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Product Variety, Price Elasticity of Demand and Fixed Cost in Spatial Models*

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Abstract

This paper explores the implications of price-dependent demand in spatial models of product differentiation. We introduce consumers with a quasi-linear utility function in the framework of the Salop (1979) model. We show that the so-called excess entry theorem relies critically on the assumption of completely inelastic demand. Our model is able to produce excessive, insufficient, or optimal product variety. A proof for the existence and uniqueness of symmetric equilibrium when price elasticity of demand is increasing in price is also provided.

Keywords: Demand elasticity; Spatial models; Excess entry theorem
JEL-Classification: L11, L13

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1 Introduction

Spatial models of product differentiation in the spirit of Hotelling (1929) and Salop (1979) have been a popular tool in Industrial Organization. They have been used to study competition in all sorts of markets and all sorts of issues.[1]

Typically, the Hotelling model has been used to study location decisions by firms while the Salop model has been used to study entry decisions and market structure. Concerning the Salop model, one prominent result is the so-called excess entry theorem. It states that in a free-entry equilibrium, there are always more firms entering into the market than would be desirable from a welfare point of view. That is, there is excessive entry into the market. As firms are usually assumed to be single product firms, the result can also be interpreted as an excess of product variety provided in the market.[2]

However, one underlying, and quite restrictive assumption in the Salop model, is that consumer demand does not depend on the price of a product. Each consumer demands a single unit of a differentiated product. In consequence, the price then constitutes a mere transfer between consumers and firms and thus has no impact on total welfare. It is the aim of the present paper to lift this assumption of completely inelastic demand. We want to investigate the consequences of this modification on the validity of the excess entry theorem. In contrast to a version of the model with completely inelastic demand, prices do have a welfare impact. Higher prices of the differentiated product now lead to lower demanded quantities of the differentiated product.

In a previous paper, Gu and Wenzel (2007), we show that this assumption of completely inelastic demand can be critical. We introduce price-dependent demand in the Salop model by assuming a specific functional form. Consumer demand takes the form of a demand function with constant elasticity. In this framework, we get excess, insufficient or optimal

[1]E.g., Anderson and Coate (2005) on media markets, Friedman and Thisse (1993) on collusion, Armstrong (2006) for a study on two-sided markets, and many more.

[2]With respect to variants of the standard Salop model, Matsumura and Okamura (2006) find this excess entry result holds for a broad class of transport and production cost functions.

entry. The outcome depends on consumer demand elasticity. If the demand elasticity is sufficiently small, we get excess entry while with large demand elasticities we get insufficient entry. The driving force of the results is the impact of the consumer reactions on the level of competition.

The present paper explores the relationship between price-dependent demand and the excess entry theorem in a more general setting. We study a setup in which consumer preferences can be represented by a quasi-linear utility function. We make a mild restriction on the resulting consumer demand function for the differentiated product, namely we assume that the price elasticity is increasing in the price. This assumption is satisfied by many demand functions, for instance, linear demand functions. It is also a common assumption in the literature.[3] In this setup, we establish existence and uniqueness of symmetric price equilibrium in Salop models for general price dependent demand functions.

Our main aim is to characterize welfare properties of the free-entry equilibrium. We show that unlike the standard Salop model with completely inelastic demand, the free-entry equilibrium may exhibit excessive, insufficient or optimal entry. The intuition behind this result lies in the degree of competition. If demand is relatively inelastic, competition is weak and hence too much firms enter. However, when demand is price-dependent firms must be more careful when setting the price as an increase in price reduces the quantity demanded. The stronger this effect, that is, the larger the demand elasticity is, the lower are prices and profits. And hence, the incentives to enter are reduced. Thus, following this intuition, we show that if the demand elasticity in equilibrium is low we get excess entry, and if it is high, we get insufficient entry. However, in our setup equilibrium demand elasticity is endogenous. Thus, we are interested in exogenous parameters that lead to a high / low demand elasticity. We find that high fixed costs of entry and low transportation costs lead a high demand elasticity and hence insufficient entry. Conversely, low fixed costs of entry and and high transportation costs lead to a low demand elasticity and hence, excessive entry.

Our result also closes, at least partially, the gap between different approaches of modeling competition in differentiated product markets. In

[3]See, e.g., Lariviere and Porteus (2001) and Ziya et al. (2003).

representative consumer models, such as Dixit and Stiglitz (1977), or in discrete choice models of product differentiation, for instance see the overview in Anderson et al. (1992), equilibrium entry can be excessive, insufficient, or optimal depending on the exact model configuration.

Further approaches to introduce price-dependent demand into spatial models are Boeckem (1994), Rath and Zhao (2001), Peitz (2002) and Anderson and de Palma (2000).[4] The first two papers consider variants of the Hotelling framework. Boeckem (1994) introduces heterogenous consumers with respect to reservation prices. Depending on the price charged by firms some consumers choose not to buy a product. The paper by Rath and Zhao (2001) introduces elastic demand in the Hotelling framework by assuming that the quantity demanded by each consumer depends on the price charged. The authors propose a utility function that is quadratic in the quantity of the differentiated product leading to a linear demand function. In contrast to those two models we build on the Salop model as we are interested in the relationship between price-dependent demand and entry into the market. Our approach is closer to Rath and Zhao (2001) as we also assume that each consumer has a downward sloping demand for the differentiated good. Peitz (2002) features unit-elastic demand both in Hotelling and Salop settings but focuses on conditions for existence of Nash equilibrium in prices. He does not consider entry decisions. Anderson and de Palma (2000) propose a model that integrates features of spatial models where competition is localized and representative consumer models where competition is global. In this model, consumer demand is elastic with a constant demand elasticity. The study focuses on the interaction between local and global competition.

The remainder of the paper is structured as follows. Section 2 outlines our model. Section 3 establishes the existence and uniqueness of the symmetric price equilibrium and analyzes its properties. In Section 4 we compare equilibrium with optimal outcomes. Section 5 concludes.

[4] A recent paper by Peng and Tabuchi (2007) combines a model of spatial competition with taste for variety in the spirit of Dixit and Stiglitz (1977). In their setup, the quantity demanded also depends on the price. However, their focus is a different one. They study the incentives of how much variety to offer and how many stores to establish. A paper by Hamilton et al. (1994) analyzes elastic demand in a model with quantity competition. In contrast to the present paper the authors employ a transportation costs per unit of quantity purchased.

2 The model

Here we set up our model. Our aim is to stay close to the original Salop model as, for instance, outlined in Tirole (1988). The only modification we make is to introduce price-dependent demand.

2.1 Model setup

There is a unit mass of consumers who are uniformly located on a circle with circumference one. The location of a consumer is denoted by x. Consumers derive utility from the consumption of a differentiated product and a homogenous product, which serves as a numeraire good. The homogenous good is produced in a competitive industry while the differentiated product is produced within an oligopolistic industry. Behavior in the oligopolistic industry is the focus of our analysis.

We assume that consumers' utility is quasi-linear. Then, a consumer, located at x, gains the following utility from consuming a differentiated product with characteristic x_i:

$$U = \begin{cases} V + v(q_D) - t|x - x_i| + q_H & \text{if the differentiated product is consumed} \\ q_H & \text{otherwise,} \end{cases} \tag{1}$$

where q_D and q_H are the quantity of the differentiated and homogenous good, respectively. The utility derived by the consumption of the differentiated good consists of three parts. There is a gross utility for consuming this good V. The second utility component depends on the quantity consumed $v(q_D)$; $v(q_D)$ is assumed to be continuous and three times differentiable with $v^{'} > 0$ and $v^{''} < 0$. Finally, consumers have to incur costs of mismatch (transportation cost) if the product's attributes do not match consumers' preferences; these costs are linear in distance and do not depend on the quantity consumed.[5] We assume the gross utility V and $v^{'}$

[5] Transport costs are one time costs independent of the quantity. As an interpretation these could be costs for driving to a shopping mall. Alternatively, one could also assume transport costs to depend on the quantity. These would be a plausible assumption if the horizontal dimension is interpreted as a taste dimension.

evaluated at 0 are large enough so that under *relevant* product prices no consumer abstains from buying the differentiated product. Note also that there is decreasing marginal utility in the quantity of the differentiated product.

Each consumer is endowed with wealth Y which she can spend on the two commodities, the differentiated product and and the numeraire good. We restrict consumers to consume only one variant of the differentiated product. Let us denote the price of the differentiated product by p and normalize the price of the numeraire good to 1. Then each consumer faces the following budget constraint:

$$Y = p * q_D + q_H. \tag{2}$$

The differentiated product is offered by an oligopolistic industry with $n \geq 2$ firms each offering a single variant. We are not interested in location patterns. Hence, we assume that these firms are located equidistantly on the unit circle. The distance between two neighboring firms then is $\frac{1}{n}$.[6]

To model competition in this market, we study the following three stage game. In the first stage firms may enter the market. In the second stage, firms compete in prices. In the third stage, consumers choose a supplier of the differentiated product and the quantity.

2.2 Demand for the differentiated product

We start by deriving individual demand for the differentiated product. Suppose a consumer has decided to choose a certain supplier i. Then, the quantity she demands is the solution to the following maximization problem:

$$\begin{aligned} \max_{q_D, q_H} u(q_D, q_H) &= V + v(q_D) + q_H \\ \text{s.t. } q_D * p + q_H &= Y \\ q_D, q_H &\geq 0. \end{aligned}$$

[6]See Economides (1989) for the existence of symmetric location equilibria in the model with unit demand.

A consumer's demand for the differentiated good is determined by maximizing utility (equation (1)) under the budget constraint (equation (2)). We further assume that Y is sufficiently large such that the demand for the homogenous good is always positive. Then, by solving $v^{'}(q_D) = p$ we get a downward sloping individual demand function for the differentiated product $q(p)$. Since $v(q_D)$ is continuous and three times differentiable, $q(p)$ is continuous and twice differentiable in $(0, Q)$ where $Q < +\infty$ is the up-bound of demand obtained when $p = 0$.

Our assumption of quasi-linearity becomes convenient when expressing indirect utility. The surplus associated with the demand function $q(p)$ when a consumer located at x buys the differentiated from a firm located at x_i at a price $p_i < \hat{p}$ is

$$\hat{U} = V + Y + \int_{p_i}^{\hat{p}} q\left(p\right) dp - t|x - x_i|, \tag{3}$$

where $\hat{p}$ denotes the minimum price where $q(p)$ becomes zero.

2.3 Marginal consumer and demand

Given the symmetric structure of the model, we seek for a symmetric equilibrium. Therefore we derive demand of a representative firm i which for convenience is designated to be located at zero. Suppose that this firm charges a price of p_i while all remaining firms charge a price of p_o. Then the marginal consumer is the consumer indifferent between choosing to buy from firm i and the neighboring firm located at $\frac{1}{n}$. Using equation (3) the marginal consumer ($\bar{x}$) is implicitly given by

$$V + Y + \int_{p_i}^{\hat{p}} q\left(p\right) dp - t\bar{x} = V + Y + \int_{p_o}^{\hat{p}} q\left(p\right) dp - t\left(\frac{1}{n} - \bar{x}\right),$$

or explicitly by

$$\bar{x} = \frac{1}{2n} + \frac{1}{2t}\int_{p_i}^{p_o} q\left(p\right) dp. \tag{4}$$

As each firm faces two adjacent firms, the number of consumers choosing to buy from firm i is $2\bar{x}$. According to the demand function, each consumer buys an amount of $q(p_i)$. Hence total demand at firm i is:

$$D_i = 2\bar{x} * q(p_i). \tag{5}$$

In contrast to the standard model with completely inelastic demand, total demand consists now of two parts: market share and quantity per consumer. When choosing prices firms have to take into account of both effects. An increase in price reduces market share as well as the quantity that can be sold to each customer. This second effect is not present in the standard model.

3 Analysis

This section analyzes the equilibrium. In a first step we characterize the price equilibrium for a given number of firms and provide conditions for the existence. In a second step, we seek to determine the number of firms that enter.

3.1 Price equilibrium

We look for a symmetric equilibrium in which all firms charge the same price. Assuming zero production costs, the profit of a representative firm i when this firm charges a price p_i and all remaining firms charge a price p_o is given by:

$$\Pi_i = D_i * p_i = \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q(p)\,dp\right] q(p_i)\,p_i. \tag{6}$$

To find profit maximizing price p_i, we first derive the first order derivative,

$$\frac{d\Pi_i}{dp_i} = -\frac{1}{t}p_i\left[q(p_i)\right]^2 + \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q(p)\,dp\right]\left[q(p_i) + p_i\frac{dq(p)}{dp}\Big|_{p=p_i}\right]. \tag{7}$$

By setting equation (7) to zero we obtain the following necessary condition,

$$[q(p_i)]^2 p_i \frac{1}{t} = \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q(p)\,dp\right]\left[q(p_i) + p_i \frac{dq(p)}{dp}\Big|_{p=p_i}\right]. \tag{8}$$

For the moment let us suppose that a symmetric price equilibrium exists. Later we will turn to this issue and provide existence conditions. Applying symmetry to the first-order condition, a symmetric price equilibrium is characterized by:

$$q(p^*)\,p^* = \frac{t}{n}\left[1 + \frac{p^*}{q(p^*)}\frac{dq(p)}{dp}\Big|_{p=p^*}\right]. \tag{9}$$

Note that the last part of equation (9) includes the price elasticity of individual demand evaluated at equilibrium price. After the following definition, we express this equilibrium condition in terms of price elasticity of demand.

Definition. Denote the absolute value of price elasticity of demand ε as

$$\varepsilon = -\frac{p}{q(p)}\frac{dq(p)}{dp}.$$

Equation (9) now can be rewritten as

$$q(p^*)\,p^* = \frac{t}{n}\left[1 - \varepsilon^*\right]. \tag{10}$$

We use this condition to state corresponding equilibrium profits. Inserting equation (9) into equation (6) we get

$$\Pi^* = \frac{t}{n^2}\left[1 + \frac{p^*}{q(p^*)}\frac{dq(p)}{dp}\Big|_{p=p^*}\right] = \frac{t}{n^2}\left[1 - \varepsilon^*\right]. \tag{11}$$

It can be seen immediately that there is negative relationship between equilibrium demand elasticity and equilibrium profit.

3.2 Equilibrium existence

Now we provide conditions that ensure existence of a symmetric price equilibrium as stated in equation (10). We start with a preliminary result:

Lemma 1. In equilibrium, demand is inelastic, that is, $\varepsilon^* < 1$.
Proof: see appendix.

This result reveals that analogous to a monopolist who sets the price to reach unit elasticity, a firm in the current model will only set price at the inelastic segment of the demand function.

To ensure existence, we have to impose some structure on the model. We introduce the following assumption:[7]

Assumption 1. The absolute value of price elasticity of demand ε is strictly increasing for $p \in (0, \hat{p})$ and $\lim_{p \to \hat{p}} \varepsilon(\hat{p}) = \lim_{p \to \hat{p}} \varepsilon|_{p=\hat{p}} \geq 1$.

When ε is strictly increasing in p, it is shown in the literature that the individual consumer revenue function $R(p) = pq(p)$ is strictly unimodal over the entire interval of strictly positive demand. For a discussion on this point see Ziya et al. (2004). Strict unimodality of $R(p)$ means that $pq(p)$ has a unique global maximum $\widetilde{p}$ in $(0, \hat{p})$ and if p_1 and p_2 are two points in $(0, \hat{p})$ such that $p_1 < p_2 < \widetilde{p}$ or $\widetilde{p} < p_1 < p_2$ then $R(p_1) < R(p_2) < R(\widetilde{p})$ or $R(p_2) < R(p_1) < R(\widetilde{p})$, respectively.[8] Apparently a profit maximizing monopolist will set $\widetilde{p}$ and it's well known that $\varepsilon(\widetilde{p}) = 1$. When price p goes down from $\widetilde{p}$, both price elasticity of the demand ε and product revenue $R(p)$ strictly decrease.

As an example, one functional form that does satisfy assumption 1 is a linear demand function of the type $q = a - bp$ or a quadratic function of the form $q = a - bp^2$, where both a and b are suitable positive constants.[9]

[7]This assumption is sufficient but not necessary for the existence of a symmetric price equilibrium.
[8]This representation follows Bertsekas (1999).
[9]In both examples, the maximum value of the elasticity is obviously larger than 1.

Under Assumption 1, we are now ready to prove the existence of a unique symmetric price equilibrium given by equation (10).

Proposition 1. For any given number of firms $n \geq 2$, there exists a unique symmetric price equilibrium identified by condition (10), namely, $q(p^*)\, p^* = \frac{t}{n}[1-\varepsilon^*]$.
Proof: see appendix.

3.3 Properties of price equilibrium

We can now study the properties of the price equilibrium. Lemma 2 below states the comparative statics effect of the number of firms active in the market and of transportation costs on equilibrium price, equilibrium price elasticity and firm profit.

Lemma 2. Comparative statics.

i) Equilibrium price, price elasticity of demand and firm profit decrease in the number of entrants, that is, $\frac{dp^*}{dn} < 0$, $\frac{d\varepsilon^*}{dn} < 0$ and $\frac{d\Pi^*}{dn} < 0$.

ii) Equilibrium price, price elasticity of demand and firm profit increase in transportation costs, that is, $\frac{dp^*}{dt} > 0$, $\frac{d\varepsilon^*}{dt} > 0$ and $\frac{d\Pi^*}{dt} > 0$.

Proof: see appendix.

Unsurprisingly, the larger the number of firms the lower the price. Profits also decrease with the number of firms in the market. Additionally, the demand elasticity decreases with the number of firms in the market. This follows from our assumption that the demand elasticity increases in the price. The impact of transportation costs on prices and profits is the same as in standard location models. Prices and Profits increase with transportation costs.

3.4 Entry

Until now the analysis has treated the number of firms which offer differentiated products as exogenously given. We now investigate the number of active firms when it is endogenously determined by a zero profit condition. We assume that to enter, a firm has to incur an entry cost or fixed cost of f. Additionally, we treat the number of entrants as a continuous variable. Setting equation (11) equal to f determines implicitly the number of firms that enter. We denote this number by n^c:

$$\frac{t}{(n^c)^2}(1 - \varepsilon^*_{n^c}) = f. \tag{12}$$

In general, it is not possible to express the number of entrants explicitly as the equilibrium demand elasticity ($\varepsilon^*_{n^c}$) depends on the number of competitors. In this paper, we have assumed that the market is viable for at least two firm. So the fixed costs must not be prohibitively high: $f \leq F = \frac{t}{4}(1 - \varepsilon^*_{n=2})$. Thus, we only consider fixed costs in $f \in (0, F)$.[10] We know from Lemma 2 that profits decrease monotonically in the number of firms. Hence, we know that a solution to equation (12) exists and is unique.

The comparative static results concerning transportation costs and fixed costs are as expected. Higher transportation costs lead to more entry while higher fixed costs to less entry, that is, $\frac{dn^c}{dt} > 0$ and $\frac{dn^c}{df} < 0$. This follows immediately from Lemma 2.

Later, it will turn out that equilibrium demand elasticity is a crucial factor for our welfare results. Thus, we are interested in its properties. With endogenous entry, equilibrium demand elasticity is essentially a function of the exogenous variables, fixed costs and transportation costs. When fixed costs are low, a large number of firms enter which decreases equilibrium demand elasticity (as shown in Lemma 2). Converse is the impact of transportation costs. High transportation costs lead to a large number of entrants, and hence, to a low demand elasticity. Formally,

[10]Alternatively, this can be re-stated in terms of transportation costs: $t > \frac{4f}{1-\varepsilon^*_{n=2}} = T$.

Lemma 3. Equilibrium price elasticity increases in fixed costs and decreases in transportation costs, that is, $\frac{d\varepsilon^*_{n^c}}{df} > 0$ and $\frac{d\varepsilon^*_{n^c}}{dt} < 0$.

Proof: $\frac{d\varepsilon^*_{n^c}}{df} = \frac{d\varepsilon^*}{dn}\frac{dn^c}{df} > 0$, as $\frac{d\varepsilon^*}{dn} < 0$ by lemma 2 and $\frac{dn^c}{df} < 0$ from above. $\frac{d\varepsilon^*_{n^c}}{dt} = \frac{d\varepsilon^*}{dn}\frac{dn^c}{dt} < 0$, as $\frac{d\varepsilon^*}{dn} < 0$ by lemma 2 and $\frac{dn^c}{dt} > 0$ from above.

Hence, because of these strictly monotone relationships, there is a one-to-one relationship between equilibrium demand elasticity and fixed costs or transportation costs, respectively. For instance, for each value of fixed cost $f \in (0, F)$ we can identify the corresponding equilibrium price elasticity $\varepsilon^*(f) \in (0, \varepsilon^*_{n=2})$, and vice versa. The same applies to transportation costs. We will make use of these relationships when expressing welfare results.

4 Welfare

This section considers the welfare implications. We ask whether there is excess entry into the market as it is the case in models with completely inelastic demand.

In contrast to models with completely inelastic demand, we have to consider prices in our welfare analysis as they have an impact on the quantity purchased and hence on welfare. We define social welfare as the sum of consumer utility and industry profits:

$$W = \underbrace{\int_p^{\hat{p}} q\left(p\right) dp + V + Y - 2n \int_0^{\frac{1}{2n}} tx\ dx}_{\text{Consumer welfare}} + \underbrace{\frac{t}{n}\left[1 - \varepsilon^*_n\right] - fn}_{\text{Industry profits}}. \tag{13}$$

We consider a first-best benchmark, in which the social planner can control prices and level of entry, that is, she maximizes total welfare with respect to p and n. From equation (13), we see that the optimal price is equal to marginal cost, in this case, $p = 0$. Inserting this into equation (13) yields

$$W = \int_0^{\hat{p}} q\left(p\right) dp + V + Y - 2n \int_0^{\frac{1}{2n}} tx\ dx - fn. \tag{14}$$

The problem for the social planner is then identical to the case with completely inelastic demand, hence reduced to a trade-off between transportation costs and fixed costs. The optimal number of entrants is

$$n^f = \sqrt{\frac{t}{4f}}. \tag{15}$$

To shape intuition, it is useful to start with a preliminary result:

Lemma 4. There is excess entry if $\epsilon^*_{n^c} < \frac{3}{4}$, insufficient entry if $\epsilon^*_{n^c} > \frac{3}{4}$, and optimal entry if $\epsilon^*_{n^c} = \frac{3}{4}$.

Lemma 4 can easily be derived by comparing equations (12) and (15). This lemma provides conditions for the existence of excessive, insufficient, and optimal entry. If equilibrium demand elasticity is sufficiently low we get excess entry as in the standard model with completely inelastic demand. If, on the other hand, equilibrium demand elasticity exceeds $\frac{3}{4}$, there is insufficient entry into the market. The intuition behind the result can be seen in equation (11). The higher equilibrium demand elasticity is, the lower the profits are; and hence the smaller the incentives to enter the market will be.

However, equilibrium demand elasticity is endogenous in this model. Thus, our aim is to state the welfare result in terms of exogenous variables. Now we can make use of the monotone relationship between equilibrium demand elasticity and fixed costs of entry. Entry is excessive (insufficient) if fixed costs are such that $\varepsilon^*_{n^c} < \frac{3}{4}$ ($\varepsilon^*_{n^c} > \frac{3}{4}$). This leads to:

Proposition 2. Welfare result.

i) Suppose $\varepsilon^*_{n=2} \geq \frac{3}{4}$ and define $\overline{f}$ as the fixed cost level that leads to equilibrium price elasticity $\varepsilon^*_{n^c} = \frac{3}{4}$. Then there is excess entry if $f < \overline{f}$, insufficient entry if $f > \overline{f}$ and optimal entry if $f = \overline{f}$.

ii) Suppose $\varepsilon^*_{n=2} < \frac{3}{4}$, then there is excess entry for all $f \in (0, F]$.

Proof: see appendix.

Proposition 2 contains the main contribution of the paper. When accounting for price-dependent demand the excess entry result of the standard model with completely inelastic demand needs not hold. In the proposition we have to consider two cases. First, if the demand function is such that $\varepsilon^*_{n=2} \geq \frac{3}{4}$. Then, if fixed costs of entry are high such that the corresponding equilibrium demand elasticity is high, entry into the market is insufficient. Conversely, if fixed costs are low, the number of firms that enter is high which leads to a low demand elasticity. And hence, entry into the market is excessive. The second case we have to consider is a demand function which has the property such that $\varepsilon^*_{n=2} < \frac{3}{4}$. As $\varepsilon^*_{n^c}$ decreases in n, $\varepsilon^*_{n^c} < \frac{3}{4}$ for all values of fixed costs $(f \in (0, F))$. And thus, there is always excess entry in this case.

Alternatively, it is also possible to restate the welfare result in terms of transportation costs. This is formally done in the appendix. There, we show that insufficient entry is possible if transportation costs are sufficiently low.

5 Conclusion

This paper has introduced price-dependent demand into the Salop model. Our analysis focuses on the welfare implications of this generalization of the original model outlined by Salop. While in the model with completely inelastic demand the excess entry result holds, this is no longer true when accounting for price-dependent demand. Results are not that clear-cut anymore. Entry or product variety, respectively, can be excessive, insufficient, or optimal.

As the Salop model is widely used in all sorts of applications, we believe that our results are of some importance. In the light of the present paper, accounting for price-dependent demand may lead to different welfare conclusions.

A Appendix

A.1 Proof of Lemma 1

Proof: Note when $\varepsilon \geq 1$ i.e., $\frac{dq(p)}{dp}\frac{p}{q(p)} \leq -1$, the first order derivative (7)

$$\frac{d\Pi_i}{dp_i} = \underbrace{-\left[q\left(p_i\right)\right]^2 p_i \frac{1}{t}}_{\text{negative}} + \underbrace{\left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q\left(p\right)dp\right] q\left(p_i\right)}_{\text{positive}} \underbrace{\left[1 + \frac{p_i}{q\left(p_i\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p_i}\right]}_{\text{non-positive}} \tag{16}$$

obtains a strictly negative value. The middle part in the right-hand side of (16) is positive because we are interested in symmetric equilibrium ($p_i = p_o$). With $\frac{d\Pi_i}{dp_i}$ being negative, whenever demand elasticity exceeds or is equal to 1, a firm wants to reduce price in order to boost demand. In equilibrium, whenever it exists, however, the F.O.C. (9) holds,

$$\begin{aligned} & 1 + \frac{p^*}{q\left(p^*\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p^*} > 0 \\ \Longrightarrow\ & \frac{p^*}{q\left(p^*\right)}\frac{dq\left(p\right)}{dp}\Big|_{p=p^*} > -1 \\ \Longrightarrow\ & \varepsilon^* < 1. \end{aligned}$$

This concludes the proof.

A.2 Proof of Proposition 1

Proof: The structure of the proof is the following. We first show that the necessary first order condition (10) admits a unique solution. Second, we prove that under the condition of symmetric price and without undercutting, firm profit is quasi-concave in the strategy variable p_i. Last we show that firms have no incentive to undercut neighbors when they are in the situation identified by the first order condition.

1) Define $\Delta(p) = q\left(p\right)p - \frac{t}{n}\left[1 - \varepsilon(p)\right]$. Because $v(q_D)$ is continuous and three times differentiable, $q(p)$ and $\varepsilon(p)$ are continuous and differen-

tiable. Hence, $\Delta(p)$ is continuous. Note that

$$\lim_{p\to 0}\Delta(p) = 0 - \frac{t}{n}\left[1 - \lim_{p\to 0}\varepsilon(p)\right] = 0 - \frac{t}{n} < 0$$

and for the individual consumer ($R(p) = pq(p)$) revenue-maximizing $\widetilde{p}$,

$$\Delta(\widetilde{p}) = q\left(\widetilde{p}\right)\widetilde{p} > 0.$$

Because of continuity, $\Delta(p) = 0$ obtains solution(s) for $p \in (0, \widetilde{p})$. Take the derivative of $\Delta(p)$,

$$\frac{d\Delta(p)}{dp} = \frac{dR(p)}{dp} + \frac{t}{n}\frac{d\varepsilon(p)}{dp}.$$

Following Assumption 1, $\frac{d\varepsilon(p)}{dp} > 0$; since $R(p)$ is strictly unimodal, for $p \in (0, \widetilde{p})$, $\frac{dR(p)}{dp} > 0$ as well. Hence, we conclude $\frac{d\Delta(p)}{dp} > 0$. Because of this monotonicity, $\Delta(p) = 0$ obtains a unique solution in $(0, \widetilde{p})$. When $p \in [\widetilde{p}, \hat{p})$, we know $\varepsilon(p) \geq 1$ which means $\Delta(p) > 0$ for $[\widetilde{p}, \hat{p})$. So the solution given by $q\left(p\right)p = \frac{t}{n}\left[1 - \varepsilon(p)\right]$ for $p \in (0, \widetilde{p})$ is the unique solution.

2) Take derivative of the F.O.C. (7),

$$\begin{aligned}\frac{d^2\Pi_i}{dp_i^2} =& -\frac{1}{t}\left([q(p_i)]^2 + 2p_i q(p_i)\frac{dq}{dp}\Big|_{p=p_i}\right) - \frac{1-\varepsilon(p_i)}{t}\left[q(p_i)\right]^2 \\ &+ \left[\frac{1}{n} + \frac{1}{t}\int_{p_i}^{p_o} q\left(p\right)dp\right]\left((1-\varepsilon(p_i))\frac{dq}{dp}\Big|_{p=p_i} - q(p_i)\frac{d\varepsilon}{dp}\Big|_{p=p_i}\right).\end{aligned}$$

We know when the first order condition under symmetric price holds, $q\left(p^*\right)p = \frac{t}{n}\left[1 - \varepsilon(p^*)\right]$. Evaluate the second order derivative at p^*,

$$\begin{aligned}\frac{d^2\Pi_i}{dp_i^2}\Big|_{p=p^*} =& -\frac{1}{t}\left([q(p^*)]^2 + 2p^* q(p^*)\frac{dq}{dp}\Big|_{p=p^*}\right) - \frac{np^*q(p^*)}{t^2}\left[q(p^*)\right]^2 \\ &+ \left[\frac{1}{n} + \frac{1}{t}\int_{p^*}^{p^*} q\left(p\right)dp\right]\left(\frac{np^*q(p^*)}{t}\frac{dq}{dp}\Big|_{p=p^*} - q(p^*)\frac{d\varepsilon}{dp}\Big|_{p=p^*}\right) \\ =& -\frac{p^*q(p^*)}{t}\frac{dq}{dp}\Big|_{p=p^*} - \frac{q^2(p^*)}{t} - \frac{np^*q^3(p^*)}{t^2} - \frac{q(p^*)}{n}\frac{d\varepsilon}{dp}\Big|_{p=p^*} \\ =& -\frac{q^2(p^*)}{t}\left(1-\varepsilon(p^*)\right) - \frac{np^*q^3(p^*)}{t^2} - \frac{q(p^*)}{n}\frac{d\varepsilon}{dp}\Big|_{p=p^*}. \qquad (17)\end{aligned}$$

Since price elasticity is increasing in price $\left(\frac{d\varepsilon}{dp}\Big|_{p=p_i} > 0 \right)$ and whenever the first order condition holds $(1 - \varepsilon(p^*)) > 0$, the right hand side of equation (17) is strictly negative for $\forall p_i \in (0, \widetilde{p})$. In consequence, any firm's profit function is necessarily strictly concave whenever condition (10) holds. Hence for all of the firms, firm payoff is strictly quasiconcave in strategy variable p_i.

3) In this step we verify if any firm would have incentive to undercut its neighbors. For a firm to undercut its closest neighbors, the price it sets has to be low enough to attract consumers with a distance further than $\frac{1}{n}$. Using consumer's indirect utility function, for $0 < p_i < p^*$ the following condition has to hold.

$$\begin{aligned} &\int_{p_i}^{\hat{p}} q(p)dp - \frac{t}{n} + Y + V \geq \int_{p^*}^{\hat{p}} q(p)dp + Y + V \\ \Longleftrightarrow &\int_{p_i}^{p^*} q(p)dp \geq \frac{t}{n}. \end{aligned} \tag{18}$$

To investigate condition (18), we first prepare an additional result (i.e., inequality (21) below) for further use. By solving (10) we will have equilibrium price p^*, the corresponding demand $q^* = q(p^*)$ and price elasticity $\varepsilon^* = \varepsilon(p^*)$. Define constant

$$\varphi = \frac{q^*}{(p^*)^{-\varepsilon^*}}.$$

We construct a demand function with constant elasticity ε^*,

$$q^\dagger(p) = \varphi p^{-\varepsilon^*}$$

which also passes through the point (p^*, q^*). With this demand function we can obtain the following closed form formula for $0 < p_i < p^*$,

$$\int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp = \frac{\varphi}{1 - \varepsilon^*} p^{1-\varepsilon^*} \Big|_{p_i}^{p^*}. \tag{19}$$

Applying the necessary condition for symmetric equilibrium $1 - \varepsilon^* =$

$\frac{n}{t}p^* q\left(p^*\right)$, equation (19) becomes

$$\begin{aligned}\int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp =& \frac{q^*}{\left(p^*\right)^{-\varepsilon^*}} \frac{t}{np^* q^*} \left(\left(p^*\right)^{1-\varepsilon^*} - \left(p_i\right)^{1-\varepsilon^*}\right) \\ =& \frac{t}{n} \frac{\left(p^*\right)^{1-\varepsilon^*} - \left(p_i\right)^{1-\varepsilon^*}}{\left(p^*\right)^{1-\varepsilon^*}} \\ =& \frac{t}{n} \left(1 - \left(\frac{p_i}{p^*}\right)^{1-\varepsilon^*}\right).\end{aligned}$$

Since $0 \leq 1 - \varepsilon^* < 1$ and $0 < p_i < p^*$, we have

$$\int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp < \frac{t}{n}, \forall\ p_i \in (0, p^*). \tag{20}$$

Note also that $q^\dagger(p) = \varphi p^{-\varepsilon^*}$ has a constant elasticity ε^* while $q(p)$ obtains elasticity ε^* at the point (p^*, q^*) but strictly lower elasticity $\varepsilon < \varepsilon^*$ when price decreases. That is, for the same percentage decrease of price, although $q(p)$ and $q^\dagger(p)$ start out at the same point (p^*, q^*), $q(p)$ increase less than $q^\dagger(p)$ does. Hence,

$$\begin{aligned} & q(p) < \varphi p^{-\varepsilon^*}, \text{ for } \forall\ p \in (0, p^*) \\ \Longrightarrow & \int_{p_i}^{p^*} q(p) dp < \int_{p_i}^{p^*} \varphi p^{-\varepsilon^*} dp, \text{ for } \forall\ p_i \in (0, p^*).\end{aligned}$$

By condition (20) we have the next result,

$$\int_{p_i}^{p^*} q(p) dp < \frac{t}{n} \text{ for } \forall\ p_i \in (0, p^*). \tag{21}$$

Now we are ready to discuss the undercutting strategy for firm i facing consumer demand function $q(p)$. To undercut its neighbors who are charging the symmetric equilibrium price p^*, condition (18) has to hold. Because of the result we established in (21), there exists no positive price that satisfies condition (18). Hence there is no firm who is able to take over neighbor's business without losing money.

4) We have shown that for any $n \geq 2$, there exists a unique solution to condition (10). Moreover, the strategy profile characterized by this condition is indeed an equilibrium because firms' payoffs are strictly

quasiconcave in own strategy and there is no incentive for firms to undercut neighbors. This concludes the proof.

A.3 Proof of Lemma 2

Proof: Take total differentiation of equation (10) with respect to the number of firms,

$$
\begin{aligned}
\frac{dq^*}{dp}\frac{dp^*}{dn}p^* + q^*\frac{dp^*}{dn} &= \frac{t}{n}\left(-\frac{d\varepsilon^*}{dp}\frac{dp^*}{dn}\right) - (1-\varepsilon^*)\frac{t}{n^2} \\
\Longrightarrow \left(\frac{dq^*}{dp}p^* + q^*\right)\frac{dp^*}{dn} &= -\frac{t}{n}\frac{d\varepsilon^*}{dp}\frac{dp^*}{dn} - \frac{t}{n^2}(1-\varepsilon^*) \\
\Longrightarrow \frac{dp^*}{dn}\left(q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}\right) &= -\frac{t}{n^2}(1-\varepsilon^*) \\
\Longrightarrow \frac{dp^*}{dn} &= \frac{-\frac{t}{n^2}(1-\varepsilon^*)}{q(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}}.
\end{aligned}
$$

Since $(1-\varepsilon^*) > 0$ and $\frac{d\varepsilon^*}{dp} > 0$, in equilibrium $\frac{dp^*}{dn} < 0$.

Also from Assumption 1

$$
\frac{d\varepsilon^*}{dn} = \frac{d\varepsilon^*}{dp}\frac{dp^*}{dn} < 0.
$$

Differentiate equation (11) with respect to n:

$$
\begin{aligned}
\frac{d\Pi^*}{dn} &= -\frac{2t}{n^3}(1-\varepsilon^*) - \frac{t}{n^2}\frac{d\varepsilon^*}{dn} \\
&= -\frac{t}{n^3}(1-\varepsilon^*)\left[2 - \frac{t}{n}\frac{d\varepsilon^*}{dp}\left(\frac{1}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}}\right)\right] \\
&= -\frac{t}{n^3}(1-\varepsilon^*)\frac{2q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} < 0.
\end{aligned}
$$

Take total differentiation of equation (10) with respect to transportation costs,

$$\begin{aligned}
\frac{dq^*}{dp}\frac{dp^*}{dt}p^* + q^*\frac{dp^*}{dt} &= -\frac{t}{n}\left(\frac{d\varepsilon^*}{dp}\frac{dp^*}{dt}\right) + (1-\varepsilon^*)\frac{1}{n} \\
\Longrightarrow \frac{dp^*}{dt}\left(q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}\right) &= \frac{1}{n}(1-\varepsilon^*) \\
\Longrightarrow \frac{dp^*}{dt} &= \frac{\frac{1}{n}(1-\varepsilon^*)}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} > 0.
\end{aligned}$$

It follows:

$$\begin{aligned}
\frac{d\varepsilon^*}{dt} &= \frac{d\varepsilon}{dp}\frac{dp^*}{dt} \\
&= \frac{d\varepsilon}{dp}\frac{\frac{1}{n}(1-\varepsilon^*)}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} > 0.
\end{aligned}$$

Differentiate equation (11) with respect to transportation costs:

$$\begin{aligned}
\frac{d\Pi^*}{dt} =& \frac{1}{n^2}(1-\varepsilon^*) - \frac{t}{n^2}\frac{d\varepsilon^*}{dt} \\
=& \frac{1}{n^2}(1-\varepsilon^*)\left[1 - \frac{t}{n}\frac{d\varepsilon}{dp}\left(\frac{1}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon}{dp}}\right)\right] \\
=& \frac{1}{n^2}(1-\varepsilon^*)\frac{q^*(1-\varepsilon^*)}{q^*(1-\varepsilon^*) + \frac{t}{n}\frac{d\varepsilon^*}{dp}} > 0.
\end{aligned}$$

A.4 Proof of Proposition 2

Proof: From Lemma 3, we know that equilibrium elasticity under free entry increases in fixed cost. From Lemma 4, we know that $n^c > n^f$ when $\varepsilon^*_{n^c} < \frac{3}{4}$, $n^c = n^f$ when $\varepsilon^*_{n^c} = \frac{3}{4}$, and $n^c < n^f$ when $\varepsilon^*_{n^c} > \frac{3}{4}$.

We have to consider two cases:

1) $\varepsilon^*_{n=2} > \frac{3}{4}$. Then, there exists a fixed cost $\bar{f}$ such that the resulting equilibrium demand elasticity is equal to $\frac{3}{4}$. Since $\varepsilon^*_{n^c}$ increases in f, for $f < \bar{f}$, $\varepsilon^*_{n^c} < \frac{3}{4}$ which leads to excessive entry by Lemma 4. Conversely, for $f > \bar{f}$, $\varepsilon^*_{n^c} > \frac{3}{4}$ which means insufficient entry.

2) $\varepsilon^*_{n=2} < \frac{3}{4}$. Then, since $\varepsilon^*_{n^c}$ decreases in n, $\varepsilon^*_{n^c} < \frac{3}{4}$ for all values of f. And hence, there is excess entry.

We can also reformulate Proposition 2 in terms of transportation costs. What we need first is to show that there is a monotone relationship between equilibrium demand elasticity and transportation costs. This is $\frac{d\varepsilon^*_{n^c}}{dt} = \frac{d\varepsilon}{dn}\frac{dn}{dt} < 0$, as $\frac{d\varepsilon}{dn} < 0$ from Lemma 2 and $\frac{dn}{dt} > 0$.

Again, we must distinguish the two cases:

1) $\varepsilon^*_{n=2} > \frac{3}{4}$. Then, there exists a transportation cost $\bar{t}$ such that the resulting equilibrium demand elasticity is equal to $\frac{3}{4}$. Since $\varepsilon^*_{n^c}$ decreases in t, for $t > \bar{t}$, $\varepsilon^*_{n^c} < \frac{3}{4}$ which leads to excessive entry by Lemma 4. Conversely, for $t < \bar{t}$, $\varepsilon^*_{n^c} > \frac{3}{4}$ which means insufficient entry.

2) $\varepsilon^*_{n=2} < \frac{3}{4}$. Then, since $\varepsilon^*_{n^c}$ decreases in n, $\varepsilon^*_{n^c} < \frac{3}{4}$ for all values of t. And hence, there is excess entry.

We state the result formally as a corollary:

Corollary 1. Welfare result in terms of transportation costs.

i) Suppose $\varepsilon^*_{n=2} \geq \frac{3}{4}$ and define $\bar{t}$ as the transportation cost level that leads to equilibrium price elasticity ε^* of $\frac{3}{4}$. Then there is excess entry if $t > \bar{t}$, insufficient entry if $t < \bar{t}$ and optimal entry if $t = \bar{t}$.

ii) Suppose $\varepsilon^*_{n=2} < \frac{3}{4}$, then there is excess entry for all $t > T$.

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