Wage Cyclicalities and Labor Market Dynamics at the Establishment Level: Theory and Evidence

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Abstract

This paper analyzes the effects of different wage cyclicalities on labor market flow dynamics at the establishment level. We derive a model that allows for heterogeneous wage cyclicalities across firms over the business cycle and confront the theoretical results with the new AWFP dataset, which comprises the entire universe of German establishments. In line with theory, establishments with more procyclical wage movements over the business cycle have a more countercyclical hires rate and employment behavior. This result is robust when we look at certain sectors and states. Wage cyclicalities do not only have the expected qualitative impact on stocks and flows, but the quantitative responses are also in line with the proposed model. More generally, our empirical results provide support for theories that lead to an effect of wage rigidities on labor market flow dynamics.

JEL classification: E24, E32, J64.

Keywords: Labor Market Flows, Wages, Administrative Data, Establishment, Matches

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1 Introduction

In labor markets models with search frictions, matched workers and firms face a bilateral monopoly. Workers and firms are better off than without the match, which allows for wage rigidities that are bilaterally efficient, as long as the wage is in between the respective reservation wages. Thus, these models do not run afoul of the Barro Critique (1977). The search and matching literature proposes different types of wage rigidities (e.g. Hall 2005, Hall and Milgrom 2008), which bring the model outcomes closer to various features of the data. When wages become less volatile over the business cycle due to real wage rigidity, the present value of a match moves by more over the cycle and job creation as well as (un)employment become more volatile.

There is a growing empirical literature on the question how rigid wages are over the business cycle (e.g. Bauer and Lochner, 2016, Haefke et al. 2013, Gertler et al. 2016, Martins et al., 2012, Carneiro et al. 2012, Stüber 2016). However — to the best of our knowledge — there is not a single paper that analyzes whether firm-specific differences in wage rigidities actually affect how establishments hire and fire over the business cycle. If wage rigidities should be a solution for the Shimer (2005) puzzle, it is not only important to find some degree of wage rigidity in the data. It is also important that different wage cyclicalities actually have an effect on the hiring behavior of firms in the data.

This issue was not analyzed so far because aggregate time series data are largely uninformative and suitable microeconomic datasets have not been available. Our paper fills this gap by using the new Administrative Worker and Labor Market Flow Panel (AWFP) dataset, which aggregates German administrative data to the establishment level for the years 1975–2014 (Seth and Stüber 2016). The dataset has detailed labor market flow, stock and wage information at the establishment level. It was designed to answer theory based questions, as the one in this paper.

To set the stage, we show some stylized facts from the AWFP dataset. Real wages comove negatively with aggregate unemployment. This is in line with the previous literature on wage cyclicalities (e.g. Stüber 2016), which analyzes wage cyclicalities at the worker level. We show that real wage growth over time is very heterogeneous across establishments. Despite this heterogeneity in real wage growth, about 90 percent of establishments with more than 10 employees hire in any given year.

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1 According to the Barro Critique, a wage rigidity is bilaterally inefficient in a neoclassical demand-supply framework because both parties would be better off without this rigidity, i.e. there is money left on the table.

2 In principal, rigid real wages in the data could simply represent the desire of risk averse workers to have smooth income flows, without affecting labor market dynamics by much.
Against this background, we propose a model with heterogeneous wage cyclicalities across firms. We use the simulated model to develop two wage cyclicality measures and two employment and hires rate cyclicality measures. Our model provides the interesting insight that standard deviation based measures or correlation-based measures would not be suitable proxies for the analysis of the effect of wage rigidities on labor market dynamics.

We estimate the effects of firm-specific wage cyclicalities on firm-specific hires rate and employment cyclicalities based on the simulated model. This establishes a reference point for the estimations based on the AWFP dataset. Interestingly, our paper finds that different wage cyclicalities at the establishment level have the theoretically predicted qualitative effects. Establishment whose wage per full-time worker comoves less with the business cycle exhibit a stronger comovement of their hiring activity and employment stock with the business cycle. Furthermore, a weaker comovement of the wage with the business cycle leads to a stronger comovement of separations with the cycle in most subperiods.

The estimation results are in a similar order of magnitude as the predictions by theory. This is both true for the simulated model and an analytical steady state version (Appendix 8.3). Thus, wage rigidities do not only have the predicted qualitative effects, but the quantitative order of magnitude is also correct.

Note that we look at the effects of different wage cyclicalities through the lens of a model with random search and labor market flows. However, we consider our paper as a starting point that establishes stylized facts, which are relevant for various other streams of the literature. These results can for example be compared to a directed search framework (e.g. Julien et al. 2009) or to the role of wage rigidities in medium scale DSGE models (e.g. Christiano et al. 2005 or Smets and Wouters 2007), where wages are adjusted in staggered manner. Thus, our results provide an interesting reference point. Although we define our wage cyclicity measures based on our model, the estimations are not structural but reduced form. This has the advantage that the estimated results can easily be compared to other model frameworks.

The rest of the paper proceeds as follows. Section 2 provides a description of the AWFP dataset, shows how wages evolve at the establishment level and provides stylized facts on the cross-sectional dispersion of real wage growth. Section 3 derives a model of heterogeneous wage cyclicalities across firms, which is able to match some key facts from the data. In section 4, we calibrate our model, show simulation results, interpret these results and develop model-based measures. Section 5 applies these model-based measures to the AWFP dataset and interprets results. Section 6 concludes.

3The latter are frictionless models where firms adjust along the intensive margin. However, it is well known that in aggregate the extensive margin dominates over the business cycle.
2 Wage Cyclicality over Time Series and in the Cross-Section

This section proceeds in three steps. First, we provide a brief description of the employed AWFP data. Second, we estimate how strongly wages at the establishment level comove with aggregate unemployment. Third, we show that there is a substantial cross-sectional dispersion of wage growth in each period of time.

2.1 Dataset and flow definition

The Administrative Worker and Labor Market Flow Panel (AWFP) aggregates German administrative wage, labor market flow and stock information at the establishment level for the years 1975–2014. The underlying administrative micro data source is mainly the Employment History (Beschäftigtenhistorik, BeH) of the Institute for Employment Research (IAB). Before aggregating the data to the establishment level, several adjustments and imputations were conducted at the micro data. For example, the administrative wages are censored at the contribution assessment ceiling of the social security system. Therefore censored wages were imputed. For more detailed information on the AWFP see Appendix 8.1 or Seth and Stüber (2016).

We use the AWFP at the annual frequency and restrict the data to West German establishments (excluding Berlin) and the years 1979–2014. For coherency, we focus on wages and flows for full-time workers.\footnote{More precisely we focus on “regular workers” according to the definition used in the AWFP (see Appendix 8.1).}

Following Davis et al. (2006), we define the hires rate \((hr_{it})\) as new full-time hires in an establishment \(i\) divided by the average number of full-time workers in year \(t\) and \(t − 1\). The separations rate \((sr_{it})\) is defined equivalently.\footnote{Stocks and flows are calculated using the “end-of-period” definition (see Appendix 8.1).}

2.2 Regression Results

There is a growing empirical literature on the question how cyclical wages move over the business cycle (e.g. Haefke et al. 2013, Gertler et al. 2016, Martins et al., 2012, Carneiro et al. 2012, Stüber 2016). Typically, worker-specific wages are regressed on an aggregate business cycle indicator (such as GDP, productivity or unemployment) and it is checked whether the wages of new matches and all matches show different dynamics.
To set the stage, we run a comparable regressions based on our establishment dataset. So far, this analysis has only been performed at the worker level for Germany (Stüber 2016). Thus, we establish new stylized facts for the wage cyclicality at the establishment level.

To analyze the wage cyclicality over the business cycle within establishments, we estimate the following two regressions:

\[
\ln w_{it} = \alpha_0 + \alpha_1 u_t + \alpha_2 t + \alpha_3 t^2 + \alpha_4 C_{it} + \mu_i + \varepsilon_{it}, \quad (1)
\]

\[
\ln w_{it}^N = \alpha_0 + \alpha_1 u_t + \alpha_2 t + \alpha_3 t^2 + \alpha_4 C_{it} + \mu_i + \varepsilon_{it}, \quad (2)
\]

where \( w_{it} \) is the real average daily wage of all matches at establishment \( i \) in year \( t \) and \( w_{it}^N \) is the real daily wage of all new matches at establishment \( i \) in year \( t \). \( u_t \) is the aggregate unemployment rate for West Germany. In addition, we add a linear and a quadratic time trend as well as an establishment fixed effect to control for time invariant heterogeneity. \( C \) contains a vector of control variables, namely the share of low skilled and medium-skilled workers at the establishment level and dummies for sectors and states.

Note that we have decided to run our regressions on the annual level. The reason is the underlying nature of the data. Wages in our dataset are calculated based on employment spells. If an employment spell lasts for the entire year, then we would not obtain any time variation at the quarterly level in this given year (but simply divide the annual wage by four). Thus, time variation on the quarterly level only comes from shorter employment spell. Due to long-lasting employment relationships in Germany, we have decided for the annual level.\(^6\)

Table 1 shows the estimated coefficients for unemployment. For calculating the error terms, we cluster at the annual level.

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>( w_{it} )</th>
<th>( w_{it}^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient (t-values): ( u_t )</td>
<td>-1.16 (-17.17)</td>
<td>-1.77 (-42.52)</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Education Share Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sectoral Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>State Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.84</td>
<td>0.67</td>
</tr>
<tr>
<td>Observations</td>
<td>45,328,084</td>
<td>24,723,379</td>
</tr>
</tbody>
</table>

\(^6\)However, we ran some of the regressions on the quarterly level and the key results were not affected.
Our regression results show that wages at the establishment level comove with the expected negative sign with unemployment. In times of lower aggregate unemployment, wages increase. In our unweighted regression, the wages for new matches at the establishment level are about 50% more cyclical than wages for all workers.

In order to make our results comparable to the existing literature on wage cyclicalities, which were estimated at the worker-level,\(^7\) we run a regression where we weight with the size of the establishment (i.e. the number of full-time workers in a particular establishment). While the estimated coefficient for all workers remains stable, the estimated coefficient for new matches drops and is in a similar order of magnitude as the one for all workers (see table 2). This shows that the strong cyclicity in table 1 is driven by small establishments.

Table 2: Wage Regressions Weighted

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>(w_{it})</th>
<th>(w^N_{it})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient (t-values): (u_t)</td>
<td>-1.18 (-115.32)</td>
<td>-1.20 (-77.04)</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Education Share Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sectoral Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>State Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.93</td>
<td>0.80</td>
</tr>
<tr>
<td>Observations</td>
<td>45,328,084</td>
<td>24,723,379</td>
</tr>
</tbody>
</table>

How do our results compare to the existing literature on wage cyclicalities for Germany? The estimated coefficients in the weighted regressions are well in line with Stüber (2016) who estimates the sensitivity of log wages to unemployment at the worker (and not the establishment) level. He estimates coefficients of -1.26 for all workers and -1.33 for new workers. As in our weighted regression, the incremental effect between these groups is small. However, Stüber’s (2016) coefficients are somewhat larger than the ones in our weighted regressions. This is in line with Solon et al. (1994) who argue that using aggregated time series data instead of longitudinal microdata leads to an underestimation of wage cyclicality due to a composition bias. Although they compare microdata to highly aggregated data (e.g. on the national level), the argument also applies to our analysis, where we use numbers that are aggregated to the establishment level.

Overall, there is a substantial negative comovement of real wages with aggregate unemployment. The search and matching literature has proposed various wage rigidity mechanisms (e.g. Hall 2005 and Hall and Milgrom 2008) in order to solve the Shimer (2005)

\(^7\)In the existing literature on wage cyclicalities, every wage realization per worker is one observation, while in our analysis based on the AWFP data every wage realization per establishment is one observation.
puzzle. Our paper does not discuss whether real wages are sufficiently rigid in order to generate strong amplification in search and matching models. This type of exercise has been done in other papers, such as Haefke et al. (2013) and Gertler et al. (2015). Instead, we use our establishment dataset to shed light on the cross-sectional dispersion of wage cyclical and the effects of different wage cyclicalities on establishment-specific labor market flow dynamics.

2.3 Cross-Sectional Dispersion

Figure 1 shows the real wage growth (for full-time workers) at the establishment level in Germany from 1979-2014 for the 10th, 25th, 50th, 75th and 90th percentile. First, the picture illustrates that the real wage growth in all these percentile is positively correlated. Second, the figure shows that the annual real wage growth is very heterogeneous in the cross-section. Interestingly, despite this heterogeneity in real wage growth across establishment, more than 66.5% of establishments hire in any given year. The number is much larger when we exclude very small establishments with just a few employees, which only hire once in a while. If we constrain to establishments with more than 10 or more than 50 employees, the share increases to 89.9 and 98.7 percent.

Would it be possible in a standard search and matching model of the Mortensen and
Pissarides (1994) type to have heterogeneous wage cyclicalities across firms, while almost all firms (above a certain size) hire in every period? Obviously, this is possible if firms with different wage cyclicalities act in different labor market segments, such as for example in Barnichon and Figura (2015). However, we also find substantial differences in wage cyclicalities once we move to the sectoral or regional level (see Appendix 8.4).

Can a standard search and matching model explain this in a given labor market segment? Imagine that establishments with different wage cyclicalities act in the same labor market segment and that they are hit by the same aggregate shock. Imagine further that the economy moves into a boom and firm A’s wage increases by more than firm B’s wage. In this case, firm B would face a higher expected present value than firm A. Given that the market tightness, the worker-finding rate and thereby the hiring costs are a market outcome, only firm B would be posting vacancies and hire, while firm A would shut down its vacancy posting and hiring activity.\(^8\) Thus, the standard random search and matching model could not yield the outcome we find in the data.

### 3 The Model

We propose a model that helps us to interpret the patterns in the data and that allows us to define relevant empirical measures. Obviously, there may be other model mechanisms, which yield similar results.\(^9\) Given that we establish our stylized facts for the effects of wage rigidities on labor market flows based on reduced-form measures (and not a structural estimation), we provide an input for future research to analyze the (dis-)accordance of different model classes with these facts.

In order to be in line with the facts from section 2, we assume that each firm obtains an undirected flow of applicants. Once workers and firms get in contact with one another, each worker-firm pairs draws from the same idiosyncratic training cost distribution. Firms choose an optimal cutoff point and thereby decide about the fraction of workers they want to select (labor selection). The cutoff point and the hiring rate depend on the wage cyclicality. Hiring will be different (but will not necessarily be shut down) if the wage cyclicality is different from the average in the economy.

\(^8\)The standard search and matching’s job-creation condition is \(\frac{\kappa}{q(\theta_t)} = a_t - w_t + E_t \delta (1 - \phi) \frac{\kappa}{q(\theta_{t+1})}\). Given that \(\frac{\kappa}{q(\theta_t)}\) is market-determined, only the most profitable firms will hire. Thus, different wage cyclicalities and joint hiring cannot coexist.

\(^9\)One possible alternative would be to assume decreasing returns in a search and matching model. With less hiring, the marginal product increases and thereby firms with different wage cyclicalities can hire at the same time. Given that Hirsch et al. (2014) find constant returns for a substantial fraction of establishments in Germany (in particular larger ones), we propose a different solution.
Note that our model setup is similar to Chugh and Merkl (2016) and Kohlbrecher et al. (2016). The key difference is that we allow for heterogeneous wage cyclicalities across establishments. Kohlbrecher et al. (2016) show that a model setup with labor selection mechanism generates an equilibrium Cobb-Douglas constant returns comovement between matches on the one hand and unemployment and vacancies on the other hand. In our paper, we abstract from vacancies because they are not important for our argument and they are not included in the AWFP dataset (where we only have stocks, flows and wages).

3.1 Heterogeneous Groups

In our model economy, there is a continuum of firms that are completely homogenous, except for their wage formation over the business cycle. Workers can either be unemployed (searching) or employed. Employed workers are separated with an exogenous probability $\phi$. In each period, unemployed workers send their application to one random firm (i.e. search is completely undirected). Thus, each firm will receive a certain fraction of searching workers in the economy. Once workers and firms get in contact with one another, the respective worker-firm pair draws an idiosyncratic match-specific training cost shock (or more generally a match-specific productivity shock) from a stable density function $f(\varepsilon_t)$. Firms of type $i$ will only hire workers below a certain threshold $\varepsilon_{it} \leq \tilde{\varepsilon}_{it}$, i.e. only workers with more favorable characteristics will be selected. In the period after the match has taken place, all worker-firm pairs are homogenous (i.e. there are no more idiosyncratic shocks).

The present value of firm of type $i$ is

$$a_t - \varepsilon_{ijt} - h - w_{ijt} + E_t \delta (1 - \phi) J_{it+1},$$

where $a_t$ is the aggregate productivity, $\varepsilon_{ijt}$ is the match-specific idiosyncratic shocks ($j$ denotes the match), $h$ are linear hiring costs that are the same for all firms, $w_{ijt}$ is the wage at firm $i$ for match $j$, $\delta$ is the discount factor, $\phi$ is the exogenous separation rate and $J$ is the future value of a job. Existing workers-firm pairs have the following recursive representation:

$$J_{it} = a_t - w_{it} + E_t \delta (1 - \phi) J_{it+1}. \tag{4}$$

Note that we assumed that the workers are homogenous for existing worker-firm pairs. Thus, there is no idiosyncratic shock term.

Each firm chooses an optimal cutoff point for the idiosyncratic match-specific shock:

$$\tilde{\varepsilon}_{it} = a_t - h - w_{ijt} + E_t \delta (1 - \phi) J_{it+1}. \tag{5}$$
A firm of type $i$ will select a certain fraction of applicants, namely:

$$\eta_{it} = \int_{-\infty}^{\tilde{\varepsilon}_{it}} f(\varepsilon_t) \, d\varepsilon_t.$$  \hfill (6)

Given that firms are homogenous (except for the their wage cyclicality), in steady state, all firms have the same selection rate $\eta$. The selection rate over the business cycle will depend on the wage formation mechanism.

The employment dynamics equations of firm $i$ is:

$$n_{it} = (1 - \phi) n_{it-1} + c_{it} \eta_{it},$$  \hfill (7)

where $c_{it}$ are the number of contacts of a particular firm type, which depends on the number of unemployed workers and firms in the economy.

### 3.2 Wage Formation

The only assumed heterogeneity across firm types is the wage formation mechanism. First, we derive the Nash bargaining solution. Second, we assume that only some firms adjust immediately to the Nash bargaining solution, while others adjust with a certain delay.

We assume that the idiosyncratic training costs and hiring costs are sunk at the time of bargaining and production.\(^{10}\) Thus, all worker firm-pairs (independently if new match or not) have the same flow value, namely $J_{it}$. The fall-back option in case of disagreement is 0.

Workers’ flow value in case of a match is

$$W_t = w_t + E_t \delta (1 - \phi) W_{t+1} + E_t \delta \phi U_{t+1}.$$  \hfill (8)

The fall-back option for workers is the value of unemployment.

$$U_t = b + E_t \delta (1 - \eta_{t+1}) U_{t+1} + E_t \delta \eta_{t+1} W_{t+1},$$  \hfill (9)

where $\eta_{t+1}$ is the aggregate probability of making a match in the next period.

Thus, the standard Nash product is

$$\Lambda_t = (W_t - U_t)^{\nu} (J_t)^{1-\nu}. $$  \hfill (10)

Maximization with respect to wages yields the following result:\(^{11}\)

---

\(^{10}\)This is in line with Pissarides (2009). Thus, the wage does not depend on the idiosyncratic component. This assumption is without loss of generality.

\(^{11}\)See Appendix 8.2 for the derivation.
\[ w_t = \nu (a_t + \delta \eta_{t+1} J_{t+1}) + (1 - \nu) b. \]  

(11)

If all firm types followed the Nash bargaining solution, they would all have the same wage cyclicity. In spirit of Blanchard and Gali (2007), we choose a very simple wage rigidity mechanism:

\[ w_{it} = \kappa_i w_t + (1 - \kappa_i) w^{\text{norm}}, \]  

(12)

where \( \kappa_i \in (0, 1] \) is the firm-specific degree of wage rigidity over the business cycle. The wage norm is the steady state value of the Nash bargain \( w^{\text{norm}} = w = \nu (a + \delta \eta J) + (1 - \nu) b \). Thus, all firms have the same wage in steady state. A firm with \( \kappa_i = 1 \) immediately implements the Nash bargaining solution. By contrast, for \( \kappa_i < 1 \), the firm converges to the Nash bargaining solution with a certain delay.

Note that our paper does not provide a theoretical foundation for wage rigidities.\(^{12}\) We take different wage cyclicalities as given and analyze their impact on hiring and employment. The heterogeneity of wage cyclicalities in the model allows us to establish a reference point compared to the data.

### 3.3 Aggregation

In order to establish an equilibrium, we have to aggregate across all firm types. The aggregate employment rate is:

\[ n_t = (1 - \phi) n_{t-1} + s_t \int_i c_{it} \eta_{it}, \]  

(13)

where \( s \) is the number of searching workers and where the number of matches is aggregated over all firm types.

In addition, unemployment workers and employed workers have to add up to 1.

---

\(^{12}\)Firms in Germany have very diverse wage formation mechanisms (see e.g. Hirsch et al. 2014). Some are subject to collective bargaining, others bargain with unions at the firm level and yet others have completely individualized contracts. If firms are subject to a collective agreements, they can always be more generous (but not less). Interestingly, Knoppik and Beissinger (2009) shows that the variation in national downward nominal wage rigidity cannot convincingly be explained by institutional variables. In addition, other institutions such as the workers’ council (legal representation of workers, especially present in larger firms) have an indirect effect on wage formation (although they are not involved directly). The analysis of bargaining institutions on wage cyclicalities and their feedback effects on flows is certainly a topic of interest, but goes beyond the scope of this paper. To analyze this, we would have to link the AWFP dataset to the IAB Establishment Panel (as e.g. Merkl and Stüber 2017) and thereby lose a major share of observations (both in the time dimension and the cross-section).
\[ n_t = 1 - u_t. \] (14)

All workers who look for a job and who are unable to match are defined as unemployed.

\[ u_t = s_t \left( 1 - \int c_i \eta_{it} \right), \] (15)
i.e. those who lost their job exogenously in period \( t \) and those searching workers who did not find a job in the previous period.

We assume that each searching worker gets in contact with one firm in each period.\(^{13}\) Thus, the number of contacts per firm type is

\[ c_{it} = \frac{u_t}{F}, \] (16)

where \( F \) is the number of firms or firm types (depending on the disaggregation level). Note that we are going to choose a discrete number of firm types in our simulation below. The firm type will be our disaggregation level (because all firms of the same type behave in the same way). As we assume that each firm type has the same size, the number of contacts per firm type is the number of searching workers divided by the number of firm types.

4 Model Based Results

4.1 Calibration

In order to analyze the effects of different degrees of wage rigidities at the establishment level and to derive model-based measures, we parametrize and simulate the model. There is a set of parameters that is absolutely standard. We set the discount factor to \( \delta = 0.99 \), given that our simulation will be performed on the quarterly level. In line with the average quarterly flow rates from the \textit{AWFP} dataset, the exogenous quarterly separations rate is set to \( \phi = 0.07 \) (see Bachmann et al. 2016 for quarterly statistics). This also pins down the economy wide hires rate, which must be equal to the separation rate in steady state.

The aggregate productivity is normalized to 1. We assume that productivity is subject to an aggregate shock with a first-order autoregressive process. The aggregate productivity shock is drawn from a normal distribution with mean zero and the standard deviation is normalized to 1. The first-order autocorrelation coefficient is set to 0.8.\(^{14}\) We assume a

\(^{13}\)This is similar to Chugh and Merkl (2016) who show how the model can be extended to multiple applications per period.

\(^{14}\)This number is both in line with the autocorrelation of labor productivity (per employed worker) in Germany from 1979-2014 and the estimated autocorrelation of productivity shocks in Smets and Wouters.
bargaining power of workers $\nu = 0.5$.

The remaining parameters are less standard. We discretize the number of different wage rigidity bins into 5 equally sized groups with $\kappa_i = [1, 0.8, 0.6, 0.4, 0.2]$. A firm with a value $\kappa_i = 1$ follows the Nash bargained wage, while a firm with $\kappa_i = 0.2$ only converges there slowly. The former firm has very flexible (cyclical) wages and the latter firm has very rigid (acyclical) wages.

The two final parameters to be pinned down are the linear hiring costs $h$ and the properties of the idiosyncratic shock distribution. For analytical tractability, we use a logistic distribution\(^{15}\) for the idiosyncratic match-specific distribution with mean zero ($\mu = 0$). We target the standard deviation of residual wages for new hires (0.17) from Kohlbrecher et al. (2016) and the average unemployment rate from 1979-2014 (0.08). In order to hit these two targets, we choose the dispersion parameter of the logistic distribution $z$ and the linear hiring costs $h$. This yields $z = 0.25$ and $h = 0.64$.

### 4.2 Numerical Results and Implications

Figure 2 shows how the five different firm types react under a positive productivity shock in the model simulation.\(^{16}\) Note that firms are inversely ordered according to their wage rigidity. Firm 1 (with $\kappa_1 = 1$) has a completely flexible wage, which moves roughly one to one with aggregate productivity, while firm 5 (with $\kappa_1 = 0.2$) has the most rigid real wage over the business cycle. For better visibility, we only show thirty quarters, although the actual simulation is longer. Given that figure 2 shows an economic upswing, we see an increase of aggregate wages and employment due to a positive aggregate productivity shock (see upper left panel). The different wage dynamics for all firm types is depicted in the lower left panel. By assumption, wages go up by a lot more for type 1 firms than for type 5 firms. All firms have an incentive to hire a larger share of their applicants because the present value of a match increases (i.e. the share of applicants selected goes up for each firm type). However, this increase is less pronounced for the firms with $\kappa = 1$ because a larger part of the increase bilateral surplus goes to workers. Thus, the selection rate $\eta$ increases most for firms of type 5 (with the most rigid wages). Interestingly, this leads to a decline of the firm-specific hires rate (defined as firm-specific matches divided by the firm-specific employment stock) and employment stock for firms of type 1 (see right hand panels). The underlying reason is that the aggregate stock of searching workers declines due to the boom\(^{2003}\).

\(^{15}\)The corresponding probability density function is $f(\varepsilon_t) = \frac{e^{\frac{\varepsilon_t - \mu}{z}}}{z(1 + e^{\frac{\varepsilon_t - \mu}{z}})^2}$.

\(^{16}\)Appendix 8.3 contains analytical derivations based on the model.
in the economy. All firms receive fewer applicants and firms of type 1 increase their selection rate less than all other firms. Thus, firms of type 1 obtain a smaller share of the shrinking stock of searching workers and thereby their hires rate and employment stock declines.

This is an important observation for constructing measures for the effect of wage rigidities on firm-specific employment. In aggregate, search and matching models with larger wage rigidities lead to stronger amplification (i.e. larger volatilities of (un)employment). This is also true in our model for the entire economy.\textsuperscript{17} However, the standard deviation (or more generally any type of volatility measure) would not be suitable for a cross-sectional analysis of the effects of wage rigidities on firm-specific employment. While wage rigidities matter for hires and employment in our model, they do not have a monotonic effect on the standard deviations of firm-specific hires rates and employment stocks. Firms of type 1 and

\textsuperscript{17}When we switch off the wage rigidity and set all firm types to $\kappa = 1$, the aggregate volatility of unemployment drops by roughly one half.
both have a larger standard deviation of employment than firms of type 3. However, their employment stocks move into different directions. This will be discussed in more details in the next section.

4.3 Model Based Regression Results

The model is useful for providing an intuition for the underlying economic mechanisms and in order to develop measures that we can apply to the data. We simulate the model and propose two measures that work well within the context of our model such that we can apply them to our dataset. But before we do so, we reject two other potential measures.

4.3.1 Considerations on the Measures

The right panel of figure 3 plots the standard deviation of wages, hires rates and employment. It is visible that the connection between the standard deviation of wages and hires rates/employment is non-monotonic. The reason is that the standard deviation does not take into account the direction of the movement, which we require for our analysis.

The left panel of figure 3 plots the correlation of firm-specific wages, hires rates and employment in groups 1 to 5 with aggregate employment (left panel). It illustrates that a correlation based measure would not be suitable for the analysis of wage cyclicalities and their effects on the hires rate and employment. The wages in all subgroups in our model have a correlation with aggregate employment of around 0.9. The reason is that wages in all subgroups increase with higher aggregate productivity. The magnitude of the wage movements in these five groups is different. However, the correlation is the same. Thereby, a correlation is not a useful measure for our analysis. We require a measure that takes into account the size of the movement.

Thus, in order to analyze the effects of wage rigidities on hires and employment, we need measures that take the direction and size of the movement into account. In the next two subsections, we propose two suitable measures.

4.3.2 Covariance Based Measures

We start by defining measures that show how the firm-specific wage and the flows/stocks move relative to the aggregate business cycle. We define $\beta_i$ to be the wage cyclicality for

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Due to the nature of the AWFP data, we performed our empirical analysis on the annual level (see section 2 for details). For comparability reasons, we aggregate our quarterly simulation to the annual level and we simulate the same number of periods, namely 144 quarters (36 years). We use the wage at the end of the fourth quarter and we calculate the number of matches at a given firm that still works at the end of the year, i.e. $c_{it}\eta_{it} (1 - \phi)^3 + c_{it+1}\eta_{it+1} (1 - \phi)^2 + c_{it+2}\eta_{it+2} (1 - \phi) + c_{it+3}\eta_{it+3}$.
Figure 3: Connection of standard deviation and correlation based measures in the model. The x-axis denotes the different wage rigidity types as shown in figure 2.

establishment $i$.

$$\beta_i = \frac{cov(\Delta \ln w_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)},$$  \hspace{1cm} (17)

where $cov$ is the covariance between the establishment-specific wage growth and the aggregate employment growth and $var$ is the variance of the aggregate employment growth. $N$ is the economy wide number of workers.\(^{19}\) A positive (negative) coefficient $\beta_i$ indicates a positive (negative) comovement between the firm-specific wage and aggregate employment. The larger $\beta_i$, the more procyclical or the less rigid is the establishment-specific wage.

Equivalently, we define the comovement of the firm-specific employment stock ($n_{it}$) and hires rate ($hr_{it}$):\(^{20}\)

$$\gamma_{ni} = \frac{cov(\Delta \ln n_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)},$$  \hspace{1cm} (18)

\(^{19}\)In the model, we could equally use GDP as measure for the business cycle, which has a strong positive correlation with employment. As discussed in section 4, using aggregate employment has advantages when using the AWFP data.

\(^{20}\)The hires rate is calculated as in the empirical model, namely matches divided by the average employment stock in this and the previous period (see section 4.1).
Figure 4: Illustration of the effects of heterogeneous wage cyclicalities on the hires rate/employment dynamics in the model. The x-axis denotes the different wage rigidity types as shown in figure 2.

\[ \gamma_{hr}^i = \frac{cov(\Delta hr_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)} . \]  \hspace{1cm} (19)

A positive (negative) \( \gamma_{hr}^i \) indicates a positive (negative) comovement between establishment-specific stocks/flows and aggregate employment. The larger \( \gamma_{hr}^i \), the stronger is the comovement. Note that we define \( \gamma_{hr}^i \) based on absolute deviations and not log-deviations. The reason is that the hires rate is already defined as a rate, namely as the number hires divided by the average number of workers.

In contrast to the standard deviation or correlation based measures from the previous subsection, our proposed measures take into account the size and direction of the movements. Figure 4 shows that there is a monotonic negative relationship between \( \beta_i \) and \( \gamma_n^i / \gamma_{hr}^i \).

We also estimate the effects \( \beta_i \) on \( \gamma_{x}^i \):

\[ \gamma_{x}^i = \alpha_0^x + \alpha_1^x \beta_i + \varepsilon_i . \]  \hspace{1cm} (20)

Table 3 shows the regression results. There is a strong negative comovement between \( \gamma_n^i / \gamma_{hr}^i \) and \( \beta_i \). All estimated coefficients are statistically significant at the 1% level. A firm with a more procyclical wage movement shows a more countercyclical employment and hires rate movement.
Table 3: Covariance Based Measures

<table>
<thead>
<tr>
<th>Estimated Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^w_1$</td>
</tr>
<tr>
<td>$\alpha^{hr}_1$</td>
</tr>
</tbody>
</table>

4.3.3 Relative Measures

As an alternative to the covariance-based measure, we define a measure that shows how far the wage growth and employment growth / hires rate change is below/above the average in the economy. As visible in figure 1, firms wages and their job-creation/employment move in inverse order. Thus, we define $w^r_{it}$ as a relative wage measure

$$w^r_{it} = \triangle \ln w_{it} - \frac{\sum_i \triangle \ln w_{it}}{E},$$ (21)

where $E$ is the number of firms and the second term on the right hand side of this equation shows the average wage growth in the economy. Thus, $w^r_{it}$ is the relative wage growth position of firm $i$ relative to all other firms. A positive (negative) number indicates a wage growth above (below) average.

We are interested in the effects of the wage growth rate on the establishment-specific employment and labor market flow dynamics. Thus, we define two measures:

$$n^r_{it} = \triangle \ln n_{it} - \frac{\sum_i \triangle \ln n_{it}}{E},$$ (22)

$$hr^r_{it} = \triangle hr_{it} - \frac{\sum_i \triangle hr_{it}}{E},$$ (23)

which all denote the position of firm-specific employment ($n_{it}$) and hires rate ($hr_{it}$) relative to the mean in the economy.

To test how well these measures work within our model, we estimate the following regression equation based on simulated data:

$$x^r_{it} = \alpha_o + \alpha_1 w^r_{it} + \sum T + \mu_i + \varepsilon_{it},$$ (24)

where $x^r_{it}$ may either be $n^r_{it}$ or $hr^r_{it}$. $\mu_i$ are establishment fixed-effects and $T$ are time dummies.\footnote{The theoretical simulation does not show a notable time trend. We estimate the time trend for comparability to the empirical results. Omitting the time trend would leave the results unaffected.}
Table 4: Relative Measures

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>$n_{it}$</th>
<th>$hr_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient (t-values): $w_{it}$</td>
<td>-0.654 (-2.10)</td>
<td>-1.093 (-6.93)</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.18</td>
<td>0.56</td>
</tr>
<tr>
<td>Observations</td>
<td>180</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 4 shows that an establishment that has a wage growth that is 1 percent above the average is associated with an employment growth that is 0.7 percent below the average and a hires rate that is 1 percentage points below the average. All estimated coefficients are at least significant at the 5 percent level. The model fit in terms of the R squared for the hires rate is very good. The hires rate has the larger R squared because it reacts more quickly to differences in wage cyclicalities, while the employment stock (as stock variable) also includes past movements. Overall, the relative measures are also suitable for our empirical analysis.

5 Empirical Results

This section uses the AWFP dataset. It analyzes how different wage cyclicalities at the establishment level affect the flow and stock cyclicalities, based on the theoretical measures from section 4.

5.1 Regression Results

5.1.1 Covariance Based Measure

We look at the data through the lens of our model and define the equivalent $\beta_i$ measure, which we repeat for convenience and in order to define the employed variables.

$$\beta_i = \frac{cov(\Delta \ln w_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)}, \quad (25)$$

$w_{it}$ are the earnings per full-time worker (i.e. the overall wage income of all full-time workers in a given establishment divided by the number of full-time workers). $N$ is the economywide number of full-time workers. Using $N$ as an indicator for the state of the macroeconomy has the advantage that we can coherently aggregate from our own microeconomic establishment data to different disaggregation levels (e.g. to the sectoral or regional level). Alternatively,

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22 As in the empirical analysis below, the standard errors are clustered per time period.

23 West Germany excluding Berlin.
we have used GDP and labor market tightness at the national level. Given that the results are very similar, we do not show them. However, they are available on request.

We construct three measures for the cyclicality of establishment-specific stocks and labor market flows $\gamma_i^x$, namely:

$$
\gamma_i^n = \frac{cov(\Delta \ln n_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)},
$$

(26)

$$
\gamma_i^{hr} = \frac{cov(\Delta hr_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)},
$$

(27)

$$
\gamma_i^{sr} = \frac{cov(\Delta sr_{it}, \Delta \ln N_t)}{var(\Delta \ln N_t)}.
$$

(28)

As we are interested in the effects of different wage cyclicalities on labor market dynamics, we estimate the following regressions based on these two measures.

$$
\gamma_i^x = \alpha_0^x + \alpha_1^x \beta_i + \varepsilon_i,
$$

(29)

where $\alpha_1$ is the coefficient of interest and $\varepsilon_i$ is the error term. As shown above, our model predicts a negative coefficient for $\alpha_1^n$ and $\alpha_1^{hr}$. An establishment with more cyclical wages (i.e. larger $\beta_i$) is expected to have less procyclical employment and hires rate dynamics (i.e. smaller $\gamma_i^n$ and $\gamma_i^{hr}$).

The estimated coefficients for the entire sample are $\hat{\alpha}_1^n = -0.124$, $\hat{\alpha}_1^{hr} = -0.197$ and $\hat{\alpha}_1^{sr} = 0.026$. Given that our sample goes from 1979 to 2014, the establishment-specific wage cyclicality may change over time. Establishments may for example leave or join the collective bargaining agreements or a different management may lead to different wage formation. In order to check for the robustness of results, we split our sample into four years time windows (starting with 1979–82 and ending with 2011–14). Table 5 shows the results.

---

24 For the empirical analysis we drop extreme outliers, i.e. observations with $\beta_i$ or $\gamma_i^x$ below / above the 1st / 99th percentile of the corresponding distribution.
Interestingly, the sign of the estimated coefficients $\alpha_n^1$ and $\alpha_{hr}^1$ are always in line with economic theory, although the order of magnitude varies somewhat. Firms with more procyclical wages over the business cycle have more countercyclical employment dynamics and hires rates. To be more illustrative: Imagine an establishment moves into a recession (a period of declining aggregate employment) and wages do not decline. This establishment would be forced to reduce the hires rate and employment by more than establishments where the wage declines in the recession.

The estimated coefficient for separations also has the expected negative sign for the entire sample period. However, at first sight it seems surprising that the estimated coefficient $\alpha_{sr}^1$ is negative for some time windows because it suggests that establishments where wages do not decline in recessions face a reduction of separations. We conjecture that this pattern in the data may be driven by worker initiated quits. When wages do not increase by much in a boom, workers may either quit into unemployment (such as in Brown et al. 2015) or move to a different employer (on-the-job search). Note that we are unable to differentiate between separations that are initiated by firms (i.e. fires) and that are initiated by employees (i.e. quits) in the AWFP data. Our results suggest that the employee initiated separations dominate in some episodes. In a recession, employees reduce their quit rates in establishments where wage cuts are less severe.

Thus, overall the results are well in line with economic theory. However, the covariance-based measures have several shortcomings. First, given that we have one cross-sectional measure for each establishment, it is impossible to control for time-variant and time-invariant heterogeneities. Second, the estimated coefficients are difficult to interpret in terms of their

---

25 Results are robust if we run the analyses separately for federal states or industry sectors. Results available upon request.
magnitude. Therefore, we test the robustness of results with a second measure.

5.1.2 Relative Measures

Equivalently to theory, we define the following four measures, which we repeat for convenience:

\[
w_{it}^r = \triangle \ln w_{it} - \frac{\sum_i \triangle \ln w_{it}}{E},
\]

\[
n_{it}^r = \triangle \ln n_{it} - \frac{\sum_i \triangle \ln n_{it}}{E},
\]

\[
h_{it}^r = \triangle h_{it} - \frac{\sum_i \triangle h_{it}}{E},
\]

\[
s_{it}^r = \triangle s_{it} - \frac{\sum_i \triangle s_{it}}{E},
\]

which all denote the position of establishment-specific employment, hires and separation rates relative to the mean in the economy.\(^{26}\)

Our model predicts that establishments with a below mean growth rate of wages should have an above mean growth rate of employment. The same holds true for the hires rate.

We specify the following regression equation:

\[
x_{it}^r = \alpha_o + \alpha_1 w_{it}^r + \alpha_2 C_{it} + \sum T + \mu_i + \varepsilon_{it},
\]

where \(x_{it}^r\) may either be \(n_{it}^r, h_{it}^r\) or \(s_{it}^r\). One advantage of this specification compared to the covariance-based one is that we can control for time-invariant heterogeneity by using establishment-fixed effects, that we can add a time trend and time-variant control variables at the establishment level (such as skill shares).

In contrast to the theoretical simulation where firms are homogenous in all other dimensions, differences in the data may also be driven by other factors such as skill composition or sectoral effects. Thus, we add a vector of control variables \(C_{it}\), which may include the share of low- and medium-skilled workers in a given establishments or dummies for states and sectors.

Our estimation coefficient suggest that a firm with a wage growth that is 1% below the average in the economy shows an employment growth that is 0.4 percent above the average in the economy (see table 6). The estimated coefficient for the relative wage position

\(^{26}\)For the empirical analysis we drop extreme outliers, i.e. observations with \(w_{it}^r, n_{it}^r, h_{it}^r\) or \(s_{it}^r\), below / above the 1\(^{st}\) / 99\(^{th}\) percentile of the corresponding distribution.
is statistically significant at the 1% level (also when we disaggregate further). Note that standard errors are clustered at the annual level. The result is robust when we control for education share dummies (i.e. the share of low and medium-skilled workers over time), sectoral and state dummies. It is well in line with our theoretical prediction. Assume that all firms in the economy are hit by a positive aggregate shock and some have a rigid wage, while others do not. Those firms with a rigid wage will increase their employment stock by more in a boom than the average firm, while firms with a flexible wage will do so by less.

Table 6: Relative Measures: Employment

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>$n_{it}^r$</th>
<th>$n_{it}^r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient (t-values): $w_{it}^r$</td>
<td>-0.407 (-30.70)</td>
<td>-0.407 (-30.97)</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Education Share Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sectoral Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>State Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>Observations</td>
<td>38,897,038</td>
<td>38,897,038</td>
</tr>
</tbody>
</table>

Next, we analyze how the different employment stock adjustment is performed in terms of flows. Table 7 shows that a wage growth that is one percent below the economy wide average leads to an increase of the hires rate by -0.4 percentage points. This suggests that the hires margin matters for the employment adjustment.

Table 7: Relative Measures: Hires Rate

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>$hr_{it}^r$</th>
<th>$hr_{it}^r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient (t-values): $w_{it}^r$</td>
<td>-0.424 (41.62)</td>
<td>-0.422 (-41.78)</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Education Share Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sectoral Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>State Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Observations</td>
<td>38,897,038</td>
<td>38,897,038</td>
</tr>
</tbody>
</table>

Our estimations for the separation rate show that a wage growth that is one percent below the economy wide average leads to a decrease of the separation rate of 0.1 percentage points (see table 8). This appears in line with what we would expect under endogenous separations (typically modelled by idiosyncratic productivity shocks). If wages increase by
less than average in a firm in a boom, the firm has an incentive to reduce firing by more than average. Given that the hires rate and the separation rate are normalized in the same way (as share of the firm-specific employment stock), we can directly compare the coefficients. Interestingly, wage rigidities have more of an effect on the hires rate than the separations rate. This may have to do with workers’ quit reaction (see discussion above). In aggregate, it is well known that quits and firings have inverse cyclicalities (e.g. Hall 2006). This may explain why separations (which are the combination of firing and quits) react by less to a different wage cyclicity than hirings.

Table 8: Relative Measures: Separations Rate

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>$sr^r_{it}$</th>
<th>$sr^s_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient (t-values): $w^r_{it}$</td>
<td>0.112 (8.18)</td>
<td>0.112 (8.15)</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Education Share Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sectoral Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>State Dummies</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Observations</td>
<td>38,897,038</td>
<td>38,897,038</td>
</tr>
</tbody>
</table>

Our empirical regressions are based on the assumption that all establishments are subject to the same economy wide business cycle. However, if the business cycle is of different magnitude in different sectors or regions, a different wage growth relative to the economy wide mean may be the result of different shocks. The sector and state dummies may be insufficient to control for these issues. In order to exclude that this generates spurious results, we run the same regressions on a sectoral and state level.

Table 9 in the Appendix 8.4 show the estimated coefficients for ten different sectors. The estimated coefficients for employment range from -0.307 to -0.521. The estimated coefficients for the hires rate range from -0.265 to -0.587. The estimated coefficients for the separations rate range from 0.060 to 0.192. While there are quantitative differences across sectors, the adjustment mechanism of employment, hires and separations appears to be similar.

Table 10 in Appendix 8.4 shows the estimated coefficients for ten different West German states. The estimated coefficients for employment range from -0.352 to -0.432. The estimated coefficients for the hires rate range from -0.389 to -0.431. The estimated coefficients for the separations rate range from 0.097 to 0.125. Interestingly, the range of estimated coefficients is much smaller on the state level than on the sectoral level.

Overall, a disaggregation to the sectoral and state level shows that our results remain robust. Thus, the economy wide results do not seem to be driven by spurious movements of
wages, which are not due to the cycle.

6 Conclusion

Our paper shows that different wage cyclicalities at the establishment level lead to a different behavior of hires, separations and employment over the business cycle. Firms with more procyclical wages hire more countercyclically. This is in line with our proposed theoretical framework.

Interestingly, we do not only find empirical support for the right qualitative responses in the data, but we also find plausible quantitative reactions relative to our proposed model. Although the coefficients in the model are more than twice as large as in the data, we consider this as a remarkably close outcome. It has to be kept in mind that the model is very simple. It has firms that are homogenous in all other dimension, the wage formation mechanism is fairly simple (in the real world, we would expect more complicated feedback effects) and that the model abstracts from all sorts of other interactions (e.g. workers’ quit behavior). Thus, we conclude that the model based measure and the empirical result are in a similar order of magnitude, although the model-based measures are somewhat larger.

This provides support for quantitative theories that assign a role to real wage rigidities. Our regression results may serve as a benchmark for different theoretical frameworks such as random search and matching models, directed search models or neoclassical frameworks with infrequent wage adjustments (e.g. Calvo wage adjustment). Similar theoretical simulation exercises can be performed and compared to our estimations results.
7 References


8 Appendices

8.1 Appendix 1: The Administrative Worker and Labor Market Flow Panel

The Administrative Worker and Labor Market Flow Panel (AWFP) aggregates German administrative wage, labor market flow and stock information at the establishment level of the years 1975–2014. All data are available at an annually and quarterly frequency.

The underlying administrative micro data source is mainly the Employment History (Beschäftigtenhistorik, BeH) of the Institute for Employment Research (IAB). The BeH comprises all individuals who were at least once employed subject to social security since 1975.\footnote{The BeH also comprises marginal part-time workers employed since 1999.} Some data packages — concerning flows from or into unemployment — use additional data from the Benefit Recipient History (Leistungsempfängerhistorik, LeH). The LeH comprises, inter alia, all individuals that receipt benefits in accordance with Social Code Book III (recorded from 1975 onwards).

Before aggregating the data to the establishment level, several adjustments and imputations were conducted at the micro data (see Seth and Stüber 2016). For example, the administrative wages are censored at the contribution assessment ceiling of the social security system. Therefore censored wages were imputed according to Schmucker et al. (2016).

For coherency, we focus on wages and flows for “regular workers”. In the AWFP a person is defined as a “regular worker” when he/she is full-time employed and belongs to person group 101 (employee s.t. social security without special features), 140 (seamen) or 143 (maritime pilots) in the BeH (see Seth and Stüber 2016). Therefore all (marginal) part-time employees, employees in partial retirement, interns etc. are not accounted for as regular workers.

According to the AWFP, stocks and flows are calculated using the “end-of-period flow” definition (see Seth and Stüber 2016):

- The stock of employees of an establishment in some year $t$ equals the number of regular workers on the last day of year $t$.

- Inflows of employees of an establishment for year $t$ equals the number of regular workers who were regularly employed on the last day of year $t$ but not so on the last day of the preceding year, $t-1$.

- Outflows of employees of an establishment for year $t$ equals the number of regular workers who were regularly employed on the last day of the preceding year ($t-1$) but
We use the AWFP at the annually frequency and restrict the data to West German establishments (excluding Berlin) and the years 1979–2014. The dataset contains on average more than 1.3 million establishments per year. For illustration purposes figure 5 shows the time series for the aggregated hires rate, separation rate, daily real wage per full-time worker (in 2010 prices), and the number of full-time workers. Hires and separations rate are calculated as sum of all hires / separations divided by the average number of full-time workers in \( t \) and \( t-1 \).

### 8.2 Appendix 2: Derivation of the Nash Wage

The Nash product is

\[
\Lambda_t = (W_t - U_t)^\nu (J_t)^{1-\nu},
\]  
\quad (35)

with

\[
W_t - U_t = w_t - b + E_t \delta (1 - \phi - \eta_{t+1}) (W_{t+1} - U_{t+1}).
\]  
\quad (36)

and
\[ J_t = a_t - w_t + E_t \delta (1 - \phi) J_{t+1}. \] (37)

Maximization of the Nash product with respect to the wage yields

\[ \frac{\partial \Lambda_t}{\partial w_t} = \nu J_t \frac{\partial W_t}{\partial w_t} + (1 - \nu) (W_t - U_t) \frac{\partial J_t}{\partial w_t} = 0 \] (38)

\[ \nu J_t = (1 - \nu) (W_t - U_t) \] (39)

After substitution:

\[ \nu (a_t - w_t + E_t \delta (1 - \phi) J_{t+1}) = (1 - \nu) [w_t - b + E_t \delta (1 - \phi - \eta_{t+1}) (W_{t+1} - U_{t+1})]. \] (40)

Using equation (39):

\[ \nu (a_t - w_t + E_t \delta (1 - \phi) J_{t+1}) = (1 - \nu) \left[ w_t - b + E_t \delta (1 - \phi - \eta_{t+1}) \frac{\nu}{1 - \nu} J_{t+1} \right]. \] (41)

\[ w_t = \nu (a_t + \delta \eta_{t+1} J_{t+1}) + (1 - \nu) b. \] (42)

### 8.3 Appendix 3: Analytical Results

The numerical results may be sensitive to the particular parametrization we have chosen. Thus, we use a simplified analytical version of our model to make more general statements.

Remember the key equations from our theoretical model. The selection rate of firm \( i \) is:

\[ \eta_{it} = \int_{\tilde{\varepsilon}_{it}}^{\infty} f(\varepsilon_t) d\varepsilon_t. \] (43)

The selection cutoff point is

\[ \tilde{\varepsilon}_{it} = a_t - w_{it} - h + \beta (1 - \phi) E_t J_{it+1}, \] (44)

with

\[ J_{it} = a_t - w_{it} + \beta (1 - \phi) E_t J_{it+1}. \] (45)

The only source of heterogeneity at the establishment level is the wage formation mechanism. For analytical tractability, we assume that wages are a function of productivity:
where $\varepsilon_{w,a} = \frac{\partial w_{it}(a) \cdot a}{\partial a}$ is the elasticity of the wage in firm $i$ with respect to aggregate productivity.

In order to make analytical statements, we rewrite everything in terms of the steady state and we normalize $h$ to 0. Thus:

$$\eta_i = \int_{\tilde{\varepsilon}_i}^{\infty} f(\varepsilon) d\varepsilon \quad (47)$$

$$\tilde{\varepsilon}_i = \frac{a - w_i(a)}{1 - \beta (1 - \lambda)} \quad (48)$$

What is the effect of different degrees of wage rigidity on the firm-specific selection rate?

$$\frac{\partial \eta_i}{\partial a} = f(\tilde{\varepsilon}_i) \frac{(1 - \frac{\partial w_i}{\partial a})}{1 - \beta (1 - \lambda)} \quad (49)$$

$$= f(\tilde{\varepsilon}_i) \frac{w \left( \frac{a}{w} - \frac{\partial w_i}{\partial a} \right)}{a \left( 1 - \beta (1 - \lambda) \right)} \quad (50)$$

$$= f(\tilde{\varepsilon}_i) \frac{w \left( \frac{a}{w} - \varepsilon_{w,a} \right)}{a \left( 1 - \beta (1 - \lambda) \right)} \quad (51)$$

In the data, we observe the reaction of the hires rate to aggregate changes $hr_i = \eta_i s_i / n_i$, where $s_i$ is the firm-specific number of applicants. Thus, everything has to be transformed into the hires rate $hr_i = \eta_i s_i / n_i$ and its elasticity with respect to aggregate productivity.

$$\frac{\partial hr_i}{\partial a} = \frac{\partial (\eta_i s_i / n_i)}{\partial a} = \frac{\partial \eta_i}{\partial a} \frac{s_i}{n_i} =$$

$$= f(\tilde{\varepsilon}_i) \frac{w \left( \frac{a}{w} - \varepsilon_{w,a} \right)}{a \left( 1 - \beta (1 - \lambda) \right) n_i} \quad (52)$$

$$= f(\tilde{\varepsilon}_i) \frac{w \left( \frac{a}{w} - \varepsilon_{w,a} \right)}{a \left( 1 - \beta (1 - \lambda) \right) n_i} \quad (53)$$

How does a different wage cyclicality affect the reaction of the hires rate to an aggregate productivity shock? For this purpose, we take the first derivative of the equation above with respect to the wage elasticity:

$$\frac{\partial \partial hr_i}{\partial \varepsilon_{w,a}} = f(\tilde{\varepsilon}_i) \frac{w}{a \left( 1 - \beta (1 - \lambda) \right) n_i} \quad (54)$$

Can we make a statement how large this expression is and compare it to our empirical
results? From the data, we know that the hires rate (in absolute deviations) is 0.5 times as volatile as output (measured as deviation from the HP-trend). Thus, on average if output goes up by 1 percent, the hires rate goes up by 0.5 percentage points \( \left( \frac{\partial hr}{\partial a} = 0.5 \right) \). Now, let’s combine equations (53) and (54):

\[
\frac{\partial}{\partial a} \frac{\partial hr}{\partial a} = -\frac{a}{w} - \varepsilon w, a
\]

(55)

This equation allows us to make aggregate statements. Typically, \( \frac{a}{w} \) is close to 1 in labor market flow models without capital. From estimations by Stüber (2016), we know that \( \phi_{w, a} = 0.5 \) for Germany. Thus, if the dynamics of the aggregate model is in line with the data (in terms of \( \frac{\partial hr}{\partial a} \)), the value (in absolute terms) for the reaction of the hires rate with respect to wage changes is roughly \(-0.5/(1 - 0.5) = -1\). This is also confirmed by our estimations based on the theoretical simulations in section 3.3, where the estimated coefficient for the hires rate is also around -1.

How does this compare to our empirical results? The estimated coefficient for the hires rate is \(-0.4\). If the wage elasticity is 1 percent below average, our estimations results predict a \(-0.4\) percentage points higher hires rate. This compares to a model-based sensitivity of around \(-1\).
### 8.4 Appendix 4: Sectoral and State Results

#### Table 9: Relative Measures for Industry Sectors

<table>
<thead>
<tr>
<th>Dependent Variable: $n_{it}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient: $w_{it}$</td>
<td>-0.358</td>
<td>-0.488</td>
<td>-0.439</td>
<td>-0.480</td>
<td>-0.443</td>
<td>-0.376</td>
<td>-0.307</td>
<td>-0.346</td>
<td>-0.521</td>
<td>-0.415</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable: $hr_{it}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient: $w_{it}$</td>
<td>-0.265</td>
<td>-0.422</td>
<td>-0.421</td>
<td>-0.459</td>
<td>-0.432</td>
<td>-0.370</td>
<td>-0.305</td>
<td>-0.405</td>
<td>-0.587</td>
<td>-0.453</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable: $sr_{it}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient: $w_{it}$</td>
<td>0.156</td>
<td>0.192</td>
<td>0.175</td>
<td>0.178</td>
<td>0.177</td>
<td>0.107</td>
<td>0.091</td>
<td>0.098</td>
<td>0.060 (**)</td>
<td>0.080</td>
</tr>
</tbody>
</table>

| N                          | 946,588 | 1,514,777 | 2,972,779 | 1,135,872 | 4,473,054 | 11,626,028 | 2,799,800 | 5,724,295 | 4,720,490 | 2,957,546 |

Notes:

1 = Agriculture; 2 = Manufacturing (1); 3 = Manufacturing (2); 4 = Manufacturing (3); 5 = Electricity & Construction; 6 = Sales, Restaurants; 7 = Transport, Finance; 8 = Services; 9 = Education, Health, 10 = Other. The numbers correspond to the first digit of the industry classification 1993.

All results are highly significant at the 1 percent level, unless marked otherwise: ** = 5 % level.

#### Table 10: Relative Measures for Federal States

<table>
<thead>
<tr>
<th>Dependent Variable: $n_{it}$</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient: $w_{it}$</td>
<td>-0.412</td>
<td>-0.352</td>
<td>-0.432</td>
<td>-0.399</td>
<td>-0.401</td>
<td>-0.301</td>
<td>-0.420</td>
<td>-0.416</td>
<td>-0.412</td>
<td>-0.404</td>
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</table>

<table>
<thead>
<tr>
<th>Dependent Variable: $hr_{it}$</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
<th>B9</th>
<th>B10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient: $w_{it}$</td>
<td>-0.418</td>
<td>-0.389</td>
<td>-0.439</td>
<td>-0.391</td>
<td>-0.408</td>
<td>-0.407</td>
<td>-0.424</td>
<td>-0.431</td>
<td>-0.430</td>
<td>-0.395</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable: $sr_{it}$</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
<th>B9</th>
<th>B10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated coefficient: $w_{it}$</td>
<td>0.118</td>
<td>0.097</td>
<td>0.125</td>
<td>0.100</td>
<td>0.116</td>
<td>0.113</td>
<td>0.119</td>
<td>0.112</td>
<td>0.108</td>
<td>0.125</td>
</tr>
</tbody>
</table>

| N                          | 1,705,015 | 1,161,885 | 4,408,210 | 402,301 | 10,139,461 | 3,624,300 | 2,396,998 | 6,575,094 | 7,834,578 | 609,711 |

Notes:

S1 = Schleswig-Holstein; S2 = Hamburg; S3 = Lower Saxony; S4 = Bremen; S5 = North Rhine-Westphalia; S6 = Hesse; S7 = Rhineland-Palatinate; S8 = Baden-Württemberg; S9 = Bavaria; S10 = Saarland.

All results are highly significant at the 1 percent level.