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The Impact of Peer Ability and Heterogeneity on Student Achievement: Evidence from a Natural Experiment

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Abstract:
This paper estimates the impact of peer achievement and variance on math achievement growth. It exploits exogenous variation in peer characteristics generated at the transition to upper-secondary school in a sample of Berlin fifth graders. Parents and schools are barely able to condition their decisions on peer characteristics since classes are newly built up from a large pool of elementary school pupils. I find positive peer effects on achievement growth and no effects for peer variance. Lower-achieving pupils benefit more from abler peers. Results from simulations suggest that pupils are slightly better off in comprehensive than in ability-tracked school systems. *JEL: I21, I28.*

Keywords: peer effects in secondary school, comparison between ability-tracked and comprehensive school, natural experiment

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1. Introduction

Peer group composition of classes plays an essential role for school choice decisions of parents. Peer effects are also important in debates on school vouchers, desegregation, ability tracking or antipoverty programs. This paper investigates the (causal) impact of peer ability and heterogeneity on student achievement in math. It exploits exogenous variation in peer group characteristics which is generated in newly built up classes at the transition of Berlin students to upper-secondary school after the fourth grade.

All studies about the impact of peer ability are aware of the existence of severe endogeneity problems. Choices of parents and school administrators are the most important sources of endogeneity because of self-selection. If, for instance, pupils of ambitious parents are more likely to be placed into classes with high-achieving and homogenous peers and if such parents also support their child's educational progress to a greater extent, then peer characteristics and student achievement are spuriously correlated. School principals could also tend to group pupils into classes strategically, particularly if school accountability systems are implemented.[1]

There are two bodies of convincing research that estimate the impact of peer ability on achievement growth: the first extensively makes use of fixed-effects frameworks whereas the second aims to exploit exogenous variation generated in (quasi-) experimental settings. Both have in common that they estimate models of educational production accounting at least for lagged student and peer achievement which is assumed to capture past family, peer, and school inputs.[2] The absence of past achievement, which is the case in cross-sectional data, yields biased estimates for the impact of peer ability on student achievement because of simultaneity: if peers have an influence on a student's achievement, that student will also affect his or her peers' achievement.[3]

Hanushek et al. (2003) and Sund (2009) belong to the first strand of research. Hanushek et al. (2003) analyze a large data set of Texas public elementary school pupils (grades three through six). Controlling for fixed student, school, and school-by-grade effects and an additional number of time-varying student, family, and school characteristics, they find that an increase

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[1] There is evidence for the existence of school gaming behavior under accountability pressure: threatened schools may reclassify low-achieving students into special education, see Figlio and Getzler (2002). As shown in Jacob and Levitt (2003), some of them even manipulate testing conditions by teacher cheating.

[2] One of the earliest studies that estimates such a model of educational production is Summers and Wolfe (1977). Using a data set of sixth-grade elementary school pupils enrolled in schools located in the Philadelphia School District, they find positive peer effects on (composite) student achievement.

[3] This problem is also referred to as the "reflection problem", see Manski (1993).

in the variance of peer achievement has no significant effect on math learning. Further, a one standard deviation increase in peer achievement leads to a roughly 0.2 increase in (standardized) math achievement which is substantial. Sund (2009) also finds positive peer effects among Swedish students who are enrolled in upper-secondary education.[4] Accounting for time, school, teacher, and individual fixed effects, he additionally shows that lower-achieving students benefit more from an increase in peer ability than their higher-achieving classmates. To improve validity, he also identifies students that are attending classes in which they have no peers from previous grades since this might bias the results. Surprisingly, students are better off in classes that are more heterogeneous.

Alternatively, the second strand of research exploits situations where students are (quasi-) randomly grouped into classes. Form the viewpoint of an ideal experiment, random grouping of pupils could uncover the causal impact of peer achievement and peer heterogeneity on achievement growth since variation in peer characteristics would be exogenous in this setting.[5] Such an event is analyzed in Carrell, Fullerton, and West (2009), where U.S. Air Force Academy freshmen are exogenously assigned to peer groups of approximately 30 students. In the first year of their university study, these students have limited ability to interact with other students outside of their assigned peer group. Therefore these peer groups might be considered as classes. The authors find peer effects in math and science courses but not in foreign language courses. No results are reported for the impact of peer heterogeneity.[6]

The results presented here are obtained from a natural experiment in Berlin upper-secondary schools. Each school year, classes at the fifth grade are newly built up with pupils from a large number of elementary schools. This situation is similar to Carrell, Fullerton, and West (2009) where peer groups are built up from the pool of Air Force Academy freshmen. The identification strategy, which will be outlined in more detail below, depends on two assumptions: (i) parents of fourth graders are not able to condition their school choice decision on peer characteristics since classes at the fifth grade are built up in the future. (ii) Upper-secondary schools have limited possibilities to group fifth graders by skill at the beginning of a school year – to do so, schools need to monitor pupils for a time.

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[4] In Sweden, compulsory education is comprehensive and lasts for nine years. Subsequently, pupils have the possibility to attend upper-secondary school.

[5] Research designs that seek to mimic such situations are strongly advocated by Angrist and Pischke (2010).

[6] Using the same identification strategy, Sacerdote (2001) reports similar results for the impact of peer effects on freshmen GPA (Grade Point Average).

In addition, this study analyses which students are better off in ability-tracked and comprehensive school systems. Some countries, e.g. Germany or the Netherlands, group students by ability into different secondary school tracks at ages between 10 and 12. By contrast, the lower-secondary school systems of Japan, Norway, the UK, and the US are comprehensive and do not track at all.

The analysis of the data suggests that pupils benefit from an increase in peer math achievement but higher-achieving students do so to a smaller extent. Peer heterogeneity, as measured by peer variance and alternative measures for heterogeneity, seems not to affect math achievement growth. The results also indicate that students learn more if their achievement is high at the beginning of a school year. Depending on the estimates, simulations show that slightly more than the majority of pupils would be better off in a comprehensive than in an ability-tracked school system. The simulations also suggest that the degree of homogeneity and mean achievement of the student body becomes somewhat higher in comprehensive school. The major shortcoming of this study is related to the external validity of the results since they have been obtained from a sample of upper-secondary pupils. These might not be representative for the whole student body which could result, for instance, in biased simulation results.

The remainder of the paper is as follows: section 2 briefly describes the data. The identification strategy is outlined in section 3. Section 4 presents the main results. Using simulated data, the winners and losers from a school system change towards comprehensive school are described in section 5. Section 6 concludes.

2. Data and summary statistics

The German educational system is governed at the state level. Generally, elementary school lasts until the fourth grade when children are 10 years old. Thereafter students are tracked by ability into three different types of secondary schools: lower-secondary (Hauptschule), middle-secondary (Realschule), and upper-secondary school (Gymnasium). Upper-secondary school is the most academic track and prepares students for university study. The Berlin educational system is somewhat different since primary education lasts six years. Some Berlin upper-secondary schools, however, allow transition already after four years of elementary school.[7] In the following, these upper-secondary schools are referred to as G5 schools. In the

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[7] In contrast, transition into lower- or middle-secondary schools is not possible after the fourth grade.

school year 2002/03 around 24,200 fourth graders attended one of 402 Berlin elementary schools. 7% of them changed to one of 31 G5 schools in the following school year.[8]

The data analyzed here is called ELEMENT.[9] It is a longitudinal survey on reading comprehension and math achievement for Berlin elementary and G5 pupils. In the primary school sample, classes at the fourth grade (primary sampling units) were randomly drawn in the school year 2002/03. If the school of a drawn class hosted other classes at the fourth grade, one of them was additionally included in the sample to allow within-school comparisons. The primary school sample contains 13% of all elementary school fourth graders (71 schools, 140 classes, 3293 pupils). This cohort of fourth graders was followed through grades four to six. Further, all fifth graders that attended a G5 school in the school year 2003/04 were included and followed through grades five to six (31 schools, 59 classes, 1700 pupils).

Participation in the standardized tests at the end of each school year was compulsory and test scores are comparable across grades and school types. Attrition rates are therefore very low and solely caused by class repetitions, absence at the time of the test or school changes of followed pupils. Only a very small share, 3.5%, of G5 pupils were not observed for these reasons. G5 fifth graders were also assessed at the beginning of the school year 2003/04 which is an important property for this study. Additional pupil information were collected from questionnaires completed by students and parents on a voluntary basis. The data contain no information about formerly attended elementary school classes of G5 pupils.

Table 1 consists of two sections. Section A presents descriptive statistics for the analyzed sample of G5 pupils. For comparisons, second B contains descriptive statistics for a representative sample of Berlin fifth graders in primary education. Math test scores across all grades and school types have been rescaled to mean zero and standard deviation one at the beginning of the fifth grade in G5 schools.[10] During a school year, G5 students' math skills increased by 0.81 standard deviations on average. Fifth graders in primary education experience a smaller increase in math skills (0.72 standard deviations). Compared to G5 pupils, their average skill level at the beginning of the fifth grade is 1.29 standard deviations lower which is equivalent to one and a half school years. This large difference implies that external validity of the results could be restricted to upper-secondary school pupils.

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[8] At that time, the number of Berlin upper-secondary schools (including G5 schools) was 111.

[9] "**E**rhebung zum **Le**se- und **M**athematikverständnis: **Ent**wicklungen in den Jahrgangsstufen 4 bis 6 in Berlin", English translation: "Survey on reading comprehension and math achievement: Developments in grades 4 through 6 in Berlin". Detailed data descriptions and a codebook (both in German) are available on the homepages of the Berlin senate department for education, science, and research (Berliner Senatsverwaltung für Bildung, Wissenschaft und Forschung).

[10] The standardization of all math test scores in the data with respect to G5 fifth graders is explained in Table A 1.

Like math test scores in G5 schools, mean peer achievement at the beginning of the fifth grade in G5 schools also equals zero. Compared to peer achievement in elementary school, however, the standard deviation in peer achievement is much smaller in G5 schools. The summary statistics for peer variance show that the degree of within-class heterogeneity is somewhat larger in G5 schools. This indicates that G5 schools might not primarily seek to build up classes that are homogenous in math skills. Additionally, one can infer from the standard deviations of peer variance that the share of classes with a very high or low degree of peer variance is larger in the elementary school sample. Most G5 pupils have a favorable socioeconomic background: 80% of their parents finished upper-secondary school and have at least 100 books at home. This could account for the large difference in mean achievement between G5 and elementary school fifth graders.[11]

3. Exogenous variation in peer characteristics

The functional relationship of interest has the general form

$$T_{i,1} = f\left(T_{i,0}, X_{i,0}, E_{-i,0}, V_{-i,0}\right).$$

$T_{i,1}$ is pupil i's math test score in time 1 which is, for instance, the end of a school year. All explanatory variables are measured in the past, as indicated by the index 0, which could be the beginning of the school year. Past achievement $T_{i,0}$ is assumed to capture sufficiently all previous school, peer, and family inputs. $X_{i,0}$ is a set of additional explanatory variables for achievement growth.

The variables of interest are peer achievement $E_{-i,0}$ and peer variance $V_{-i,0}$, both measured at the class level in time 0. Peer achievement $E_{-i,0}$ is the average test score of i's classmates. The calculation of this average excludes $T_{i,0}$ as emphasized by the subscript $-i$. Computing $E_{-i,0}$ in this manner rules out the possibility that any correlation between $E_{-i,0}$ and $T_{i,1}$ is caused by $T_{i,0}$.[12] Peer variance $V_{-i,0}$ is calculated in a similar way.[13]

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[11] A cross-country comparison in Woessmann (2004) shows that family background has strong effects on student achievement in the US and Western European countries. Among all countries, the strongest effects are found for the UK and Germany. A detailed analysis for the impact of socioeconomic background variables on educational attainment of Swiss pupils is given in Bauer and Riphahn (2007).

[12] To make this point clear, let $e_{i,0}$ be the mean achievement in i's class (including i's own test score) which implies $e_{i,0} = e_{j,0}$ if two (different) pupils i and j attend the same class. Further, let $\rho(a, b)$ be the correlation coefficient between two variables a and b. Obviously, $\rho\left(e_{i,0}, T_{i,0}\right) > 0$. If $\rho\left(T_{i,1}, T_{i,0}\right) > 0$ is also true, then $T_{i,1}$ and $e_{i,0}$ must be positively correlated.

[13] The computation of peer achievement and variance is explained in more detail in Table A 1.

The experiment that could ideally be used to capture the causal effect of peer achievement and peer heterogeneity on student achievement would be random grouping of pupils into classes. Pupils who enter a G5 school after four years of primary education are in a similar situation since parents and schools have limited capabilities to condition their school-choice decisions on peer ability and heterogeneity and G5 schools might also not be interested in to do so.

In primary education, the mean and variance of peer achievement could be endogenous for several reasons. If ambitious parents systematically keep their child away from attending classes or schools with underperforming or very heterogeneous peers, and if they are more likely to support the educational progress of their child (e.g. homework assistance, private lessons), then achievement growth and peer characteristics might be correlated. Endogeneity caused by omitted variables could also arise in elementary schools if certain school principals are interested in raising mean achievement by ability-grouping of pupils into classes. To do so, however, parents and school principals need to monitor pupils for a certain time.

These problems do not arise or can be accounted for in G5 schools at the beginning of the fifth grade. Parents have to apply for a G5 school six months in advance of which implies that they cannot condition their application (and registration) at least on peer heterogeneity of the future G5 class. However, parents can expect that peer achievement in math is higher in G5 schools with an emphasis on science.[14] Therefore all regressions for the impact off peer characteristics on math growth solely exploit within-school variation in the dependent and explanatory variables.

When G5 schools newly build up classes at the fifth grade, there is no reason to assume that the mean or variance of peer achievement is a determinant in this process: (i) the pool of applicants consists of high-achieving pupils with favorable learning environments. Therefore ability-grouping should not play an important role in G5 schools. (ii) Even if G5 schools aim at grouping pupils by skill, they have insufficient information to do so. Selection into G5 schools mainly depends on a pupil's ability which is measured by school grades from elementary school. Compared to standardized achievement tests, school grades are far from a perfect measure of skills.[15] At best, school grades allow comparisons of pupils that attended the same class in elementary school. However, the number of elementary schools is 13 times larger than the number of G5 schools such that most fifth graders in G5 schools previously attended

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[14] G5 schools differ: some of them focus more on science, others on humanities.

[15] For example, Dardanoni, Modica, and Pennsi (2009) find for 14 of 16 OECD countries that schools with high shares of underperforming students tend to set lower grading standards.

different classes in different elementary schools.[16] Even if pupils are homogenous in terms of school grades, they might still be heterogeneous in skills. (iii) The descriptive statistics in Table 1 suggest that G5 schools do not seek to sort pupils by skill since G5 classes are more heterogeneous in math achievement than classes in elementary education. When building up new classes at the fifth grade, it is more likely that G5 schools try to balance the shares of boys and girls across classes. As shown in OECD (2004), girls outperform boys in reading and underperform in math.[17] Consequently, it is difficult to build up mixed classes that are homogenous in math and reading achievement at the same time.

This reasoning motivates the identification strategy: within-school variation in the mean and variance of peer achievement is assumed to be exogenous and estimates of their coefficients may be interpreted causally. To test the exogeneity of peer achievement and variance at the fifth grade in G5 schools, these two variables are regressed on math achievement and other explanatory variables in Table 2. All variables are measured at the beginning of the school year. For easier interpretation, the dependent variables have been rescaled to mean zero and standard deviation one. Columns 1 and 3 do not account for school fixed effects – these are controlled for in columns 2 and 4.

In the case of perfect randomization, one would expect insignificant estimates with absolute values close to zero. One can infer from Table 2, column 1 that higher-achieving students are more likely to have abler peers. The point estimate is large and highly significant. Thus results from regressions that exclude school fixed effects might be biased because of self-selection of high-achievers into classes with abler peers. As already mentioned, mean math achievement could be higher in G5 schools that put an emphasis on science and lower in G5 schools focusing more on humanities. The results from column 2 seem to confirm this hypothesis: Once school fixed effects are accounted for, the point estimate for math achievement remains significant, but turns into negative with an absolute value close to zero. Similarly, peer variance and student achievement are correlated, but point estimates in columns 3 and 4 are also close to zero.

Regarding the remaining explanatory variables, there seems to be no systematical self-selection into classes or schools with specific levels of peer achievement or variance since

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[16] Since class size is smaller in elementary schools, the number of elementary school classes exceeds the number of G5 classes to the 17-fold.

[17] This pattern can be found for most OECD countries that participated in PISA 2003 (Programme for International Student Assessment). Average gender-differences in math and reading achievement are reported in OECD (2004), Figures 2.18. and 6.6, respectively.

most related coefficients are either insignificant or close to zero. Further, the few significant estimates are not robust since they are sensitive to the in- or exclusion of school fixed effects. Summing up, endogeneity cannot be ruled out completely, but is likely to play a minor role once school fixed effects are taken into account.

4. Results

The impact of peer characteristics on achievement growth is estimated with the following baseline-model of educational production:

$$T_{i,1} = \alpha_1 E_{-i,0} + \alpha_2 V_{-i,0} + \alpha_3 R_{i,0} + \alpha_4 T_{i,0} + \beta' X_i + \gamma_s + \varepsilon_i$$

$T_{i,1}$ is pupil i's math test score at the end of the fifth grade as indicated by the subscript 1. All variables on the RHS are indexed with 0 which stands for "beginning of fifth grade". The explanatory variables are therefore predetermined which rules out simultaneity. Peer achievement $E_{-i,0}$ is the average test score of i's classmates at the beginning of the fifth grade. By assumption, $E_{-i,0}$ sufficiently captures past school and family inputs of the peers. Since G5 classes are built up from a large pool of elementary school fourth graders, the probability is small that pupils who are grouped into a G5 class previously attended the same elementary school classes. Therefore $E_{-i,0}$ should not be biased because of peer-interactions in the past. $V_{-i,0}$ is the variance of i's peers.

$R_{i,0}$ is pupil i's class percentile rank in math test scores. By definition, $R_{i,0} \in [0,1]$. Within classes, the highest-achieving pupil has rank one, the median-achiever has rank 0.5 and the lowest-achiever has rank zero. $T_{i,0}$, the math test score at the beginning of the fifth grade, is the most important control variable. It captures i's past educational inputs and additionally accounts for the correlation between $R_{i,0}$ and $T_{i,0}$.[18] X_i is a column-vector which contains a constant and a set of additional controls, namely: age, a girl dummy, and the highest educational background of parents. γ_s is a school fixed effect. Disturbances ε_i allow for correlated residuals among students that attend same G5 classes.

Table 3 reports estimates for the impact of peer characteristics on math achievement growth in G5 schools. It is divided into a top and a bottom section. Section A excludes additional controls X – these are taken into account in section B. Across all specifications (columns 1-4)

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[18] Pupils with high test scores are likely to have a high rank. If past achievement $T_{i,0}$ is left out in a regression of present achievement $T_{i,1}$ on past rank $R_{i,0}$, then past rank would pick up the correlation between past and present achievement. On the other hand, two pupils with same test score may have different ranks if they attend different classes.

the results are not sensitive to the inclusion of X but become somewhat more precise. For easier interpretation, peer achievement and peer variance have sample mean zero and standard deviation one.

Estimates in column 1 suggest that higher ranked pupils learn more during a school year.[19] This is consistent with Cullen, Jacob, and Levitt (2006), who also find that a student's relative position among his or her peers is an important determinant of his or her academic success. The point estimate for peer variance is close to zero and insignificant. The impact of peer achievement is positive and significant in section B only, the significance test yields a p-value of 0.13 in section A.

In both sections, the effect of peer achievement is estimated with low precision since highly ranked pupils benefit less from an increase in peer achievement (column 2). Depending on the estimates in section B, column 2, the total derivative with respect to peer achievement and class percentile rank is:

$$\mathrm{d}T_{i,1} = \left(0.476 - 0.177E_{-i,0}\right)\mathrm{d}R_{i,0} + \left(0.210 - 0.177R_{i,0}\right)\mathrm{d}E_{-i,0}$$

The first term indicates that students benefit from an rank increase, however, the effect is smaller in classes with high peer achievement.[20] Regarding the second term, all pupils benefit from an increase in peer achievement since $R_{i,0} \in [0,1]$, but highly ranked pupils do so to a smaller extent.[21] One common explanation for this pattern is that low-ability students might learn from better-achieving peers during a school year. Since highly ranked pupils do not have this advantage, their returns to an increase in peer achievement are diminishing. Further, one can infer from these findings that placing an (average) pupil into a class with low peer achievement is not necessarily harmful: on the one hand, that pupil's educational progress is lowered by its peers, on the other, that pupil is likely to benefit from an increase in its percentile rank.

The third column in Table 3 checks weather pupils also respond differently to changes in peer variance. Like in previous specifications (columns 1-2), peer variance does not harm G5 fifth graders. The fully interacted model (column 4) confirms the findings from the second specification: pupils benefit from an increase in peer achievement or their rank and peer variance is irrelevant. These patterns (significance levels, relations among estimates) remain the same if

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[19] All regressions in Table 3 and Table 4 control for student achievement at the beginning of the fifth grade.

[20] The effect in the first term becomes negative if $E_{-i,0} > 2.69$, which is extremely high.

[21] This relation is also found in Sund (2009) and Zimmer (2003).

controls for missing or imputed values are left out from the regressions, but absolute values of the estimates become about 10% larger in that case.

So far, the results show that peer heterogeneity does not harm student achievement. To test this finding, Table 4 reports results for alternative measures for heterogeneity. Column 1 controls for peer variance. These estimates are therefore identical with the fourth column in Table 3, section B. In the second column of Table 4, peer standard deviation instead of peer variance is used to measure heterogeneity. The additional alternative measures (columns 3-6) are constant among pupils that attend the same classes – these measures therefore only vary across classes. Colums 3 and 4 report results for the "common" variance and standard deviation (including pupil i), respectively. Columns 5 and 6 show results for two measures for the range. The range is a test score difference between two pupils that attend the same class. Range(100/0) is the test score difference between the most able and the least able student whereas range(90/10) captures the test score difference of the two pupils with percentile ranks 0.90 and 0.10.

All estimates for the impact of peer heterogeneity and the interaction between rank and peer heterogeneity are (jointly) insignificant in Table 4 and the main findings from Table 3 are confirmed. The results in Table 4 are virtually unaffected by the in- or exclusion of additional controls X and control variables for missing or imputed values. If peer achievement and various measures for peer heterogeneity are not interacted with the percentile rank, the results become very similar to the findings reported in Table 3, column 1.

5. Are pupils better off in a comprehensive or ability-tracked school system?

This section aims to identify which groups of pupils are better off in ability-tracked compared to comprehensive school systems. Ability-tracking implies that the heterogeneity in student achievement within tracks is smaller than the heterogeneity of the whole student body. Further, mean achievement of students enrolled in the highest-level track exceeds mean achievement in the lower-level tracks. Compared to ability-tracking, classes in comprehensive schools are similar in terms of mean achievement and tend to be more heterogeneous. Hanushek and Woessmann (2006) provide empirical evidence that the heterogeneity of the student body increases in countries with tracked school systems. They also find that early tracking might reduce mean performance.

Simulations are a tool for identifying students that would benefit and suffer from a school-system change from ability-tracking towards comprehensive school. Using the estimates from the previous section, one could predict and compare each student's achievement in both school systems. The setup of the simulation implemented here is simple: there are two points in time, beginning and end of the school year which are referred to as time 0 and time 1. In time 0, a pupil's initial ability is drawn from the standard normal distribution. These pupils are then either grouped by ability into three different tracks or randomly placed into classes.[22] For both scenarios (tracked or comprehensive school), the rank and peer achievement are computed for time 0. In time 0, the expected value of mean achievement in the middle track equals the expected value of mean achievement in any comprehensive school class. Using the estimates in Table 3 (column 2, section B), each pupil's potential achievement growth in both school systems is then predicted.[23] Predictions solely depend on the rank, peer achievement and the interaction between rank and peer achievement.

The winners and losers from a school system change towards comprehensive school are described in Table 5. In columns 1 and 2, pupils are assigned to six groups (column 1) depending on their initial achievement in time 0. Initial achievement is drawn from the standard normal distribution and pupils are ordered by their percentiles in the achievement distribution (column 2). For instance, all pupils in group 1 and 2 belong to the bottom third of the achievement distribution.[24]

Columns 3 to 5 contain information about each group's situation in the ability-tracked scenario. One can infer from column 3, that all pupils from group 1 and 2 would have attended lower-secondary school, and all pupils from the top third of the achievement distribution (groups 5 and 6) would be enrolled in upper-secondary education. In an ability-tracked system, peer achievement is heterogeneous among secondary school types (column 4). For example, average-achievers (groups 3 and 4) are expected to attend middle-secondary school where peer achievement is expected to be zero which is the median of the standard normal distribution. Consequently, peer achievement in lower-secondary or upper-secondary school is expected to be smaller or greater than zero, respectively. Regarding their ranks, pupils from groups 2, 4, and 6 are expected to have high ranks within their assigned secondary school track (column

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[22] Without any loss in generality, the simulated data used here solely consists of 90 pupils that are grouped (by ability or randomly) into three classes of size 30. The number of replications is 10,000 which ensures the robustness of the results. For each replication, the random-number seed is set to the current value of the replication counter.

[23] The results presented in this section are not sensitive to the used set of estimates from column 2 (section A or B, with or without controls for missing and imputed values).

[24] The numerical values of the thresholds in column 2 are obtained from a large number (10,000) of replications. For each replication, these thresholds somewhat differ from the values in column 2. Column 2 reports average values of thresholds across all replications.

5).[25] For instance, pupils from group 4 do not belong to the highest-achievers in the whole student body (see column 2). Within middle-secondary school, however, column 5 shows that these pupils are likely to have high ranks.

Columns 6 to 8 display each groups' situation in the comprehensive school scenario. Regardless of their initial ability (column 2), all pupils would have attended comprehensive school (column 6). Peer achievement in comprehensive school is expected to be zero which is the mean of the standard normal distribution (column 7). Since pupils are grouped randomly into classes, each class might be considered as a representative sample of the whole student body. Therefore each pupil's rank (column 8) and percentile in the achievement distribution (column 2) are expected to be very similar in comprehensive school. For instance, the highest-achievers among all students (group 6, column 2) are expected to have the highest ranks in comprehensive school too.

As already mentioned in the previous section, placing an (average) pupil into a class with low peer achievement is accompanied by two adverse effects: on the one hand, that pupil suffers from the low achievement of its peers, on the other, that pupil is likely to benefit from an increase in his or her percentile rank. To identify the winners and losers from a school system change towards comprehensive school, one first needs to predict student achievement in time 1 for both scenarios:

$$T^s_{i,1} = 0.476R^s_{i,0} + 0.210E^s_{-i,0} - 0.177R^s_{i,0}E^s_{-i,0}$$

Where the superscript $s \in \{t, c\}$ indicates the type of school system (tracked or comprehensive). The coefficients are from Table 3, section B, column 2. From the viewpoint of the simulation, a school system change simply implies a change in peer achievement and class ranks in time 0:

$$\Delta T_{i,1} = T^c_{i,1} - T^t_{i,1} = 0.476\Delta R_{i,0} + 0.210\Delta E_{-i,0} - 0.177\Delta R_{i,0}E_{i,0}$$

Here, a change towards comprehensive school is considered. Therefore $\Delta T_{i,1}$ is defined as $T^c_{i,1} - T^t_{i,1}$. Same reasoning applies for the remaining variables. The six groups of pupils in Table 5 are differently affected by a change of the school system from ability-tracking towards comprehensive school. Column 9 shows the expected changes in peer achievement

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[25] The numerical values of the thresholds in column 5 are calculated from the values in column 2: $0.39 = (0.13 - 0.00) \cdot 3$, $0.50 = (0.50 - 0.3\bar{3}) \cdot 3$, and $0.64 = (0.88 - 0.6\bar{6}) \cdot 3$.

$\Delta E_{-i,0}$: For instance, all pupils that would have attended lower-secondary school (groups 1 and 2) experience higher levels of peer achievement in comprehensive school, therefore a school system change towards comprehensive school would imply an increase in peer achievement for these pupils. However, potential upper-secondary school pupils (groups 5 and 6) are confronted with less abler peers in comprehensive school as indicated by the negative values of $\Delta E_{-i,0}$.

Column 10 shows the expected change in the rank $\Delta R_{i,0}$. Pupils that would have attended upper-secondary school (groups 5 and 6) clearly benefit from a rank increase in the case of school system change towards comprehensive school. On the other hand, potential lower-secondary school pupils (groups 1 and 2) are ranked lower in comprehensive school. Among all subgroups, pupils from the second or fifth group are expected to experience the largest changes in their rank.

The main finding from the simulations is reported in column 11: Pupils that would be ranked around 0.50 or below in secondary school (column 5) are the winners from a change in the school system.[26] This result differs from Zimmer (2003) who suggests that that detracking schools has positive effects on all lower-achieving students but no effects on high-achievers.

Since student achievement in time 1 can be predicted for the ability-tracked and comprehensive school scenario, one can quantify the shares of winners (and losers) form a change towards comprehensive school. The expected value for the share of winners is 53% which indicates that slightly more than the majority of students are (expected to be) better off in comprehensive school.[27] Further, all replications show that mean achievement of the whole student body (in time 1) is slightly larger in the comprehensive school scenario. The student body also becomes a bit more homogenous in time 1 if the school system changes towards comprehensive school. These findings therefore provide a possible explanation for the results reported in Hanushek and Woessmann (2006).

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[26] To be more precise, the winners are potential lower-secondary pupils with tracked percentile rank below 0.39 (group 1), potential middle-secondary pupils with tracked percentile rank below 0.50 (group 3), and potential upper-secondary pupils with tracked percentile rank below 0.64 (group 5).

[27] $53\% = \frac{1}{T}\sum_{t=1}^{T} b_t$, where $T = 10{,}000$ is the total number of replications and b_t is the share of winners in replication t. Consequently, $(1 - b_t)$ is the related share of losers. The simulations show that $\forall\, t \in T\colon b_t \in [0.39, 0.63]$, and $\Pr(0.50 < b_t \leq 0.63) = 0.75$, $\Pr(b_t = 0.50) = 0.09$, and $\Pr(0.39 \leq b_t < 0.50) = 0.16$. The three probabilities sum up to 1.

6. Concluding remarks

This paper intends to estimate the causal impact of peer achievement and of various measures for peer heterogeneity on math achievement. Exploiting a natural experiment in a sample of Berlin fifth graders in upper-secondary school, the results indicate that pupils benefit from abler peers, but pupils with high class percentile ranks do so at diminishing rates. Holding other things constant, a one standard deviation increase in peer achievement at the beginning of a school year improves a median-ranked student's test score by roughly 0.12 standard deviations. Further, peer heterogeneity seems not to harm achievement growth.

Depending on the estimates, results from simulations suggest that pupils are slightly better off in comprehensive than in ability-tracked school systems. To be more precise, all pupils that would have been below-median-achievers in their assigned track benefit from a school system change towards comprehensive school. In that case, however, the other half of students experiences a decrease in achievement growth. This differs from Zimmer (2003) who finds that detracking schools has positive effects on all lower-achieving pupils but no effects on high-achievers. The simulations also show that both, the degree of homogeneity and mean achievement of the student body, become somewhat higher in the comprehensive school scenario. Regarding external validity, all results might be representative for upper-secondary school pupils only.

Basically, this study made a before-after comparison: how will the outcome variable look like in time 1 if explanatory variables are changed in time 0? It did not make an attempt to uncover the mechanisms that operate during a school year, which is challenging because of, for instance, the presence of simultaneity. As mentioned by Hanushek et al. (2003), "The role of peers can be complex. Influences may come from friends or role models, or peer group composition may alter the nature of instruction in the classroom… The most common perspective is that peers, like families, are sources of motivation, aspiration, and direct interactions in learning." Further research could also address the question why students additionally benefit from their class percentile rank even if past student achievement is taken into account.

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Tables

Table 1: Summary statistics for G5 and elementary school fifth graders

Section:	A: G5 schools			B: Elem. schools	
	Mean	s.d.	Missings	Mean	s.d.
Student achievement:					
Math achievement (end)	0.81	0.96	0.00	-0.57	1.06
Math achievement (beginning)	0.00	1.00	0.03	-1.29	1.02
Peer characteristics:					
Peer math achievement (beg.)	0.00	0.33	0.03	-1.29	0.48
Peer math variance (beginning)	0.90	0.28	0.03	0.81	0.35
Other pupil characteristics:					
Age	11.32	0.43	0.01	11.57	0.59
Girl	0.52		0.00	0.49	
Books at home:					
0-25	0.07		0.11	0.30	
26-100	0.16		0.11	0.29	
101-200	0.18		0.11	0.14	
more than 200	0.59		0.11	0.27	
Parental education:					
Lower-secondary	0.02		0.13	0.13	
Middle-secondary	0.19		0.13	0.45	
Upper-secondary	0.79		0.13	0.42	
Class/school characteristics:					
Class size	29.45	3.43	0.00	23.23	3.35
Located in East Germany	0.38		0.00	0.37	
N(pupils)	1642			3081	
N(classes)	59			138	
N(schools)	31			71	

Standard deviations not reported for dummy variables. The column "missings" reports the share of missing values. "Math achievement (end)/(beginning)" is the student math achievement at the end/beginning of the fifth grade. The original test scores in the data have been rescaled to mean zero and standard deviation one at the beginning of the fifth grade in G5 schools (see Table A 1 for additional information). "Peer achievement" and "peer variance" are the achievement mean and variance of a pupil's classmates (both measured at the beginning of the fifth grade). Parental education contains the highest secondary school degree of parents.

Table 2: Exogeneity of peer achievement and peer variance (G5 schools, fifth grade)

Dependent Variable:	Peer achievement		Peer variance	
	(1)	(2)	(3)	(4)
School fixed effects:	no	yes	no	yes
Pupil characteristics:				
Math achievement	0.220***	-0.029**	-0.089***	-0.068***
	(0.03)	(0.01)	(0.03)	(0.02)
Age	-0.025	-0.046*	-0.135**	-0.033
	(0.06)	(0.03)	(0.06)	(0.04)
Girl	0.066	0.053**	-0.014	-0.049
	(0.05)	(0.02)	(0.05)	(0.04)
Books at home: (Ref. category: more than 200 books)				
101-200 Books	0.011	0.011	-0.073	-0.049
	(0.07)	(0.03)	(0.07)	(0.05)
26-100 Books	-0.146*	-0.006	-0.009	0.056
	(0.08)	(0.04)	(0.08)	(0.06)
0-25 Books	0.019	0.132**	-0.010	-0.135
	(0.11)	(0.05)	(0.12)	(0.08)
Parental education: (Ref. category: upper-secondary)				
Middle-secondary	-0.161**	0.006	-0.058	-0.039
	(0.07)	(0.03)	(0.07)	(0.05)
Lower-secondary	-0.311	-0.093	0.116	0.175
	(0.19)	(0.09)	(0.20)	(0.14)
R^2 (adj.)	0.0603	0.7921	0.0125	0.4960

Significance levels: * 10%, ** 5%, *** 1%. Standard errors in parentheses. Used data: ELMENT fifth graders in G5 schools (1592 Obs.). For easier interpretation, the dependent variables have been rescaled to mean zero and standard deviation one. All variables are measured at the beginning of the fifth grade. All regressions include a constant and controls for missing or imputed values.

Table 3: Impact of peer characteristics on student achievement (G5 schools, fifth grade)

Dependent variable:	Math achievement (end of fifth grade)			
	(1)	(2)	(3)	(4)
Section A: No additional controls				
Rank (1=best)	0.434*	0.491*	0.540**	0.576**
	(0.254)	(0.255)	(0.252)	(0.253)
Peer achievement	0.102	0.200***	0.11	0.200***
	(0.067)	(0.068)	(0.067)	(0.067)
Peer variance	-0.005	-0.005	-0.072	-0.06
	(0.019)	(0.019)	(0.044)	(0.045)
Rank*p-achievement		-0.178***		-0.165***
		(0.057)		(0.058)
Rank*p-variance			0.129*	0.107
			(0.076)	(0.080)
R^2 (adj.)	0.3961	0.3988	0.3972	0.3994
Section B: Including additional controls				
Rank (1=best)	0.418*	0.476*	0.513**	0.549**
	(0.249)	(0.251)	(0.250)	(0.251)
Peer achievement	0.114*	0.210***	0.120*	0.210***
	(0.065)	(0.065)	(0.065)	(0.064)
Peer variance	-0.006	-0.005	-0.065	-0.053
	(0.018)	(0.018)	(0.045)	(0.047)
Rank*p-achievement		-0.177***		-0.165***
		(0.055)		(0.057)
Rank*p-variance			0.115	0.094
			(0.076)	(0.081)
R^2 (adj.)	0.4006	0.4033	0.4014	0.4036

Significance levels: * 10%, ** 5%, *** 1%. Robust standard errors that allow for correlated residuals among students in the same class are in parentheses. Used data: ELEMENT fifth graders in G5 schools (1642 observations). "Additional controls" are: age, a girl dummy, books at home and parental education. All explanatory variables are measured at the beginning of the fifth grade. All regressions include a constant, student achievement in math (beginning of the school year), school fixed effects and controls for missing or imputed values. "Rank" is a pupil's class percentile rank in math. By definition, the rank lies between zero and one. "Peer achievement" and "peer variance" are the achievement mean and variance of a pupil's classmates. For easier interpretation, "Peer achievement" and "Peer variance" have sample mean zero and standard deviation one. "Rank*p-achievement" is an interaction term between pupil Rank and peer achievement. "Rank*p-variance" is constructed in the same manner.

Table 4: Impact of peer heterogeneity (alternative measures) on student achievement

Dependent variable:	Math achievement (end of fifth grade)					
	(1)	(2)	(3)	(4)	(5)	(6)
Rank (1=best)	0.549**	0.545**	0.601**	0.598**	0.482*	0.485*
	(0.251)	(0.253)	(0.263)	(0.264)	(0.254)	(0.264)
Peer achievement	0.210***	0.210***	0.215***	0.215***	0.217***	0.211***
	(0.064)	(0.065)	(0.065)	(0.065)	(0.062)	(0.063)
Rank*p-achievement	-0.165***	-0.166***	-0.163***	-0.163***	-0.175***	-0.178***
	(0.057)	(0.057)	(0.057)	(0.058)	(0.056)	(0.056)
Peer variance	-0.053					
	(0.047)					
Rank*p-variance	0.094					
	(0.081)					
Peer standard deviation		-0.049				
		(0.045)				
Rank*peer standard dev.		0.086				
		(0.079)				
Variance			-0.056			
			(0.049)			
Rank* variance			0.098			
			(0.083)			
Standard deviation				-0.054		
				(0.048)		
Rank*standard deviation				0.096		
				(0.080)		
Range(100/0)					0.029	
					(0.044)	
Rank*range(100/0)					0.01	
					(0.067)	
Range(90/10)						-0.033
						(0.055)
Rank*range(90/10)						0.013
						(0.104)
Joint p-value	0.5069	0.5447	0.4937	0.4927	0.4879	0.2727
R^2 (adj.)	0.4036	0.4035	0.4036	0.4036	0.4035	0.4032

Significance levels: * 10%, ** 5%, *** 1%. Robust standard errors that allow for correlated residuals among students in the same class are in parentheses. Used data: ELEMENT fifth graders in G5 schools (1642 observations). " Joint p-value" is the p-value for the joint significance of the measure of heterogeneity and its interaction. "Peer variance" is the achievement variance of a pupil's classmates. "Variance" and "standard deviation" are the within-class variance and standard deviation in student achievement. The range is a test score difference between two pupils that attend the same class. "Range(100/0)" is the test score difference between the most able and the least able student whereas "range(90/10)" captures the test score difference of the two pupils with percentile ranks 0.90 and 0.10. All regressions include a constant, student achievement in math (beginning of the school year), school fixed effects, age, a girl dummy, books at home, parental education and controls for missing or imputed values.

Table 5: Expected winners and losers of a school system change from ability-tracking towards comprehensive school

Initial student achievement in time 0		Scenario A: Ability-tracked school system			Scenario B: Comprehensive school			Change from ability-tracking towards comprehensive school		
Gr. (1)	Achievement percentile P_0 (2)	School type (3)	Peer ach. E_0^t (4)	Rank R_0^t (5)	School type (6)	Peer ach. E_0^c (7)	Rank R_0^c (8)	Change in E_0 (9)	Change in R_0 (10)	net effect in time 1 (11)
1.	$0.00 \leq P_0 \leq 0.13$	lower-sec.	$E_0^t < 0$	$0.00 \leq R_0^t \leq 0.39$	comp.	$E_0^c = 0$	$0.00 \leq R_0^c \leq 0.13$	$\Delta E_0 > 0$	$\Delta R_0 \leq 0$	positive
2.	$0.13 < P_0 \leq 0.3\overline{3}$	lower-sec.	$E_0^t < 0$	$0.39 < R_0^t \leq 1.00$	comp.	$E_0^c = 0$	$0.13 < R_0^c \leq 0.3\overline{3}$	$\Delta E_0 > 0$	$\Delta R_0 \ll 0$	negative
3.	$0.3\overline{3} < P_0 \leq 0.50$	middle-sec.	$E_0^t = 0$	$0.00 \leq R_0^t \leq 0.50$	comp.	$E_0^c = 0$	$0.3\overline{3} < R_0^c \leq 0.50$	$\Delta E_0 = 0$	$\Delta R_0 \geq 0$	positive
4.	$0.50 < P_0 \leq 0.6\overline{6}$	middle-sec.	$E_0^t = 0$	$0.50 < R_0^t \leq 1.00$	comp.	$E_0^c = 0$	$0.50 < R_0^c \leq 0.6\overline{6}$	$\Delta E_0 = 0$	$\Delta R_0 \leq 0$	negative
5.	$0.6\overline{6} < P_0 \leq 0.88$	upper-sec.	$E_0^t > 0$	$0.00 \leq R_0^t \leq 0.64$	comp.	$E_0^c = 0$	$0.6\overline{6} < R_0^c \leq 0.88$	$\Delta E_0 < 0$	$\Delta R_0 \gg 0$	positive
6.	$0.88 < P_0 \leq 1.00$	upper-sec.	$E_0^t > 0$	$0.64 < R_0^t \leq 1.00$	comp.	$E_0^c = 0$	$0.88 < R_0^c \leq 1.00$	$\Delta E_0 < 0$	$\Delta R_0 \geq 0$	negative

This figure displays the expected situation of the student body in two different school systems: ability-tracking (columns 3 to 5) and comprehensive school (columns 6 to 8) which is also indicated by the superscripts t and c. Pupils are assigned to six groups (column 1) depending on their initial achievement in time 0. Initial achievement is drawn from the standard normal distribution and pupils are ordered by their percentile P_0 in the achievement distribution (column 2). In both scenarios (ability-tracking or comprehensive school) pupils from different groups are confronted with certain peers (columns 4 and 7) and have certain ranks (columns 5 and 8). The impact of a school system change from ability-tracking towards comprehensive school for different groups is reported in columns 9 to 11. "Change in R_0" is defined as follows: $\Delta R \equiv R_0^c - R_0^t$ and $\Delta E_0 \equiv E_0^c - E_0^t$. The "net effect in time 1" indicates which groups of pupils are better off in time 1 if they would be in a comprehensive school system.

Appendix

Table A 1: Computation of various variables

Task	Description
Standardization of original values of math test scores across all grades and school types	Let $T^*_{i,g,s,t}$ be pupil i's original math test score in the data (as indicated by the star superscript) enrolled at grade $g \in \{4,5,6\}$ in school $s \in \{0,1\}$ in time $t \in \{0,1\}$. $s = 0$ if i attends elementary school and $s = 1$ if i attends a G5 school. $t = 0$ if T^* has been measured at the beginning of the school year, otherwise $t = 1$ (end of the school year). Further, let $\mu^*_{i,5,1,0}$ and $\sigma^*_{i,5,1,0}$ be the mean and standard deviation of T^* of G5 fifth graders in the beginning of the school year. Math test scores across all grades and school types are then rescaled as follows: $T_{i,g,s,t} = \left(T^*_{i,g,s,t} - \mu^*_{i,5,1,0}\right)/\sigma^*_{i,5,1,0}$. Peer achievement $E_{-i,0}$ and peer variance $V_{-i,0}$ are then computed from $T_{i,g,s,t}$.
Computation of peer achievement $E_{-i,0}$ and peer variance $V_{-i,0}$	$E_{-i,0} \equiv \frac{1}{n_c-1}\sum_{j\in I_c\setminus\{i\}} T_{j,0}$, where I_c is the set of pupil IDs that attend class c and n_c denotes the number of elements in I_c. Similarly, $V_{-i,0} \equiv \frac{1}{n_c-1}\sum_{j\in I_c\setminus\{i\}}\left(T_{j,0} - E_{-i,0}\right)^2$.

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