefficient
constant	-2.21***
$\log \theta$	0.36***
t	-0.0003***

Table 5: Matching function estimation with negative time trend (deterministic simulation).

The intuition for this negative time trend is best illustrated in a model with a degenerate contact function where the job-finding rate is solely driven by the optimal cutoff point of idiosyncratic productivity. When vacancy posting costs drop, the number of va-

[30]It is important to distinguish ex-ante hiring costs and ex-post hiring costs in search and matching models. Given that vacancy posting costs are divided by the probability of filling a vacancy, these are costs prior to hiring a worker.

[31]Productivity follows the same AR(1) process as before. As we are interested in the nonlinear solution, the shocks to productivity are deterministic (i.e. we pick one particular productivity path in the simulation).

cancies increases (see free-entry condition). However, with a degenerate contact function this has no effect on matches and unemployment. Thus, the job-finding rate (matches divided by unemployment) remains constant, while the market tightness increases (vacancies divided by unemployment). In other words, it looks as if the estimated matching function has become less efficient. The job-finding rate remains constant although the market tightness has increased. If vacancy posting costs drop over a long time horizon, this leads to a negative time trend in the matching function.[32] If both a traditional contact function and the selection mechanism are at work, this effect is sustained as can be seen in Table 5.

The negative time trend can also be found in our estimation in Table 1. Interestingly, a negative time trend is a common feature of matching function estimations. According to Poeschel (2012), the sudies surveyed in Petrongolo and Pissarides (2001) that include a time trend "clearly suggest that there is a highly significant negative time trend, implying that labour market performance appears to deteriorate over time." For Germany, Fahr and Sunde (2004) have documented a negative time trend.[33]

Our paper shows that the decline in ex-ante hiring costs due to new technologies rationalizes why many matching function estimations may generate a negative time trend. In the absence of idiosyncratic productivity shocks, this negative time trend would be a sign for a worrisome instability of the contact function and a deteriorating labor market performance. We provide an explanation how a stable contact function and an estimated negative time trend can be reconciled. As long as there are no reliable proxies for the development of vacancy posting costs over time, it is impossible to control for this omitted variable bias, which is captured by the time trend. However, our paper provides an explanation that makes the observed time trend less worrisome.

5.2. Business Cycle Asymmetries

The analytical results in Section 3 are based on a steady state analysis (i.e. a first-order approximation). The numerical results in Section 4.3 use a second-order approximation. Thus, in both cases higher order effects are not taken into account. Figure 3 displays

[32]The coefficient on market tightness slightly deviates from 0.35. This is due to the fact that we simulate the nonlinear transition from one steady state to another. The selection rate is slightly lower in the final steady state, the weight on vacancies thus higher. Intuitively, due to lower vacancy posting costs, firms can afford to be more selective.

[33]Note that some empirical studies suggest that there was a recent increase of matching efficiency in Germany (e.g. Fahr and Sunde, 2009; Klinger and Rothe, 2012; Hertweck and Sigrist, 2013). This is often attributed to the recent German labor market reforms. In contrast to the time trend, which is found in many different studies and for different observation periods, the labor market reforms seem to have implied a permanent upward shift of the matching efficiency.

the reactions of the combined model in response to a one percent (upper panel) and a three percent (lower panel) productivity shock (with autocorrelation coefficient 0.95), which are solved deterministically and fully nonlinearly. In order to facilitate comparison of the quantitative responses, the responses are all in absolute terms (i.e. normalized to positive numbers). The upper panel illustrates that the nonlinear responses to a regularly sized productivity shock show only minor asymmetries. Thus, our analytical results and the stochastic solution method appear to be appropriate approximations in a normal business cycle environment.

However, the lower panel of Figure 3 reveals large asymmetries of both the job-finding rate and unemployment to large business cycle shocks. The responses to a negative 3% productivity shock are more than twice as large as the responses to an equally sized positive productivity shock.

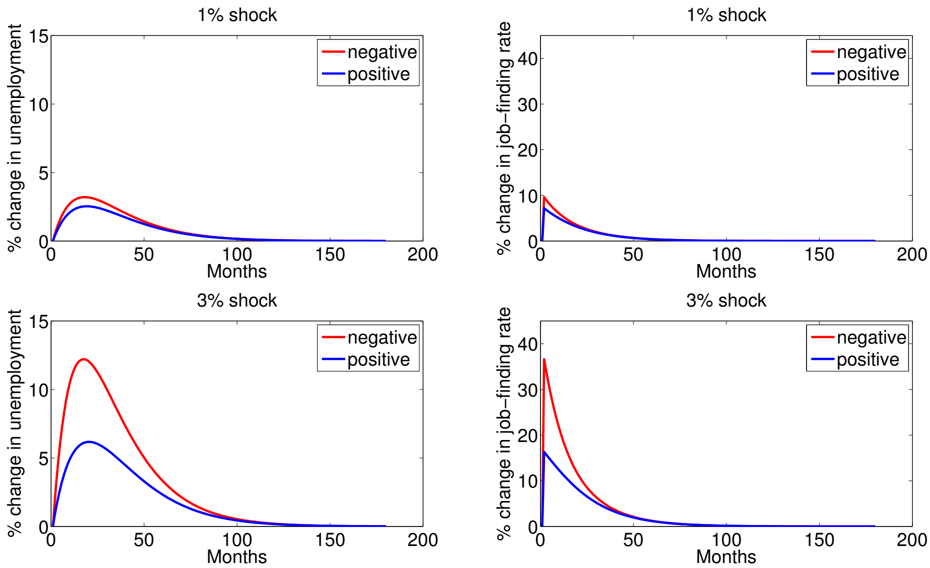

Figure 3: Response of unemployment and job-finding rate to productivity shock. Responses are in percent deviation from steady state. For expositional convenience, the response of unemployment to a positive shock and the response of the job-finding rate to a negative shock are flipped upside down.

To isolate the driving forces of this asymmetry, we use our baseline calibration and impose a degenerate contact and degenerate selection mechanism respectively. More precisely, we assume that the contact rate (middle panel in Figure 4) and the selection rate (right panel) are constant over the business cycle. All other parameter values of the model remain unchanged. Figure 4 shows that both the degenerate contact and the degenerate selection model generate some asymmetries in response to a 3% productivity shock. These asymmetries are particularly strong for the degenerate contact model. Or in different words: the selection mechanism is the key driver for this asymmetry.

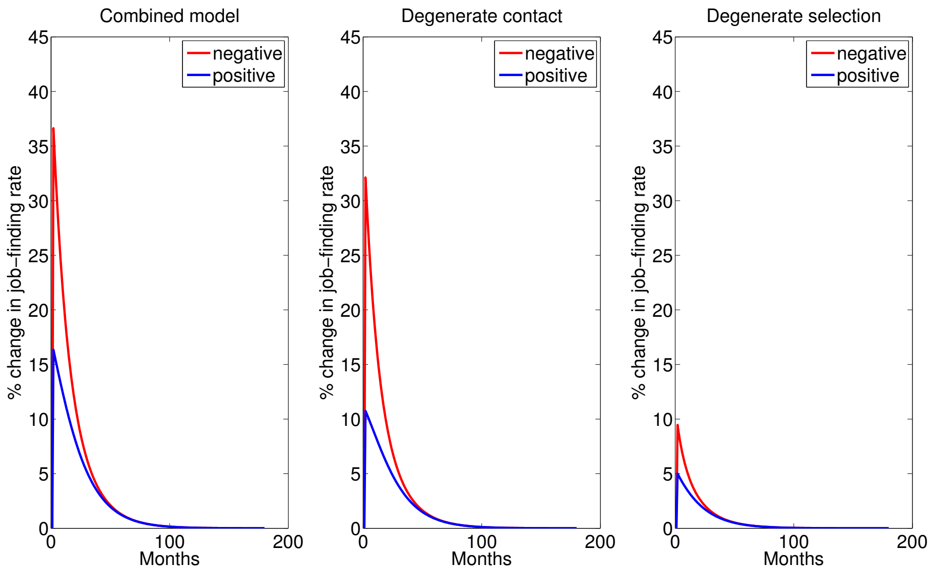

Figure 4: Response of the job-finding rate to a 3% productivity shock with degenerate contact/selection. Responses are in percent deviation from steady state. For expositional convenience, the response of the job-finding rate to a negative shock is flipped upside down.

The asymmetry is straightforward to see in a model with a degenerate contact function, where the dynamics of the job-finding rate is exclusively driven by the selection rate:

$$\eta_t = \int_{\tilde{\varepsilon}_t}^{\infty} f_{\varepsilon}\left(\varepsilon_t\right) d\varepsilon_t. \tag{35}$$

Although the cutoff point $\tilde{\varepsilon}_t$ moves symmetrically over the business cycle, the selection condition generates large asymmetries. The reason is the shape of the underlying idiosyncratic productivity distribution. In our calibration, a positive aggregate productivity shock moves the cutoff point to a part in the distribution with less density. By contrast, a negative productivity shock pushes the cutoff point to a part of the distribution with more density. It is very important to emphasize that we have calibrated our distribution to residual wage data (see Figure 2) and that the fit of the logistic distribution is very accurate at the left hand side of the distribution. This is the part of the distribution which is relevant for the asymmetric selection mechanism and explains the strong asymmetric labor reaction to large symmetric aggregate shocks.

Why does the contact function also generate asymmetries? This is straightforward to see with degenerate selection. In this case the dynamics of the job-finding rate is exclusively driven by the contact rate. In the steady state version, the contact rate is:

$$f = \left(\frac{a - w}{\kappa\left(1 - \beta\left(1 - \sigma\right)\right)}\right)^{\frac{\xi}{1-\xi}}. \tag{36}$$

Thus, there may be an asymmetry for the contact rate due to the exponent. Note that ξ is the elasticity of the contact function with respect to vacancies. With $\xi = 0.5$, this asymmetry would be absent and positive and negative productivity shocks would generate the same quantitative reactions, with opposite signs. In the combined model, we have calibrated $\xi = 0.09$. Thus, the exponent is 0.10. This generates a non-negligible additional asymmetry (see right panel in Figure 4).

Keep in mind that we have performed our entire analysis about asymmetries using the combined model and switching the contact and selection channel off respectively. If we did not take into account the selection mechanism and used the standard practice of parametrizing the contact function with parameters from our matching function estimation, we would have to set $\xi = 0.35$. As a consequence, the exponent in equation (36) would increase from 0.10 to 0.54. The shown asymmetries would be reduced. Asymmetries in a standard search and matching model (e.g. Shimer, 2005) in response to a 3% productivity shock are thus very small.[34]

To sum up, when we combine contact function and selection, the asymmetry is driven by two forces. First and most importantly, the curvature of the idiosyncratic productivity matters for the nonlinear dynamics of the selection rate. According to our calibration to wage data, this is a very powerful mechanism. Second, the behavior of the contact function becomes more nonlinear (due to the lower ξ compared to a standard parametrization).

To illustrate that the asymmetry in the combined model is very meaningful, Figure 5 shows the impact reaction of the job-finding rate in response to productivity shocks ranging from -3% to 3% in fully nonlinear simulations.[35] The larger the shock, the larger is the discrepancy between the response to a negative and the response to a positive shock.

Kohlbrecher and Merkl (2016) build on the model mechanism in this paper and demon-

[34] As described, this is due to the larger elasticity of the contact function with respect to vacancies. In addition, in Shimer's (2005) calibration, amplification to aggregate productivity shocks is small, which makes it even more difficult for asymmetries to show up (because the expected present value of a worker does not fluctuate a lot in response to aggregate productivity shocks). The combined model has stronger amplification. First, the selection mechanism generates extra amplification and extra asymmetry (as can be seen in Figure 4). Second, the contact function is also somewhat more volatile than in Shimer's (2005) calibration because the calibration of the idiosyncratic productivity shocks reduces the average size of the surplus.

[35] As the job-finding rate is a forward looking variable, the reaction is largest in the impact period.

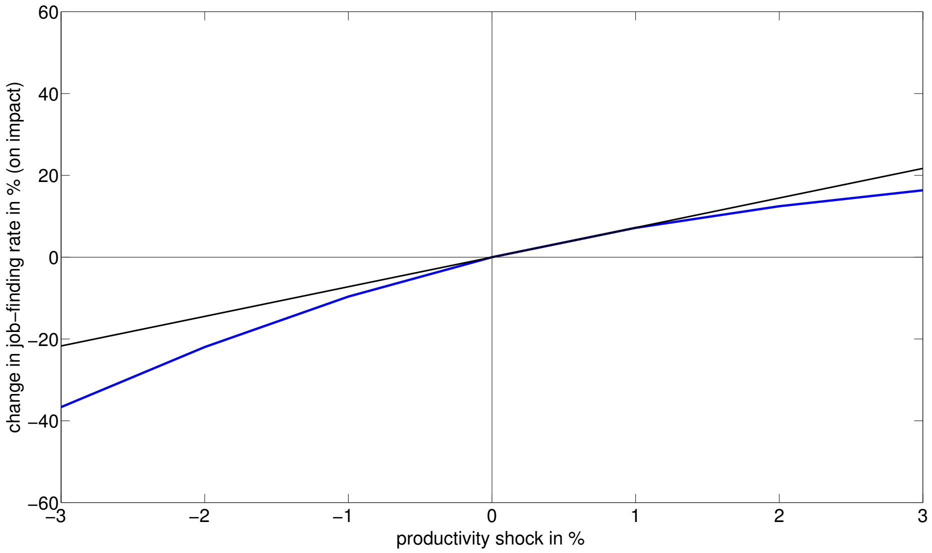

Figure 5: Impact response of job-finding rate to productivity shock. The blue line represents the nonlinear impact response of the job-finding rate to a x-percent productivity shock. The dashed line represents the same for a first-order Taylor approximation.

strate that labor selection is able to explain several stylized facts of the United States labor market. An advantage of the U.S. data over German data for the study of asymmetries is the availability of longer time series with several severe recessions. Kohlbrecher and Merkl (2016) show that apart from generating large asymmetric responses of unemployment and the job-finding rate, selection can also explain why the U.S. Beveridge curve shifts outwards in recessions and why measured matching efficiency appears to fall in economic downswings.

Our paper provides a new explanation for a phenomenon that was already debated a long time ago in the empirical literature (e.g. Long and Summers, 1986; Neftçi, 1984). Huizinga and Schiantarelli (1992) explain in a dynamic insider-outsider model why unemployment may react asymmetrically to business cycle shocks. They endogenize the reservation wage level in a general-equilibrium version. Thus, their mechanism is very different from ours where the curvature of the idiosyncratic productivity shocks generates the asymmetries. More recently, McKay and Reis (2008) and Abbritti and Fahr (2013) provide evidence for asymmetric labor market reactions in the U.S. and other countries (including Germany) respectively. Abbritti and Fahr (2013) propose a model with downward wage rigidity to explain this phenomenon. Our model mechanism does not require any wage rigidity and is thus complementary to theirs. In McKay and Reis (2008) convex hiring costs play an important role to explain the violence of recessions.

While this mechanisms shows similarities to ours, it is not embedded in a search and matching framework and is thus silent on the movement of vacancies and the matching function over time.

Petrosky-Nadeau and Zhang (2013) show that a search and matching model (without idiosyncratic shocks) can also generate nonlinearities if calibrated according to the proposition by Hagedorn and Manovskii (2008).[36] Their mechanism is complementary to ours, but can for example not explain the time trend of the matching function. Generally, the paper by Petrosky-Nadeau and Zhang (2013) and ours have the common message that nonlinearities are important and that they should be taken seriously within macro-labor models.

Ferraro (2016) proposes in a very recent paper how a search and matching framework can be modified to generate labor market asymmetries. He uses heterogeneity in permanent skills as a modeling device. This approach is different from ours where asymmetries arise within a homogenous labor market segment.

Let us also emphasize that our full model economy with idiosyncratic shocks shows substantial amplification to aggregate shocks. The positive 3% productivity shock generates an approximate 16% impact increase of the job-finding rate and the corresponding negative shock a 36% impact decrease (see Figure 5). Thus, the job-finding is about 5 times more volatile than productivity in a strong boom and 12 times in a severe recession, i.e. the model generates substantial amplification.[37] This finding complements the results from alternative approaches to generate larger amplification such as in Hagedorn and Manovskii (2008), Mortensen and Nagypal (2007), and Zanetti (2011).

6. Conclusion

We show that a wide class of models with idiosyncratic productivity (labor selection) generates a positive equilibrium relationship between matches on the one hand and unemployment and vacancies on the other hand even if the contact function is degenerate. This comovement is consistent with a model with a Cobb-Douglas matching function with constant returns. We argue that a large part of the observed elasticity of the job-finding rate with respect to market tightness could be driven by selection effects. A combined model with traditional contact function and idiosyncratic productivity shocks has interesting implications such as asymmetric responses to large aggregate shocks.

Our paper provides important insights for future theoretical and empirical research.

[36] As it is well known, the small surplus calibration has some unappealing side effects.

[37] Solving the model with a first-order Taylor approximation delivers amplification that is in between these two numbers (see Figure 5).

We have focused on a quantitative analysis with a careful calibration to high quality German data. One of our implications is that under the presence of idiosyncratic productivity shocks, the actual elasticity of the contact function with respect to vacancies is much lower than the number resulting from matching function estimations. Against the background of the Hosios' (1990) rule, the standard bargaining power in Nash bargaining must be much lower in order to establish constrained efficiency. It is certainly an interesting topic for future research to evaluate efficiency and the optimal use of policy instruments (e.g. fiscal policy) in the presence of idiosyncratic productivity shocks for new contacts.

In addition, our paper has shown that the decline in matching efficiencies in aggregate matching function estimations may be spurious in the presence of idiosyncratic productivity shocks and a time trend for vacancy posting costs. This also sounds a cautionary note on the conventional practice to use matching function estimations to quantify the effects of certain policy measures, such as unemployment benefit reforms.

Our paper also offers an interesting laboratory to analyze the quantitative effects of different policy interventions. Vacancy posting subsidies would have much smaller effects in the combined model with idiosyncratic productivity than in a plain search and matching model without a selection margin. In addition, we expect government spending to generate larger fiscal multipliers in severe recessions when the cutoff point is at a part of the idiosyncratic shock distribution with more density. This would complement theoretical results by Michaillat (2014) and empirical results by Auerbach and Gorodnichenko (2012). A detailed quantitative analysis is left for future research.

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A. Illustration

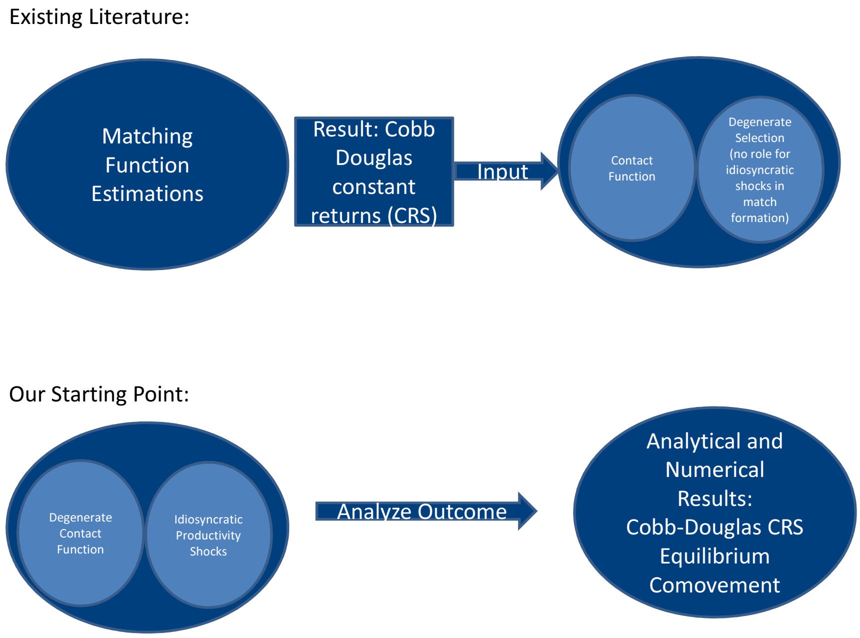

Figure 6: Illustration of matching function relation: standard procedure and selection model.

B. Theory: Derivations

This Appendix proceeds in three steps. First, we show the intermediate steps for the results in Section 3. This corresponds to the case where the idiosyncratic shock is only drawn during the first period of employment. Second, we show that the result also holds for a model with an *iid* shock in each period of employment, i.e. a model with endogenous separations (an assumption conventionally used in search and matching models with endogenous separations). Third, we show that the result does not change when workers draw an idiosyncratic shock realization at the beginning of their employment span and this realization does not change over time (an assumption conventionally used for the wage offer distribution in search models).

B.1. Baseline Results

We account for very general wage formations, with w^I denoting the wage net of contemporaneous idiosyncratic productivity.

$$w^E(\varepsilon) = w^I + \gamma\varepsilon \tag{37}$$

with

$$w^I = g(a, \eta, \theta, x). \tag{38}$$

This specification nests the Nash bargaining solution in the main text. However, it is more general. w^I is a function g of all the endogenous variables. x could be a vector of exogenous shocks and parameters, such as unemployment compensation. We exclude the trivial cases in which the wage comoves one to one with aggregate and/or idiosyncratic productivity, i.e. we assume $g'_a < 1$[38] and $\gamma < 1$. Note that our theoretical results do hold for $\gamma = 0$, although in this case we could not use the wage data to identify the distribution for idiosyncratic productivity.

In addition to the wage, we need the three equations for the cutoff point, the selection rate, and the vacancy free-entry condition:

$$\tilde{\varepsilon} = \frac{w^I - a}{(1 - \beta(1 - \sigma))} + \gamma\tilde{\varepsilon}, \tag{39}$$

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f_\varepsilon(\varepsilon)\, d\varepsilon, \tag{40}$$

$$\theta = \frac{f\eta}{\kappa}\left(\frac{a - w^I}{1 - \beta(1 - \sigma)} + \frac{(1 - \gamma)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_\varepsilon(\varepsilon)\, d\varepsilon}{\eta}\right). \tag{41}$$

We can simplify the equations for the cutoff point and market tightness further:

$$\tilde{\varepsilon} = \frac{w^I - a}{(1 - \beta(1 - \sigma))(1 - \gamma)}, \tag{42}$$

$$\theta = (1 - \gamma)\frac{f\eta}{\kappa}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_\varepsilon(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon}\right). \tag{43}$$

Using the implicit function theorem, we can derive the derivatives of all the endogenous variables with respect to productivity. The first derivative of the selection rate with

[38] g'_y denotes the partial derivative of g, the wage function, with respect to the variable y.

respect to productivity is

$$\frac{\partial \eta}{\partial a} = -f_\varepsilon\left(\tilde{\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial a}. \tag{44}$$

Thus, the elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln\left(f\eta\right)}{\partial \ln a} = \frac{-f_\varepsilon\left(\tilde{\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta}. \tag{45}$$

The first derivative of market tightness with respect to productivity is

$$\begin{aligned}\frac{\partial \theta}{\partial a} = &-(1-\gamma)\frac{f}{\kappa}f_\varepsilon(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_\varepsilon\left(\varepsilon\right)d\varepsilon}{\eta} - \tilde{\varepsilon}\right)\\ &+(1-\gamma)\frac{f\eta}{\kappa}\left(\frac{-\tilde{\varepsilon}f_\varepsilon(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}\eta + f_\varepsilon(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_\varepsilon\left(\varepsilon\right)d\varepsilon}{\eta^2} - \frac{\partial \tilde{\varepsilon}}{\partial a}\right).\end{aligned} \tag{46}$$

Simplified:

$$\frac{\partial \theta}{\partial a} = -\left(1-\gamma\right)\frac{f\eta}{\kappa}\left(\frac{\partial \tilde{\varepsilon}}{\partial a}\right). \tag{47}$$

Thus, the elasticity of market tightness with respect to productivity is

$$\frac{\partial \ln\theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_\varepsilon(\varepsilon)d\varepsilon}{\eta} - \tilde{\varepsilon}}. \tag{48}$$

The first derivative of the cutoff point with respect to productivity is given by:

$$\frac{\partial \tilde{\varepsilon}}{\partial a} = \frac{g'_a - 1}{(1-\gamma)(1-\beta(1-\sigma)) + f_\varepsilon(\tilde{\varepsilon})g'_\eta + (1-\gamma)g'_\theta}. \tag{49}$$

This term should be strictly smaller than zero for interesting cases. Imagine, for example, that the wage is given by $w^I = \gamma(a+\kappa\theta) + (1-\gamma)b$. In this case $\partial\tilde{\varepsilon}/\partial a = (-1/((1-\beta(1-\sigma)) + \kappa\alpha)) < 0$.

We can now combine (45) and (48) to obtain the elasticity of the job-finding rate with respect to market tightness:

$$
\begin{aligned}
\frac{\partial \ln (f\eta)}{\partial \ln \theta} &= \left(\frac{-f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta}\right) \Big/ \left(\frac{-\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta} - \tilde{\varepsilon}}\right) \\
&= \frac{f_{\varepsilon}(\tilde{\varepsilon})}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)\,d\varepsilon}{\eta} - \tilde{\varepsilon}\right) \\
&= \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}.
\end{aligned}
$$

B.2. Endogenous Separations

With endogenous separations, the cutoff point is

$$
\begin{aligned}
\tilde{\varepsilon} = w^I - a + \gamma\tilde{\varepsilon} + \beta(1-\sigma(\tilde{\varepsilon}))\left(w^I - a - \frac{(1-\gamma)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)\,d\varepsilon}{(1-\sigma(\tilde{\varepsilon}))}\right) \\
+ \beta^2(1-\sigma(\tilde{\varepsilon}))^2\left(w^I - a - \frac{(1-\gamma)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)\,d\varepsilon}{(1-\sigma(\tilde{\varepsilon}))}\right) + \ldots
\end{aligned}
\tag{50}
$$

$$
\tilde{\varepsilon} = \frac{\left(w^I - a - \beta(1-\sigma(\tilde{\varepsilon}))(1-\gamma)\frac{\int_{-\infty}^{\tilde{\varepsilon}}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{1-\sigma(\tilde{\varepsilon})}\right)}{(1-\beta(1-\sigma(\tilde{\varepsilon})))(1-\gamma)}. \tag{51}
$$

As usual, the selection rate is

$$
\eta = \int_{\tilde{\varepsilon}}^{\infty} f_{\varepsilon}(\varepsilon)\,d\varepsilon. \tag{52}
$$

The elasticity of the job-finding rate with respect to productivity is

$$
\frac{\partial \ln (f\eta)}{\partial \ln a} = \frac{-f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta}. \tag{53}
$$

With endogenous separations, market tightness is:

$$
\begin{aligned}
\theta &= \frac{f\eta}{\kappa}\left(\frac{a - w^I + \frac{(1-\gamma)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{(1-\sigma(\tilde{\varepsilon}))}}{1-\beta(1-\sigma(\tilde{\varepsilon}))}\right) \\
&= (1-\gamma)\frac{f\eta}{\kappa}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)\,d\varepsilon}{(1-\sigma(\tilde{\varepsilon}))} - \tilde{\varepsilon}\right).
\end{aligned}
\tag{54}
$$

Taking into account that $\eta = 1 - \sigma$ in this setting, we obtain:

$$
\begin{aligned}
\frac{\partial\theta}{\partial a} &= -(1-\gamma)\frac{f}{\kappa}f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}\left(\varepsilon\right)d\varepsilon}{\eta}-\tilde{\varepsilon}\right) \\
&\quad +(1-\gamma)\frac{f\eta}{\kappa}\left(\frac{-\tilde{\varepsilon}f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\eta+f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial\tilde{\varepsilon}}{\partial a}\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}\left(\varepsilon\right)d\varepsilon}{\eta^{2}}-\frac{\partial\tilde{\varepsilon}}{\partial a}\right).
\end{aligned}
\tag{55}
$$

After some algebra:

$$
\frac{\partial\theta}{\partial a}=-\left(1-\gamma\right)\frac{f\eta}{\kappa}\left(\frac{\partial\tilde{\varepsilon}}{\partial a}\right). \tag{56}
$$

Thus, the elasticity of market tightness with respect to productivity is

$$
\frac{\partial\ln\theta}{\partial\ln a}=\frac{-\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta}-\tilde{\varepsilon}}. \tag{57}
$$

Combining equations (53) and (57), we obtain the heterogeneity based matching function:

$$
\begin{aligned}
\frac{\partial\ln\left(f\eta\right)}{\partial\ln\theta} &= \left(\frac{-f_{\varepsilon}\left(\tilde{\varepsilon}\right)\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\eta}\right)\Bigg/\left(\frac{-\frac{\partial\tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta}-\tilde{\varepsilon}}\right) \\
&= \frac{f_{\varepsilon}\left(\tilde{\varepsilon}\right)}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}\left(\varepsilon\right)d\varepsilon}{\eta}-\tilde{\varepsilon}\right) \\
&= \frac{\partial\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta}}{\partial\tilde{\varepsilon}}.
\end{aligned}
$$

B.3. Same Idiosyncratic Shock for the Entire Employment Span

If the idiosyncratic shock is constant during the entire employment span, the cutoff point is:

$$
\begin{aligned}
\tilde{\varepsilon} &= w^{I}-a+\gamma\tilde{\varepsilon}+\beta\left(1-\sigma\right)\left(w^{I}-a-\left(1-\gamma\right)\tilde{\varepsilon}\right) \\
&\quad +\beta^{2}\left(1-\sigma\right)^{2}\left(w^{I}-a-\left(1-\gamma\right)\tilde{\varepsilon}\right)+...
\end{aligned}
\tag{58}
$$

This expression can be simplified:

$$
\tilde{\varepsilon}=\frac{w^{I}-a}{1-\gamma}. \tag{59}
$$

The selection rate is thus given by:

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f_{\varepsilon}(\varepsilon)\, d\varepsilon. \tag{60}$$

The elasticity of the job-finding rate with respect to unemployment is

$$\frac{\partial \ln (f\eta)}{\partial \ln a} = \frac{-f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\int_{\tilde{\varepsilon}}^{\infty} f_{\varepsilon}(\varepsilon)\, d\varepsilon}. \tag{61}$$

Market tightness:

$$\begin{aligned} \theta &= \frac{f\eta}{\kappa}\left(\frac{a - w^I + \frac{(1-\gamma)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon}{(1-\sigma)}}{(1-\beta(1-\sigma))}\right) \\ &= \frac{(1-\gamma)}{1-\beta(1-\sigma)}\frac{f\eta}{\kappa}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon)\, d\varepsilon}{(1-\sigma)} - \tilde{\varepsilon}\right). \end{aligned} \tag{62}$$

Using $\eta = 1 - \sigma$, we get:

$$\begin{aligned} \frac{\partial \theta}{\partial a} =& -\frac{(1-\gamma)}{1-\beta(1-\sigma)}\frac{f}{\kappa} f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon}\right) \\ &+ \frac{(1-\gamma)}{1-\beta(1-\sigma)}\frac{f\eta}{\kappa}\left(\frac{-\tilde{\varepsilon} f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}\eta + f_{\varepsilon}(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon)\, d\varepsilon}{\eta^2} - \frac{\partial \tilde{\varepsilon}}{\partial a}\right). \end{aligned} \tag{63}$$

After some algebra, we obtain:

$$\frac{\partial \theta}{\partial a} = -\frac{(1-\gamma)}{(1-\beta(1-\sigma))}\frac{f\eta}{\kappa}\left(\frac{d\tilde{\varepsilon}}{da}\right). \tag{64}$$

Thus, the elasticity is

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\left(\frac{d\tilde{\varepsilon}}{da}\right) a}{\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon}\right)}. \tag{65}$$

Combining equations (61) and (65), we obtain the heterogeneity based matching function:

$$
\begin{aligned}
\frac{\partial \ln (f\eta)}{\partial \ln \theta} &= \left(\frac{-f_{\varepsilon}\left(\tilde{\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\eta}\right) \Bigg/ \left(\frac{-\frac{\partial \tilde{\varepsilon}}{\partial a}a}{\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta} - \tilde{\varepsilon}}\right) \\
&= \frac{f_{\varepsilon}\left(\tilde{\varepsilon}\right)}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}\left(\varepsilon\right)d\varepsilon}{\eta} - \tilde{\varepsilon}\right) \\
&= \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}.
\end{aligned}
$$

C. Data Description and Mincer Regression

C.1. Data Description

The German administrative database provides coherent definitions of the matching function variables. We use monthly data over the time period from 1993 to 2007. Matches and unemployment are obtained from the Sample of Integrated Labor Market Biographies (SIAB). The SIAB is a 2% random sample of all German residents who are registered by the Federal Employment Agency because of paying social security contributions or receiving unemployment benefits (see Dorner et al., 2010). Unemployment benefits may cover contribution-based benefits, means-tested benefits, and income maintenance during training. We use an adjusted measure of unemployment benefit receipt according to Fitzenberger and Wilke (2010) to determine the unemployment stock. Matches are defined as transitions from unemployment to employment subject to social security. Even though marginal employment has become subject to social security since 1999, we do not consider this kind of employment as it is often ascribed to a stepping stone into regular jobs. The number of matches is calculated continuously, i.e. we take into account every daily transition. Hence, we do not neglect any job findings that are reversed within a month. See Nordmeier (2014) for more details on the time series.

Vacancies are taken from the official statistics and cover the number of open positions with an intended employment duration of at least seven days that are reported to the Federal Employment agency. The reported vacancies account for about 30-40% of the total number of vacancies in Germany as shown by the IAB Job Vacancy Survey. However, an upscaling of the reported vacancies would not affect our estimation results because the reporting rate does not show a cyclical pattern in our observation period. Table 6 provides an overview of the control variables used in the matching function estimation.

For our calibration exercise, we exploit the wage information included in the SIAB. Wages are shown as the employee's gross daily wage in Euros, which was calculated from the fixed-period earnings reported by the employer and the duration of the employment period in calendar days. Because we focus on new full-time jobs, we only consider wages above the marginal part-time income threshold. We use the consumer price index (CPI) from the National Accounts to obtain real daily wages.

Variables	Extracted series	Definition
Unemployment duration	*long*	Share of long-term unemployed, i.e. unemployment duration $\geq$ 1 year
Age	*young* *old*	Share of unemployed with age $\leq$ 25 years Share of unemployed with age $\geq$ 55 years
Education	*low-skilled* *high-skilled*	Share of unemployed without vocational training (acc. to Fitzenberger et al., 2005) Share of unemployed with university degree (acc. to Fitzenberger et al., 2005)
Nationality	*foreign*	Share of unemployed with immigration background (see Wichert and Wilke, 2012)
Gender	*female*	Share of female unemployed
Family status	*married* *child*	Share of married unemployed Share of unemployed with at least one child
Benefit receipt	*UB I*	Share of contribution-based unemployment benefits recipients (unemployment benefits I)

Table 6: Description of control variables for matching function estimation. Data source: SIAB.

C.2. Mincer Regression

The dependent variable in our Mincer regression is the real wage (in logs) paid to worker i in month t for new employment spells. Note that we only include wages for new employment spells for workers that moved from unemployment (see definition in Appendix C.1) to employment. We constrain ourselves to full time employed workers. The wage is obtained from the respective employment spell. If a worker is employed for the entire year, the wage is calculated as the wage for the entire year divided by the number of working days. If the worker is employed for a shorter time period (e.g. just the last three months of the year), the wage is obtained from this spell.

In the Mincer regression, we control for a number of observables, namely education (with dummy variables for different degrees), overall employment experience (number of years, as levels and squared term), age, tenure in same firm (number of years from previous employment spells in the same firm, as levels and squared term), gender, a dummy for married workers, a dummy if the worker has children, a dummy if workers were unemployed for longer than one year, a dummy if workers did not receive any unemployment benefits during unemployment, the level of unemployment benefits I (Arbeitslosengeld I, ALG I) during the previous unemployment period, the number of remaining days of entitlement to ALG I during the unemployment spell, dummies for other unemployment benefit types (e.g. ALG II), dummies for occupation groups and industry classifications, and dummies for the establishment size (<20, 20 to 99, 100 to 499, >500), dummies for the type of region (classified from large metropolitan area to rural area: 10 categories), a dummy for East Germany. In addition, we included monthly dummies for the start of the employment spell. Although the monthly wages are calculated based on the entire employment spell until the end of the year (which may be the remainder of the year), there may be certain seasonal patterns. For example, workers that were employed at the beginning of the year are likely to be eligible for the Christmas wage bonus, while this is not the case for workers who are employed towards the end of the year. Given that these payments are included in the wage, the seasonal dummies also control for these type of effects. Finally, we include a time dummy for each year (except for the starting year 1993).

In addition, we control for the last wage in the previous employment spell w_i^I. From a theoretical perspective, we expect that larger past wages are associated with positive unobservable characteristics. The estimated coefficient on $\ln w_i^I$ is indeed positive (0.4) and statistically highly significant.

D. Non-Parametric Results

Weight on vacancies based on equation (26) and a non-parametric fit of the wage distribution for entrants (normal kernel):

	5th percentile	10th percentile
$\log \theta$	0.15	0.25

Table 7: Coefficient on vacancies based on steady state approximation. Results are calculated numerically from the non-parametric fit of the distribution using equation (26).

E. Parametrization of the Model

We parametrize the model to a monthly frequency. For an overview of targets and parameters see Tables 8 and 9. Aggregate productivity in steady state is normalized to 1. In all dynamic versions of the model, productivity follows an AR(1) process with a correlation coefficient of 0.95 and a standard deviation for the shock of 0.44%. We have estimated these values from productivity data from the German National Accounts.[39] The discount factor is $0.99^{\frac{1}{3}}$. We set the value of non-work to 0.8. Unemployment benefits for short-term unemployed in Germany are 60 or 67% of the last net wage. Our value takes into account that there is a value of home production. Although this value is high compared to standard U.S. calibrations, most workers are far from indifferent between working and not working.

In line with our data we set the separation rate to 1% and target an overall job-finding rate of 5% per month. Market tightness in the data is 0.09. Vacancy posting costs are chosen to hit this target. The value of market tightness in our data seems very low. This is partly due to the fact that we only consider reported vacancies. However, the level of market tightness, and hence kappa, is a matter of scaling only and does not affect any of our results. The relatively high values of vacancy posting costs are also due to the very low flow rates of the German labor market and to the fact that we abstain from hiring costs in this model. In all model versions with a Cobb-Douglas contact function we target an overall elasticity of matches with respect to vacancies of 0.35 in line with our estimated matching function. Taking the distribution of the idiosyncratic shock, market tightness, and the job-finding rate as given, this determines the value of the contact efficiency parameter.

Wages are determined by Nash bargaining, which ensures private match efficiency. We set the bargaining power of workers to 0.5. Most of our findings are robust to the exact value of the bargaining power. It primarily matters for the calibration of the variance of idiosyncratic productivity, and hence for amplification, as well as the wage cyclicality of entry wages.

To calibrate the distribution of idiosyncratic productivity, we use the distribution of the residual entry wages. Given the proportionality between wages and idiosyncratic productivity, the distribution of the latter is a scaled version of the former. We fit a logistic distribution to the residual wage data. Note that we use the exponential of the regression residuals instead of the levels, as we estimate the Mincer regression in

[39] Productivity: Output per hours worked from the Federal Statistical Office (*Statistisches Bundesamt*), 1991Q1 to 2013Q1.

logs.[40] A nice feature of the logistic distribution is that the derivative of the conditional expectation - the term that determines the elasticity of matches with respect to vacancies - is uniquely determined by the cumulative density to the right of the cutoff point (i.e. the selection rate). The 5th and 10th percentile of our wages correspond to selection rates of 0.938 and 0.877 in the fitted distribution. Note that these do not necessarily correspond to the real selection rate as we do not know the number of workers to the left of the distribution. However, for the dynamics of our model it is irrelevant whether we have a low selection rate with a high contact rate or vice versa. What matters is the shape of the idiosyncratic shock distribution at and to the right of the cutoff point which we calibrate with wage data. The contact rate is set to match the empirical job-finding rate of 5% per month is steady state.

We determine the standard deviation of the idiosyncratic shock such that the cross-sectional (conditional) standard deviation of wages in our model matches the conditional standard deviation of wages in the data, which is based on the logistic fit of our residual wage distribution. The corresponding value is 0.17 which implies an unconditional standard deviation of the idiosyncratic shock of 0.4. The mean of the idiosyncratic productivity distribution is determined endogenously to match the relevant selection rate. We interpret the mean of the entrant productivity as fixed ex-post hiring/training costs.

Note that the mean of idiosyncratic productivity seems unrealistically low in our parametrization. Two comments are in order. First, our baseline model is very simple. The mean of the idiosyncratic shock could equally be interpreted as a fixed training or hiring cost. In addition, the influence of unions may lead to larger average wages and there could be fixed costs of production. Including these features would lead to a larger average calibrated idiosyncratic productivity in the first period of employment. Second, our results for the elasticity of the matching function with respect to vacancies are independent of our parametrization strategy. The particular combination of mean and variance for a given selection rate does not matter for our key results.

[40]The results are not very different if we did not use the exponent.

Parameter	Value	Source
Aggr. productivity	1	Normalization
AR-coef. productivity	0.95	National Accounts data
SD productivity	0.0044	National Accounts data
Discount factor	$0.99^{\frac{1}{3}}$	Set
Value of leisure	0.8	Set
Bargaining power	0.5	Set
Separation rate	0.01	SIAB data
Job-finding rate	0.05	SIAB data
Market tightness	0.09	SIAB and Vacancy data

Table 8: Common parameters and targets.

Parameter	5th	10th	Source and/or target
SD of log dist.	0.4	0.4	SIAB data (wages)
Selection rate	0.938	0.877	SIAB data (wages)
Mean of log dist.	-13.20	-13.61	Selection rate
Vacancy posting cost	0.18	0.15	Market tightness
	Combined model		
Contact elast. wrt vac.	0.22	0.09	Match elast. wrt vac.
Contact efficiency	0.09	0.07	Contact rate

Table 9: Parameters (depending on percentile for cutoff wage).