Discussion Papers in Economics

No. 10/2016

Operating a Swing Option on Today's Gas Markets - How Least Squares Monte Carlo Works and Why it is Beneficial

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ISSN 1867-6707

Friedrich-Alexander-Universität Erlangen-Nürnberg
Institute for Economics
http://www.iwqw.wiso.uni-erlangen.de/forschung/iwqw-discussion-paper-series.shtml

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ABSTRACT

We investigate, if it pays off for a company to invest into complex swing option algorithms. We first introduce least squares Monte Carlo as a complex valuation algorithm and explain in detail how it works. Using a simulation study and two backtest scenarios we compare the output of this method with a simple myopic approach, and evaluate the results also from a business point of view. We find that myopic operation performs fairly well, but given a certain contract size and a certain contract flexibility, LSMC clearly prevails.

Keywords and phrases: Swing Option; Spot Optimization; Least Squares Monte Carlo

1 Introduction

Modern energy industry is becoming more and more challenging for its participants, especially in Germany. Increased photovoltaic and wind production, swift changes in the regulatory environment (e.g. the "Energiewende"), and distortions following the decline of global oil prices are just a few examples that make long-term investments in this business more insecure than ever. In the short run, players are under pressure to generate maximum revenues from existing facilities like gas storages, whose value is, by the way, fading due to dwindling spreads between summer and winter forward prices. In this context, take-or-pay contracts, also called swing options, were created to cater consumers with variable demand. These are contracts over a flexible amount of gas taken by the buyer. Given some boundaries, e.g. daily or monthly limits, the buyer decides when and how much gas to take from the contract, whereby the possible price formulas range from a fixed flat price up to monthly indices or may include an oil formula as well. Each day, for example, the buyer faces the same question: execute the contract or wait in order to generate a higher future cash flow. Municipal utilities are confronted with a similar task, as they face customers' demand which – apart from the temperature-related component – is uncertain and needs to be forecasted. Either these companies hedge this demand-shifts perfectly (and thus earn only little money) or they are taking some degree of risk by trading them out on spot and futures' markets. All these problems are basically very similar: How to operate under various degrees of uncertainty, e.g. price development or customer offtakes? Here, we exemplary concern with swing options, whereby the given solution is – safe some minor modifications – applicable to the whole class of examples mentioned above.

In an environment of decreasing sales margins, decreasing summer-winter spreads, but more overall liquidity, common low-risk strategies like intrinsic operation, i.e. fixing revenues by instant forward market transaction, or rolling intrinsic operation, which is the same but with continuous readjustment of positions[1], yield only small revenues that are often not enough to cover fixed costs. This fact is especially visible on the gas storage market, where the spread between the upcoming winter and summer season on the Dutch Title Transfer Facility (TTF) hub touched the lower boundary of 1 EUR/MWh in March 2016. Hence, players are urged to venture into the fields of a more active, i.e. more short-term, operation of flexible assets or products. Apart from the significantly higher risk, these rather spot price oriented strategies are more complex, hence require more expertise. One solution is to purchase knowledge in form of a software, and there is plenty of (costly) choice on the market. This, again, might turn out to be dangerous as algorithms are complex: One has to understand how they work in order to properly evaluate and use their results – which is all the more important, because algorithms cannot consider all potential influence factors (liquidity just up to certain point, for example). To base an irreversible trading decision under insecurity on an algorithm output requires trust or – more likely – a certain mathematical background. As not all traders or portfolio managers do have such knowledge, they understandably refrain from applying complex algorithms.

Therefore, in this paper, we intend to facilitate the first steps with a spot-based trading strategy by explaining one widely used algorithm in detail, whereby we especially refer to the detailed example in Section 3. Thereby, we focus on the short-term optimization of a contract, i.e. how to trade the spot price versus, say, the front month price. The choice of algorithms to solve this problem is quite substantial. There is, for example, the approach to generate scenario trees in order to evaluate future price movements and their effect on the swing option or storage value (see Felix & Weber, 2012, Jaillet et al., 2004). General, energy market-related applications are analyzed e.g. in Heitsch & Römisch (2009). The idea of generated scenario trees is a very intuitive one, however, it is quite computationally intensive (especially if you increase the complexity of the price model or include more contract constraints) and therefore less useful for daily optimizing of a flexible contract. Alternatively, partial differential equations may be chosen (see e.g. Thompson et al., 2009). This is a computational-wise very efficient approach, but is – due to the functional form – restricted to Brownian motion-based price processes, i.e. one is limited when modeling the dynamics of the underlying price process. Alternatively, we could apply regression-based methods, e.g. the one proposed by Carmona & Ludkovski (2010) or the approach of Boogert

[1] See Géman (2005) for a closer description of both strategies.

& de Jong (2008) who generalize the American option valuation of Longstaff & Schwartz (2001). For an overview over various regression models, see Tsitsiklis & Van Roy (2001). LSMC proves to be the model of choice for us: it is computationally efficient, and, which is a major asset, price model and option valuation are separated, hence we are not limited in designing the price dynamics, which are discussed in Section 2.1.

We structure the paper as follows. First, we present an adequate price model, then the LSMC algorithm is explained (Section 2). In Section 3 we show how LSMC works using a simple example including all required computational steps[2]. Eventually, in the context of a case study (Section 4), we evaluate if the additional complexity and risk of spot price operation really pays off. As a benchmark model we apply the intuitive myopic[3] approach: the option is instantly exercised if the current spot price is larger then the strike price. We find that despite LSMC prevails in most scenarios, the myopic approach performs fairly well as well. Thereby the success of LSMC significantly depends on the level of contract flexibility. Below a certain degree of flexibility, a myopic operation delivers fairly good results without requiring the additional effort (and costs) of applying LSMC. In this case, contract size can help to benefit from the slightly larger payoffs of LSMC. Eventually, we see that bearish trends in the operation period pose a threat to LSMC payoffs. Section 5 concludes the paper summarizing the major findings.

2 Theory

LSMC requires scenarios of possible futures spot price developments, whose generation is based on a mathematical model to explain the development of prices over time. Section 2.1 concerns with this question, whereby Section 2.2 presents the algorithm itself.

2.1 A Price Model for Spot and Forward Prices

There is ample research about the development of commodity prices in general, however, regarding natural gas price behavior in particular, the choice is much smaller and mainly focused on parametric stochastic models. Thereby, solutions differ regarding practicability, comprehensibility, and robustness, all being relevant factors in daily business life. For the spot price, Schwartz & Smith (2000) provide a widely-used suggestion for a stochastic process, which splits the price into a long-term and a short-term component. This is very intuitive as gas prices indeed seem to contain both a long-term and a short-term, more trading-originated, component. However, model calibration is not that easy (they suggest

[2]On request we can provide a more extensive example.
[3]For information about myopic decision rules, see Eastwell et al. (1990).

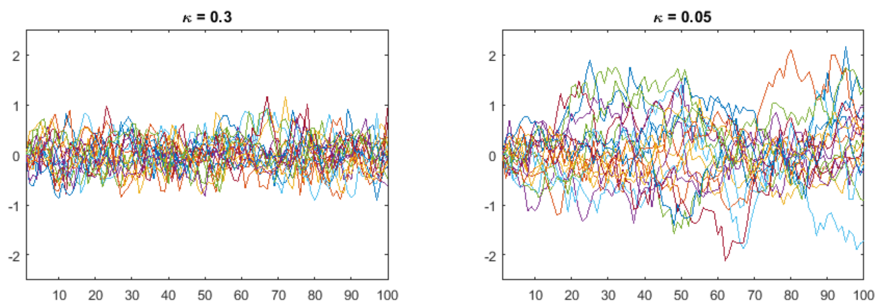
the Kalman filter, Schlüter, 2014, provides an alternative). Alternatively, a regime-switching approach, proposed e.g. by Chen & Forsyth (2010), may be applied. One might also include fundamental factors, i.e. other prices like oil, electricity, or temperature into the model. Müller et al. (2015) extend the temperature-dependent model of Stoll & Wiebauer (2010) and include oil as an exogenous factor as well. A more complex suggestion is the structural vector autoregressive model of Nick & Thoenes (2014). Here, storage levels and electricity are included as well. However, considering the idleness of German gas fired power plants, one might question, if the model specification still holds. Besides that, more data means less robustness and the user has to spend more time on validating and maintaining the database. Because of this and because we mainly focus on the short-term interaction between spot and front month prices, we favor the very popular Ornstein-Uhlenbeck type model (see e.g. Boogert & De Jong, 2008) for the spot price process $(P_t)_{t\in\mathbb{R}}$,which includes a mean reversion component with the long-term part being represented by the front month price $(F_t)_{t\in\mathbb{R}}$. Mean reversion is an accepted and widely-used model feature of gas prices (see e.g. Schwartz & Smith, 2000, Cartea & Figueroa, 2005, Boogert & De Jong, 2008). The forward price process F_t, again, is represented by a diffusion process. There are more complex models like the one-factor model of Clewlow & Strickland (1999) or the two-factor generalization of Kiesel et al. (2009), for example. They use an exponential volatility function to capture the Samuelson effect, i.e. that the existence of mean-reversion in the spot price causes its volatility to increase towards the forward product's maturity (Samuelson, 1965). However, for front month contracts we consider this effect to be negligible. Our model reads as follows:

$$\begin{aligned} d\log F_t &= \epsilon_{t,1}, \\ d\log P_t &= \kappa(\log F_t - \log P_t)dt + \epsilon_{t,2}, \end{aligned} \tag{1}$$

where $\kappa > 0$, and $(\epsilon_{t,1}, \epsilon_{t,2}) \sim \Sigma\,\mathcal{N}(0,1)$, with $\Sigma \in \mathbb{R}^{2\times 2}, \Sigma(1,1) = \sigma_1^2, \Sigma(2,2) = \sigma_2^2, \Sigma(1,2) = \Sigma(2,1) = \rho\sigma_1\sigma_2, \sigma_1, \sigma_2 > 0, \rho \in [-1,1]$. Thereby ρ denotes the correlation between long- and short-term prices. Volatility can be designed as time-varying, a step that significantly increases model complexity. Because of this we consider it as not adequate for an introduction to swing options; moreover, the basic features that determine the option value are still captured by Eq (1). An intuitive approach would be to introduce a summer/winter or quarter-wise dummy. A more advanced model for discrete-time processes is the GARCH model of Engle (1982), which is widely used in finance. GARCH stands for General Autoregressive Conditional Heteroscedasticity (GARCH) and is basically an autoregressive model for Σ. For a detailed introduction please refer to McNeil et al. (2015).

Economic-wise we motivate the mean reversion κ with existing short-term imbalance between supply and demand: For a few days, prices can drop or spike until supply and demand return to the long-term equilibrium. Given a certain κ, the spot prices P_t need $\ln 2/\kappa$ days to cover half the price difference back to the front month price F_t. The effect of κ is plotted in Figure 1, where you see 20 simulated price paths each over 100 days (volatility is fixed): The stronger the mean reversion, i.e. the larger the κ, the faster the spot returns to the front month level in Eq. (1), i.e. the smaller the oscillation of the price spread.[4]

Figure 1: Simulated Spreads between Spot and Front Month Prices for Different κ



2.2 The Theory of Least Squares Monte Carlo

A standard natural gas swing option is characterized by three parameters: a global maximum offtake, a global minimum offtake and a maximum single offtake each time the contract is exercised. Usually, a contract covers one year, hence we speak of an maximum annual contract quantity (ACQ) with a minimum offtake, the take-or-pay quantity (ToP). Option rights are exercised on a daily basis, i.e. we have a maximum daily contract quantity (DCQ). The maximum number of exercise days (Ed) on which a maximum offtake is possible amounts to $Ed[d/a] = \left\lfloor \frac{ACQ[MWh/a]}{DCQ[MWh/d/a]} \right\rfloor$. The difference between ToP and ACQ is the optional (flexible) quantity, where the holder of the option can decide whether to use it or not. The question now is: When to exercise the option and how to take into account the stochastic behavior of spot prices? To address this problem, we apply LSMC and thereby mainly stick to the approach of Boogert & De Jong (2008). Their algorithm essentially consists of four steps: 1. Discretize time and cumulative offtake levels, 2. Formulate the

[4] Due to lack of practical relevance we omit to discuss risk neutrality, a principle which postulates that forward prices are a proper estimate for spot prices (i.e. $E_t(P_T) \stackrel{!}{=} F_T$). For more information please refer to Hull (2014) or, for a very fundamental paper, to Cox et al. (1979).

decision problem, 3. Apply the LSMC algorithm, 4. Find the optimal strategy using forward induction.

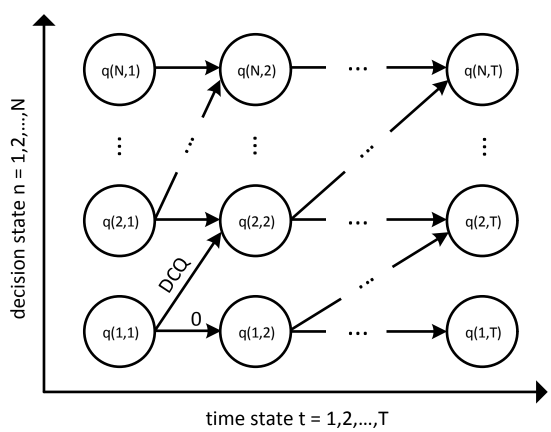
(1) Discretize time and cumulative offtake levels. For this purpose, we generate a two-dimensional discrete time-volume grid with cumulative offtakes $q_{n,t} \in Q_t$, $n = 1, 2, ..., N \in \mathbb{N}$, at time instances $t \in 1, \ldots, T \in \mathbb{N}$. Thereby, Q_t represents the set of all possible cumulative offtakes at time t. When moving from time instance t to $t+1$ the holder of the swing option can exercise a swing right. If he does, we have a state transition $q_{n,t} \Rightarrow q_{n+1,t+1}$, else we have $q_{n,t} \Rightarrow q_{n,t+1}$ (see Figure 2). For each transition, we have to choose an action $a_t \in A_t$, where

$$A_t := \begin{cases} DCQ_t & \text{if option is exercised,} \\ 0 & \text{else.} \end{cases}$$

The logic behind that is: If it makes sense to exercise the option, than we take the maximum possible amount to generate the maximum possible profits.

(2) Formulate the decision problem. Note that every action a_t leads to a cash flow

Figure 2: Discretization I



$CF_t(a_t, S_t) = a_t \cdot S_t$ depending on the spread $S_t = CP_t - P_t$ between contract price CP_t and spot price P_t. Consequently, the sum of cash flows over the life time of a swing option, i.e. the total option value, amounts to

$$\sum_{t=1}^{T} CF_t(a_t, S_t).$$

According to Bellman's principle of optimality (Bellman, 1957) the optimal decision rule reads as follows:

$$\Pi_t(S_t, q_t + a_t) := \arg\max_{a_t \in A_t} \Big\{ CF_t(a_t, S_t) + E_{t+1}[S_t, q_t + a_t] \Big\}, \tag{2}$$

where $E_{t+1}[\cdot]$ is the expected (residual) value of the swing contract from $t+1$ until maturity. This value, which is also called continuation function, includes all information available at time t regarding the future spread evolution and the optimal reaction upon this development. Hence, $E_{t+1}[\cdot]$ equals the expected sum over all future cash flows from $t+1$ up to T. This means that, based on the decision rule mentioned above, we should choose action a_t such that the value of the decision function Π_t is maximized. In practice, $E_{t+1}[\cdot]$ is unknown and thus has to be approximated.

(3) Apply LSMC to approximate $\hat{E}_{t+1}[\cdot]$. Based on our model from Section 2.1 we generate $j = 1, 2, ..., J$ price (spread) paths using Monte Carlo simulation. We apply least squares regression techniques to compute the approximated expectation of the cumulative cash flows at time $t+1$, i.e. $ACCF^j_{t+1} = CF^j_{t+1} + CF^j_{t+2}$ (= dependent variable), against the sum of $k = 1, 2, ..., K$ basis functions γ^j_k of the price spreads S^j_t (= independent variable). The formula reads as follows:

$$min \stackrel{!}{=} \sum_{j=1}^{J} \left(ACCF^j_{t+1}(a_{t+1}, S_{t+1}) - \sum_{k=1}^{K} (\beta_k \cdot \gamma^j_k) \right)^2 \tag{3}$$

As a regression function we use power polynomials of degree $K = 3$, as own empirical studies show that such polynomials produce sufficiently good results, i.e.

$$\gamma^j_k = (S^j_t)^{k-1}. \tag{4}$$

For a detailed discussion of alternatives, see Longstaff & Schwartz (2001). The least squares method is available in almost every econometric software and especially in *MS Excel* with the RGP function. Using the estimated regression parameters $\hat{\beta}$, we approximate $\hat{E}_{t+1}[S_t, q_t +$

$a_t]$:

$$\hat{E}_{t+1}[S_t, q_t + a_t] = \sum_{k=1}^{K} (\hat{\beta}_k \cdot \gamma_k^j). \tag{5}$$

This procedure runs backwards in time for every grid point $q_{n,t} \in Q_t$ in the state grid for $t = T, T-1, T-2, ..., 1$. Using the decision function $\Pi_t(S_t, q_t + a_t)$, it is possible to select the optimal action $a_t \in A_t$. The regression parameters are stored in $q_{n,t}$.

(4) Use forward induction. We start at grid point $q_{n=1,t=1}$ and move forward in time $t = 1, 2, ..., T$ in order to find the optimal operational strategy for the swing option. Therefore we insert the realized price spread S_t into Eq. (4), and calculate $\hat{E}_{t+1}[S_t, q_{n,t} + a_t]$ using $\hat{\beta}_k$. Thereby we select the optimal decision using $\Pi_t(S_t, q_t + a_t)$. We then store the state transition $q_t \Rightarrow q_{t+1}$, move on to $t + 1$, and repeat the procedure, etc.

3 A Numerical Example

For a better understanding of the LSMC algorithm we present a simple numeric example which is reproducible in MS EXCEL. The example is based on the parameters found in Table 1. Given these values, $Ed = 4/1 = 4$.

Table 1: Contract Parameters

DCQ = 1 [MWh/d]	ACQ = 4 [MWh]
ToP = 1 [MWh]	days: T = 4 [d]

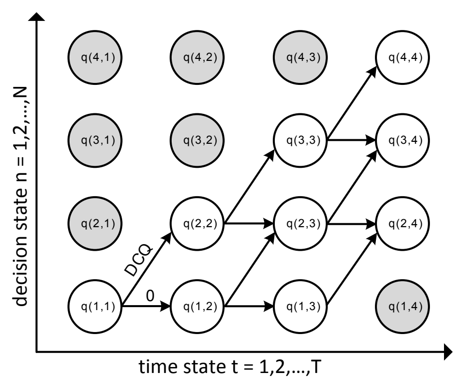
We use five paths of simulated price spreads shown in Table 2. For the discretization step we opt for a (fictive) 4×4 state grid, as we have 4 time steps and 4 steps in the quantity dimension. Figure 3 illustrates the discretization for our sample parameters, where the white circles represent the allowed states in the grid and the gray circles represent the states which cannot be reached given the decision problem's constraints. Let's, for example, start in $q_{n=1,t=1}$. From there we cannot step e.g. to $q_{n=4,t=2}$, as only two state transitions are possible, namely $q_{n=1,t=1} \Rightarrow q_{n=1,t=2}$ and $q_{n=1,t=1} \Rightarrow q_{n=2,t=2}$.

Once the discretization is done we apply the backward LSMC algorithm. We start in state $q_{4,4}$ and step backwards in time $t = T, T-1, T-2, ..., 1$. For all states in T, no further state

Table 2: Simulated Price Spreads

	$t=1$	$t=2$	$t=3$	$t=4$
$j=1$	-0.03	-0.03	0.10	-0.12
$j=2$	-0.19	-0.29	-0.41	-0.44
$j=3$	0.08	0.05	0.13	0.00
$j=4$	-0.25	-0.36	-0.32	-0.23
$j=5$	-0.20	-0.14	0.12	-0.13

Figure 3: Discretization II



transitions are possible, and we have

$$a_t^j = 0, \quad CF_t^j = 0 \quad ACCF_{t+1}^j(q_t^j + a_t^j, S_{t+1}^j) = 0.$$

Then, we step back to $t = 3$ and there exemplary to $q_{n=3,t=3}$ ($q_{n=4,t=3}$ is not possible). Here, both $a_t = DCQ_t$ and $a_t = 0$ are possible – with a state transition $q_{n=3,t=3} \Rightarrow q_{n=4,t=4}$ or $q_{n=3,t=4} \Rightarrow q_{n=4,t=4}$. In order to choose between both actions, we use the decision function $\Pi_t^j(S_t^j, q_t^j + a_t^j)$. From $ACCF_{t=4}^j(q_{t=3}^j + a_{t=3}^j, S_{t=4}^j) = 0$ follows $\hat{E}_{t=4}^j[S_{t=3}^j, q_{t=3}^j + a_{t=3}] = 0$. Consequently, $a_{t=3} = DCQ_t$, if $S_{t=3}^j > 0$, otherwise we don't exercise the contract, i.e. $a_{t=3} = 0$. Assume that we are in state $q_{n=2,t=2}^j$, were we could now choose between $a_t = DCQ_t$ or $a_t = 0$. Using Table 2 we see that for $q_{n=3,t=3}^j$ and $q_{n=2,t=3}^j$:

$$A\vec{CC}F_{t=2}((q_{n=2,t=2} + a_{t=2}), S_{t=3}) = (0.10, \;\; 0, \;\; 0.13, \;\; 0, \;\; 0)^T.$$

We apply Eq.(3), i.e. we perform a least squares regression, using a power polynomial of degree $K = 3$ which includes a constant term:

$$min \stackrel{!}{=} \sum_{j=1}^{5} \left(\begin{pmatrix} 0.10 \\ 0 \\ 0.13 \\ 0 \\ 0 \end{pmatrix} - \sum_{k=1}^{4} \beta_k \cdot \begin{pmatrix} -0.03 \\ -0.29 \\ 0.05 \\ -0.36 \\ -0.14 \end{pmatrix}^{k-1} \right)^2$$

with $(\beta_1, \beta_2, \beta_3, \beta_4) = (0.10, 0.70, 0.31, -2.55)$. The result is:

$$\hat{E}_{t=3}[S_{t=2}, q_{t=2} + a_{t=2}] = (0.082, \ \ -0.013, \ \ 0.138, \ \ 0.006, \ \ 0.016)^T,$$

which is inserted into the decision function for $a_{t=2} = DCQ$:

$$\Pi_{t=2}(S_{t=2}, q_{t=2} + a_{t=2}) := \arg\max_{a_t \in A_t} \left\{ \begin{pmatrix} a = 0 \\ a = 0 \\ a = 1 \\ a = 0 \\ a = 0 \end{pmatrix} \cdot \begin{pmatrix} -0.03 \\ -0.29 \\ 0.05 \\ -0.36 \\ -0.14 \end{pmatrix} + \begin{pmatrix} 0.082 \\ -0,013 \\ 0.138 \\ 0.006 \\ 0.016 \end{pmatrix} \right\}.$$

As a result, only in $j = 3$ is $a = DCQ$. The path-wise decisions are now used for calculating $ACCF^j_{t=2}(q^j_{t=2} + a^j_{t=2}, S^j_{t=2})$:

$$ACCF_{t=2}(q_{t=2} + a_{t=2}, S_{t=2}) = (0.10, \ \ 0, \ \ 0.19, \ \ 0, \ \ 0)^T.$$

This process is executed for the states in the state grid until state $q_{n=1,t=1}$ is reached. In Table 6 the regression parameters and in Table 7 the cumulative cash flows are shown. Finally, the estimated regression parameters and the decision function are used in the forward algorithm. The sample path in Table 8 shows the resulting decision trajectory. The decision matrix is displayed in Table 9.

4 Case Study

Using a case study we now compare LSMC to the much simpler myopic operation, which doesn't consider future price developments. In Section 4.1 we explain the setup before turning to the data in Section 4.2. Results are given in Section 4.3.

4.1 Setup of the Case Study

We compute swing values for two different contract periods: The gas year 2013/2014 with a calibration period of 2011/10/01 − 2013/09/30 and the gas year 2014/2015 with a calibration period of 2012/10/01 − 2014/09/30. Thereby, our study is centered around a base case. Its parametrization and the alternatives (including different levels of contract flexibility) are summarized in Table 3. In the base case, only one set of price simulations is used for both the forward and backward run of the LSMC algorithm. To analyze model sensitivity, we alternatively test different sets of price simulations for both parts. For the corresponding myopic strategy, as it requires only one set of simulations, we use those of the forward run in order to have comparable results (see Table 4). A backtest against realized prices is done as well in order to verify our findings.

Table 3: Parameters of the Base Case

Option type	Call
Contract price	Month ahead index
DCQ	1 MWh/d
Ed	50d; 100d; 150d (base); 200d
ACQ	Ed x DCQ
ToP	75%× ACQ (base); 100%× ACQ
Contract life time	gas year (only business days)

More levels of constraints, e.g. quarterly or seasonal boundaries, are possible as well. However, we refrain from this step due to the dilemma of dimensions, which describes the fact that by increasing the number of constraints we would increase the dimensionality of the solution space, which, in turn, means significantly higher computational effort (see Bellman & Dreyfus, 1962).

Table 4: Cases

Case	ToP	Ed	Same simulations (forward/backward)
1	**75%**	**150**	**yes**
2	75%	150	no
3	100%	50	yes
4	100%	100	yes
5	100%	150	yes
6	100%	200	yes

The base case is indicated by bold letters

4.2 Data Section

We use day ahead (DA) and month ahead (MA) prices from the TTF hub as published on the PEGAS website[5]. Day ahead means a (physical) delivery of a certain amount of gas – priced and measured in EUR/MWh – to be delivered on the next working day of the current trading day. The MA denotes the same but for the month that follows the current month[6]. Weekends are not considered. Both time series range from January 1st, 2008, to April 4th, 2016, and are depicted in Figure 4. The spikes in Feb 2012 and the weather-related spiky period in March 2013 are clearly visible in the spot prices. Apart from that we see the mean reverting behavior of the DA prices, whereby the MA serves as a long-term price level. From mid 2015 on the influence of the oil price decline is visible as gas prices – also due to a more than sufficient supply situation and warm winters – started into a continuous decline.

Table 5 gives some descriptive statistics for both calibration windows. In the first period, for example, with an estimated κ of 0.071, it takes roughly 9 days for the DA price to cover half the distance back to the MA price. As expected, DA prices show a distinctly higher volatility than MA prices. Interestingly, from Period 1 to 2, long-term volatility increased while the spot volatility dropped from about 40% to 32%. Correlation between long- and short-term prices varies also over time.

[5]As the TTF is the major, hence most liquid, Contintental European trading hub, we assume that the classic requirements like market completeness, for example, are given.

[6]If the next business day that follows today is already in the next month, then the next month is covered by another product (the Balance of the Month) and the month ahead denotes the month after the next one. For a more detailed instruction please refer to www.powernext.com.

Figure 4: TTF Day Ahead and TTF Front Month Prices

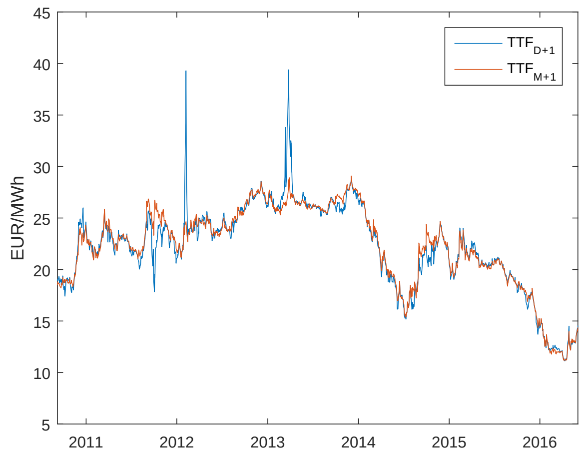


Table 5: Descriptive Statistics for both calibration windows

	Calibration window	κ	ρ	σ_{DA}	σ_{MA}
1	2011/10/01 – 2013/09/30	0.071	0.455	41.80%	21.52%
2	2012/10/01 – 2014/09/30	0.041	0.720	31.73%	22.01%

Volatilties are annualized (250 trading days)

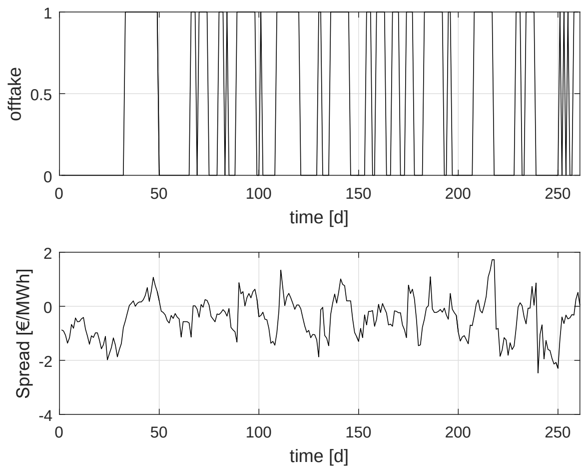
4.3 Results

Figure 5 exemplary shows the offtakes in the backtest for the first period compared to the development of the price spreads. We see that at the end of the period, the algorithm exercises more often in order to guarantee the required minimum number of offtakes, which is obviously not optimal. However, we can also see that extraordinarily big spreads are utilized as well.

In Table 10 we summarize the quantiles as well as backtest results for all analyzed cases. Thereby we prefer the 50% quantile, i.e the median, over the mean as the resulting distributions are asymmetric[7]. In general, the performance of the LSMC strategy prevails, however, there are parameter combinations where a myopic strategy yields higher payoffs,

[7]Spot price innovations are log-normally distributed.

Figure 5: Operation of a Swing Option



Parameters: $Ed = 150,\ ToP = 75\%,\ T = 261$

especially taking other costs into account. Our main results are:

1. Simulation results differ between Period 1 **and** 2 indicating that the price process changed. We also see that using different simulations for the forward and backward run of the LSMC algorithm has no significant impact. Interestingly, for Cases 1 and 2 and for both periods, the median values of a myopic strategy and LSMC are almost identical, but LSMC's backtest results in Period 1 are clearly better. Differences are largest for Cases 3 and 4 where the contract has the highest flexibility. Notably, in Case 3 in Period 1, the myopic approach performs significantly better than LSMC. Overall, it is remarkable that a strategy so simple as a myopic approach produces fairly good results in most cases.

2. Bearish times are bad times for LSMC, as (short-term) trends are not considered in the price model. They pose a threat to LSMC results and favor a myopic strategy, simply because the latter one would execute early than the other. Short-lived bullish trends, again, favor the LSMC strategy where time-to-maturity influences the option value as well. The larger a contract's flexibility the larger the effects on the overall values. In Period 1, for example, LSMC produces a by 0.2 EUR/MWh (Case 3) smaller backtest result than using the myopic operation. This is due to the fact that prices show no trend in the calibration period, but are falling in the operation period. LSMC cannot anticipate such developments (without recalibration), hence, it wouldn't adjust operation. On the other hand, in the backtest for Period 2, the LSMC result is about 0.16 EUR/MWh higher than the payoff of a myopic strategy (see Table 10). In Case 3, LSMC prevails with respect to the distribution, displayed in Figures 6 and 7. On average the plus of a LSMC strategy to the myopic approach amounts to 0.86 EUR/MWh (Period 1) or 0.49 EUR/MWh (Period 2). Here we see the effect of considering the time value, i.e. the "value of waiting". Whereas a myopic strategy exercises at the first fifty positive spreads, LSMC might wait.

3. A myopic strategy handles reduced flexibility significantly better than LSMC, albeit, still, LSMC produces better results. Comparing median values in Period 1 for Cases 3 and 4 we see that payoffs of a myopic strategy are the same while LSMC results drop from 2.18 EUR/MWh to 1.50 EUR/MWh. The effects are similar for Period 2 as well as for other quantiles. Backtest results differ, though. Here, the myopic payoff in Period 1 drops from 0.36 EUR/MWh to barely 0.07 EUR/MWh whereas LSMC remains on a stable 0.17 EUR/MWh (from 0.16 EUR/MWh). The same tendency can be seen when comparing Cases 4 and 5. In Period 1, the LSMC strategy produces better median results, with a difference of 0.18 EUR/MWh in Case 4 and 0.02 EUR/MWh in Case 5. Eventually, for the lowest assumed flexibility (Case 6) LSMC performs significantly better than a myopic approach in Period 1 (on average 0.16 EUR/MWh). For Period 2, overall results are the similar.

4. Using LSMC makes sense above a certain level of flexibility or contract size in order to compensate fixed costs e.g. for software licences or quantiative staff, which would not appear on the payroll in a myopic world.

5 Conclusion

In this paper we discuss two different operational strategies for natural gas swing options, namely LSMC and a myopic approach. Thereby we focus on the aspects of usability and financial advantages. Our results are based on Monte Carlo simulation based on a widely-used price model, which assumes that short-term spot prices are mean reverting around the price of the front month contract. For the sake of simplicity, volatility is left constant. Contract-wise we consider different flexibility configurations for two different gas years.

We see that the higher a swing option's flexibility the higher the (positive) difference between the average LSMC payoff and the average myopic result. For testing the robustness of both strategies we compute backtest cash flows for both analyzed periods. Considering that these are just two exemplary results we identify a few indications when to stay with a myopic strategy and when to choose a more advanced LSMC algorithm: First, contract size should be above a certain level to cover the additional costs of LSMC (compared to a myopic approach). Second, LSMC's performance clearly depends on the contract's flexibility. A certain degree of optionality is required to financially justify the application of such a more advanced method. Third, if prices show no trend or are bullish in the calibration period, but are expected to be bearish in the operation period, myopic operation seems more robust simply because the option is exercised earlier. On the other side, if prices are bearish in the calibration period and bullish or showing no trend in the operation period (backtest period), LSMC prevails.

LSMC performance clearly relies on a similar price behavior in both calibration and operation period. To cope with this fact we recommend a regular recalibration of the price model also during the operation period. It might also be beneficial to combine the myopic approach with LSMC to cope with appearing trends. To find suitable market signs for switching between the both strategies and the resulting improvement, further research is required.

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A Tables and Graphs

Table 6: Regression Parameters

State		Regression parameters			
$n = 4$	β_4				0.00
	β_3				0.00
	β_3				0.00
	β_2				0.00
	β_1				0.00
$n = 3$	β_4		-10.55	-2.55	0.00
	β_3		-2.41	0.31	0.00
	β_2		0.51	0.70	0.00
	β_1		0.11	0.10	0.00
$n = 2$	β_4		-5.14	-2.55	0.00
	β_3		0.15	0.31	0.00
	β_2		0.84	0.70	0.00
	β_1		0.12	0.10	0.00
$n = 1$	β_4		-15.24	-33.77	
	β_3		-5.92	-13.73	
	β_2		1.25	0.69	
	β_1		0.13	0.14	
Time		$t = 1$	$t = 2$	$t = 3$	$t = 4$

Table 7: Cumulative Cash Flows

Sim		$t=1$	$t=2$	$t=3$	$t=4$
$n=4$	$j=1$				0.00
	$j=2$				0.00
	$j=3$				0.00
	$j=4$				0.00
	$j=5$				0.00
$n=3$	$j=1$			0.10	0.00
	$j=2$			0.00	0.00
	$j=3$			0.13	0.00
	$j=4$			0.00	0.00
	$j=5$			0.00	0.00
$n=2$	$j=1$		0.10	0.10	0.00
	$j=2$		0.00	0.00	0.00
	$j=3$		0.19	0.13	0.00
	$j=4$		0.00	0.00	0.00
	$j=5$		0.00	0.00	0.00
$n=1$	$j=1$	0.10	0.10	0.10	
	$j=2$	-0.19	-0.29	-0.41	
	$j=3$	0.27	0.19	0.13	
	$j=4$	-0.25	-0.32	-0.32	
	$j=5$	-0.20	-0.14	-0.12	

Table 8: Decisions for the Sample Path

Spread (EUR/Mwh)	-0.10	0.02	0.60	0.00
Decision	0	0	1	0

Table 9: Decision Matrix

Time (t)	1				2				3				4			
State(n)	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
$j = 1$	0	-	-	-	0	0	-	-	1	1	1	-	-	0	0	0
$j = 2$	1	-	-	-	1	0	-	-	1	0	0	-	-	0	0	0
$j = 3$	1	-	-	-	1	1	-	-	1	1	1	-	-	0	0	0
$j = 4$	1	-	-	-	0	0	-	-	1	0	0	-	-	0	0	0
$j = 5$	1	-	-	-	1	0	-	-	1	0	0	-	-	0	0	0

Table 10: Results

Period 1												
	Myopic strategy [EUR/MWh]						LSMC strategy [EUR/MWh]					
	Case						Case					
Quantile	1	2	3	4	5	6	1	2	3	4	5	6
0.05	0.18	0.22	0.78	0.48	−0.27	−0.58	0.36	0.35	0.83	0.43	0.06	−0.32
0.25	0.79	0.83	1.05	1.09	0.48	−0.01	0.81	0.81	1.67	1.09	0.63	0.22
0.50	1.14	1.18	1.32	1.32	0.99	0.43	1.14	1.14	2.18	1.50	1.01	0.59
0.75	1.49	1.50	1.65	1.63	1.45	0.88	1.52	1.53	2.63	1.91	1.43	0.98
0.95	2.00	2.01	2.33	2.16	1.97	1.56	2.09	2.09	3.25	2.50	1.98	1.54
Backtest	−0.05	−0.05	0.36	0.07	−0.20	−0.33	0.16	0.17	0.16	0.17	0.03	−0.15

Period 2												
	Myopic strategy [EUR/MWh]						LSMC strategy [EUR/MWh]					
	Case						Case					
Quantile	1	2	3	4	5	6	1	2	3	4	5	6
0.05	−0.01	−0.01	0.44	0.11	−0.28	−0.48	0.09	0.09	0.32	0.11	−0.12	−0.30
0.25	0.45	0.48	0.61	0.59	0.23	−0.06	0.46	0.46	0.92	0.60	0.33	0.09
0.50	0.69	0.70	0.78	0.80	0.61	0.25	0.69	0.69	1.27	0.89	0.62	0.36
0.75	0.94	0.93	1.01	1.01	0.91	0.57	0.98	0.97	1.62	1.21	0.90	0.62
0.95	1.32	1.29	1.47	1.37	1.30	1.06	1.35	1.35	2.01	1.60	1.27	0.99
Backtest	0.48	0.48	0.79	0.57	0.45	0.24	0.47	0.47	0.95	0.67	0.43	0.24

Figure 6: Distribution of Results for the Myopic Approach and LSMC in Period 1

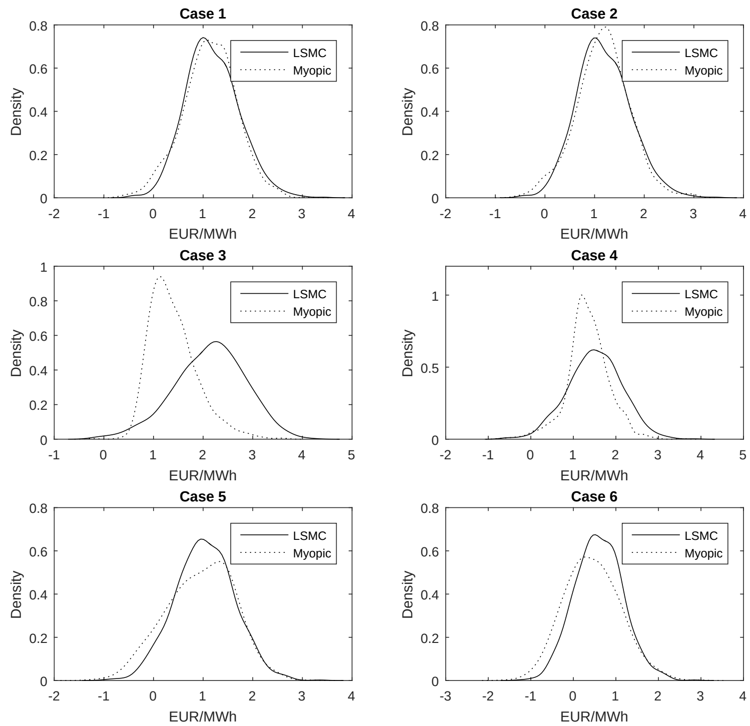

Figure 7: Distribution of Results for the Myopic Approach and LSMC in Period 2