IWQW

Institut für Wirtschaftspolitik und Quantitative
Wirtschaftsforschung

Diskussionspapier
Discussion Papers

No. 06/2014

WHERE AND WHEN TO PRAY? - OPTIMAL MASS PLANNING AND EFFICIENT RESOURCE ALLOCATION IN THE CHURCH

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ISSN 1867-6707

WHERE AND WHEN TO PRAY? - OPTIMAL MASS PLANNING AND EFFICIENT RESOURCE ALLOCATION IN THE CHURCH

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ABSTRACT. We will present a theoretical model for an integrated planning of Eucharistic masses and Liturgies of the Word assuming different priests, assistant priests and lay persons. This all-integer vector optimization problem can be seen as an economic, spacial and inter-temporal resource allocation problem taking the challenges in the context of parish clusters into account. Having provided some theoretical insights into the mass planning problem, numerical results show the relevance of our model for an efficient decision-making in religious practice. We demonstrate that innovative team work arrangements in the church can substantially improve the efficiency, fairness and acceptance of mass allocations.

1. INTRODUCTION

A lot of western countries suffer from a drastically decreasing number of priests. This *shortage of priests* yields an increased priest workload and given the dependence on priests confering the sacrament of the Eucharist results in a reduced availibility of Eucharistic masses. Therefore, a weekly Eucharistic mass cannot be guaranteed anymore in every church of a parish with the need of lowering the frequency of Eucharistic mass celebrations. This lowering raises the question, in which church of his parish a priest should offer an Eucharistic mass throughout the different weeks of a year. This *mass planning problem* can be seen as an economic, spacial and inter-temporal resource allocation problem with priests and their mass supply being the limiting factor that has different possible uses. Economic parish management and organizational rules can help in deciding on where and when masses should be celebrated in order to maximize optimization criteria like fairness and efficiency. As such, the present paper can enhance the objectivity, quality, acceptance and fairness of already existing mass allocations by attaining the best possible outcomes for the involved economic players (priests, catholics, lay persons, ...). In the current research project which is part of a cooperation with the Archbishopric of Bamberg (Germany) we tackle the complexity of Eucharistic mass plans in the context of parish clusters. A parish cluster is an association of individual parishes that form a new organisational unit responsible for a joint planning of ecclesial services (including mass celebrations). Even though the concept of parish

Date: June 22, 2014.
JEL Classification. C61,D63,L31,M54;
Key words and phrases. Allocation: Resources, Church, Job Design and Cooperation, Labor and Personnel Management, Normative Criteria, Optimization Models;
We would like to thank Helga Deinhardt, Sarah Frembs, Michael Gehret, Konrad Göller, Andreas Henkelmann, Johannes Müller, Ulrich Ortner, Bernhard Saffer, Hans Schieber, Gero P. Weißhaupt and Peter Wünsche for valuable comments and discussion.

clusters is not the same in every country, in general parish clusters can be seen as an attempt to restructure every day working routines through deeper inter-parish cooperation and team work arrangements. With efficient work organization being an important issue in personnel economics (see for instance Bailey and Cohen (1997), Deery and Iverson (2005), Galbraith (1977) or Mohrman et al. (1995)), our mass planning outcomes directly benefit from the cooperation of priests who traditionally used to structure and plan their ecclesial services separately for their own parish. However, this joint organisation makes the planning of Eucharistic masses much more complex. Parish clusters were introduced amongst others in Austria, Belgium, France, Germany, Italy, Switzerland, the Netherlands and in the USA.

In contrast to purely continuous planning problems, the discrete structure of mass allocation allows us to formulate an *all integer vector optimization problem* modelling our proposed new form of work organization in the church. We will discuss important model properties as well as various objectives like fairness or contiguity in the presence of integer variables (representing the number of masses to be celebrated by the different priests). Apart from introducing our theoretical mass allocation model, we will also show how to compute a pareto efficient solution to this problem. Therefore, our approach can directly help in assisting the church's every day planning and decision making in economic practice. Even though optimization approaches are a core element in the field of economics dating back to the first applications of linear programming for an optimal design of resource allocation mechanisms (see for instance Dantzig (1955) or Koopmans (1951)), they have not been used in efficiently allocating resources in the church. As such, the present paper can be seen as a first attempt to bridge the two disciplines economics and mathematical optimization with the church. Most of the existing studies analyze the priest shortage, its consequences and possible methods of resolution like alternative parish management systems on the basis of empirical evidence (see Hoge (1987), Schoenherr and Young (1990), Schuth (2009), Wilson (1986), Zech (1992, 1994)). Often experiences with different organisational structures are discussed (see also Seitz (2011) for a critical reflection), however, with no reference to theoretical models that are applicable for everyday planning in the church. Other studies can rather be seen as guidelines for modern parish leadership in practice (see for instance Hendricks (2009), Howes (1998), Jewell (2010), Mogilka and Wiskus (2009)).

The present paper is organized as follows. Section 2 presents data on the priest shortage in Germany and other western countries. It discusses the consequences of the priest shortage for the planning of Eucharistic masses as well as possible methods of resolution. These methods include for instance the deployment of assistant priests or the integrated planning of Liturgies of the Word. To get some first insights into the planning of Eucharistic masses, Section 3 presents a first mass planning model that assumes a single parish and a responsible priest resembling traditional mass planning of the past. Based on this simplified model we will introduce our multi-parish, multi-priest parish cluster model in Section 4. This model will allow for the above mentioned methods of resolution, finding an optimal mix of the methods used. A case study is presented in Section 5 that illustrates our concepts. The last Section 6 summarizes our main findings.

2. The Shortage of Priests

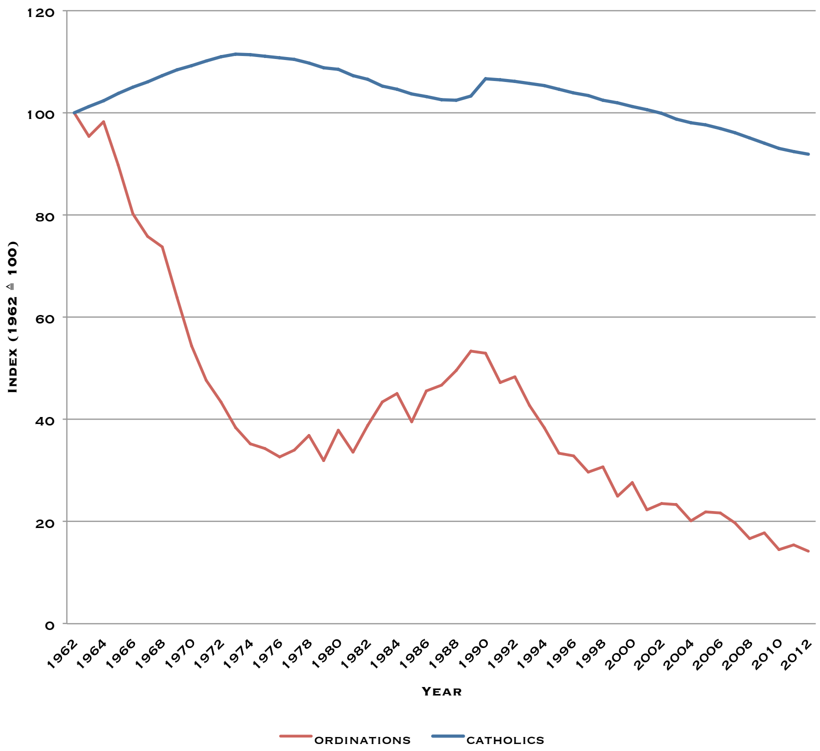
2.1. **Current Situation.** In Germany the number of (new) priests is falling. While in 1962 there have been over 550 ordinations to the priesthood, by 2012 this number has dropped to less than 80. Figure 1 depicts the annual development of the number of ordinations to the priesthood as well as the development of the number of catholics in Germany between 1962 and 2012. As can be seen, given the drastic drop in new priests, the fall in the number of catholics during this 50 year period is only moderate. The decline in new priests ultimately yielded a gap between the number of priests and the number of catholics, which is commonly known as the *shortage of priests*.

The decrease in the number of ordinations already began during the Second Vatican Council (1962-1965) and seemed to be stopped during the 1980s. However, the fall in ordinations to the priesthood continued after the reunion of Germany in the 1990s. An immediate consequence of this shortage is the increased workload for each active priest. Therefore, Figure 2 depicts the current situation in the different (arch)bishoprics of Germany, where workload is measured by i) the average number of catholics per priest and ii) the average number of parishes per priest. We note that only diocesan priests are considered in this analysis. It can be seen from Figure 2 that the current priest workload varies considerably among the diffent (arch)bishoprics of Germany. Since there seem to be 3 different types of (arch)bishoprics, we use a k-means cluster analysis to detect the members of these 3 groups. The results of this analysis indicate that the first group of (arch)bishoprics is characterized by both relatively low average parish and catholic values (blue dots). As can be seen, only 4 (arch)bishoprics belong to this group with a comparatively low level of workload. In the second group (red dots) one priest is on average responsible for few parishes (1 or less) with the average number of catholics per priest being relatively high. Obviousely, this increased catholic number yields additional pastoral care. We will refer to this group as (arch)bishoprics with a medium workload. The last group is also characterized by relatively high average catholic values but has in contrast to the second group a comparatively high average value of parishes (green dots). This might yield additional workload compared to the second group given various administrative tasks in the different parishes a priest is responsible for. This last group characterized by a comparatively heavy workload is with 14 members the largest group of (arch)bishoprics. Obviousely, this group size underlines the importance of new decision and planning methods to efficiently handle the problem of an increasing priest workload.

Having discussed the development of the number of priests in Germany, we will now focus on the question, whether the priest shortage is only a German phenomenon. Interestingly, while in 1972 there have been around 270.000 diocesan priests worldwide, this number slightly increased to almost 280.000 by the year 2010. Even though this seems to contradict the development in Germany, one also has to consider the number of catholics which almost doubled from around 680 million to nearly 1.1 billion during the same time period. This suggests that on average the workload per priest increased over the last 40 years. However, as indicated by Table 1, the development is not the same in all continents. As can be seen, the number of priests decreased in Australia, Europe and North America between 1972 and 2010. In contrast, especially African countries are characterized by a hugh increase in their number of priests. Between 1972 and 2010 the number of priests in Africa more than quadrupled. South America and Asia both show an increase by around 100-150 percent. This continent analysis indicates that the shortage of priests can in general be seen as a problem of the western world. However, apart from these inter-continental differences, Table 2 shows that there are also intra-continental differences. As depicted in Table 2, among countries with a falling priest number their respective percentage decrease is very different. Drops range from around 24 percent in Italy to more than 60 percent in the Netherlands, with Germany lying between these two countries. These enormous decrease rates again highlight the need in the western world for new and innovative organisational structures and arrangements tackling the problems that come along with the constantly decreasing number of priests.

2.2. **How to Tackle these Developments?** A lot of ecclesial services like the celebration of a Eucharistic mass are priest dependent. Given the shortage of priests in different western countries, the priests' increased workload ultimately yields to the problem that a weekly Eucharistic mass cannot be guaranteed anymore in every church of a priest's parish. This makes the planning of both Eucharistic mass times and sites more complex than before. In this subsection we will discuss different methods how to deal with the problem of an efficient Eucharistic mass planning in the

FIGURE 1. Number of Ordinations and Catholics in Germany



Source: German Bishops' Conference and kirchliches handbuch 97/98 (own calculations)

TABLE 1. Percentage Change in the Number of Priests between 1972 and 2010: Continents

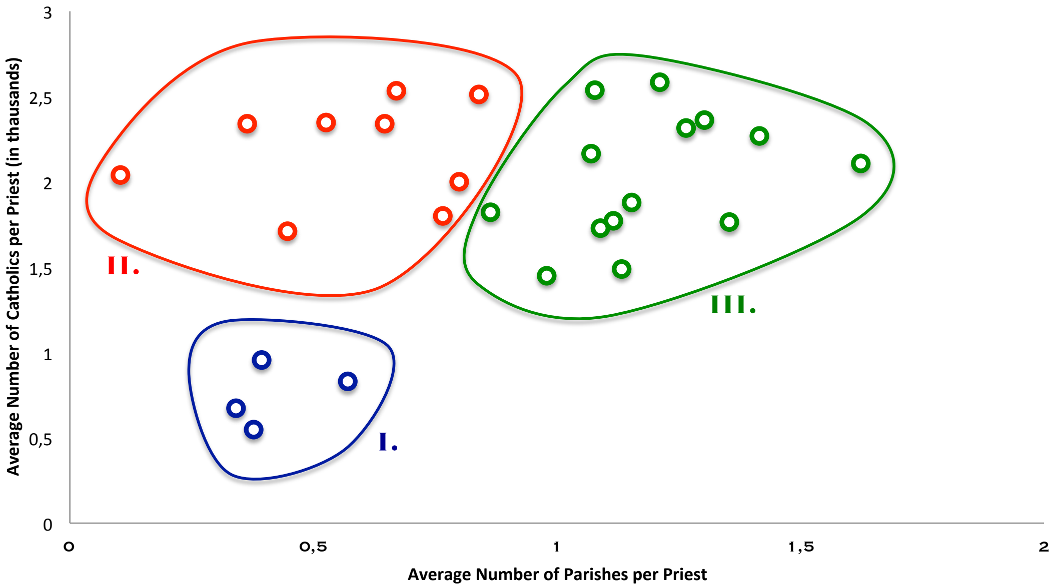
Africa	+431%
North America	-20%
South America	+105%
Asia	+151%
Europe	-27%
Australia	-21%

Source: Statistical Yearbook of the Church (1972 and 2010)

context of the priest shortage. Figure 3 depicts two main types of methods of resolution that we will refer to as direct and indirect measures.

Direct methods immediately influence the number of masses that can be celebrated in the different churches of a parish. As such, these methods directly affect total mass supply or shift the masses that are available between the different churches. One obvious way to directly increase the number of masses is to enhance the deployment of assistant priests. This increased deployment will positively affect the number of masses that can be celebrated in a given week reducing the

FIGURE 2. Workload per Priest in Different German (Arch)Bishoprics in 2012



SOURCE: GERMAN BISHOPS' CONFERENCE (OWN CALCULATIONS)

TABLE 2. The Priest Shortage in Different Western Countries (1972 vs. 2010)

NORTH AMERICA	USA	-16%
	CANADA	-38%
EUROPE	ITALY	-24%
	IRELAND	-25%
	AUSTRIA	-30%
	SPAIN	-32%
	GREAT BRITAIN	-32%
	GERMANY	-33%
	PORTUGAL	-36%
	HUNGARY	-49%
	FRANCE	-61%
	BELGIUM	-62%
	THE NETHERLANDS	-66%
OCEANIA	AUSTRALIA	-21%

SOURCE: STATISTICAL YEARBOOK OF THE CHURCH (1972 AND 2010) workload of the other priests. Another direct measure concerns the shift of masses between different churches of a parish. Here, the frequency of Eucharistic mass celebrations is lowered in some churches of a parish to ensure that in other 'prioritized' churches a weekly mass can still be ensured. Such priorizations are usally decided by the responsible priests in cooperation with local church bodies.

Indirect methods can be seen as accompaining or complementary measures. To give a first example of an indirect method, let us consider the case where the frequency of mass celebrations has been lowered and a weekly sunday mass cannot be guaranteed anymore in a given church. Then, the Code of Canon Law offers the possibility of integrating celebrations of Liturgies of the Word (Sacra Verbi Dei Calebratio) as some kind of complementary or indirect offer. Liturgies of the Word have been supported since the Second Vatican Council (Sacrosanctum Concilium, Nr. 35, 4) and are explicitly recommended by the Code of Canon Law in the case where a participation in an Eucharistic mass celebration seems impossible (see Can. 1248 §2 for details). From a theological point of view it is important to point out that if an Eucharistic mass can be celebrated at a given church in a given week, then there must not be a Liturgy of the Word at this church. This regulation directly links the planning of Eucharistic masses with the planning of Liturgies of the Word making it more complex than before. A second indirect measure is the introduction of larger administrativ and organisational units called pastoral or parish clusters (Pfarrverband, Seelsorgeeinheit, Unità Pastorale). During the last few years parish clusters were introduced amongst others in Austria, Belgium, France, Germany, Italy, Switzerland, the Netherlands and in the USA. With this structural reform dioceses tackle the problem of the priest shortage by jointly organizing parishes of the same parish cluster. The main advantage of introducing parish clusters is that an imbalanced priest-catholics relation can be better resolved by a deeper inter-parish cooperation without the need of abolishing existing structures and borders completely. Parish clusters are as a whole responsible for diverse ecclesial services including the planning of Eucharistic mass celebrations. To give an example for an introduction of parish clusters, the German Archbishopric of Bamberg (713.781 catholics) introduced 95 parish clusters formed by 310 parishes in 2006. As another recent example, the US American Archdiocese of Boston plans to cluster its 290 parishes into 125 groups.

3. Planning of Eucharistic Masses: A Single Parish Model

In this section we will assume a single parish and a responsible priest. We will introduce a first model that allocates the Sunday masses (including the Saturday evening masses) celebrated by the priest throughout the different weeks of the planning horizon to the churches of his parish. Obviousely, time and space are the two basic demand and supply decision parameters here. This simple model will help us to get some first insights into the (traditional) planning of Eucharistic masses and its underlying problem structure. In the following chapter we will relax many of the assumptions made allowing for a more realistic mass planning model.

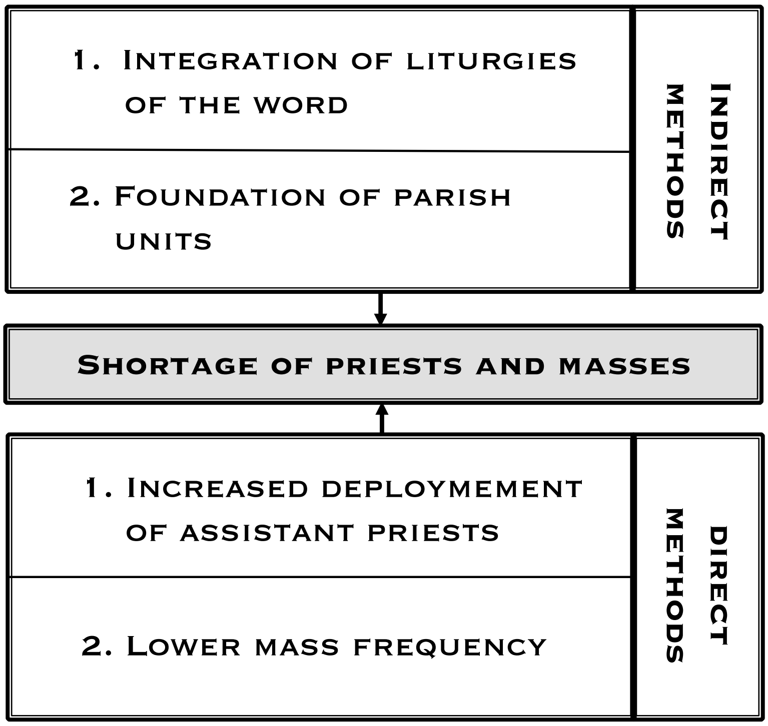
3.1. **Model Framework.**

Churches and Eucharistic Mass Demand. Let C be the set of churches in the parish and W the set of all relevant weeks to be planned. We will introduce the following binary variable x_{cw}:

$$x_{cw} = \begin{cases} 1, & \text{if an Eucharistic mass is celebrated at church } c \text{ in week } w \\ 0, & \text{otherwise} \end{cases}$$

Let further be given the frequency $f_c \in \mathbb{Z}$ of Eucharistic mass celebrations for each church $c \in C$, e.g., the total number of masses that are celebrated at this church throughout the entire planning

FIGURE 3. The Shortage of Eucharistic Masses: Possible Methods of Resolution



period:

$$\sum_{w \in W} x_{cw} = f_c, \ \forall c \in C \tag{3.1}$$

This *mass demand* of church $c \in C$ will be met by deciding in which weeks $w_c^1, w_c^2, \ldots, w_c^{f_c}$ of the planning horizon W the given number of masses f_c will be celebrated at church c (inter-temporal mass allocation). As observed in practice, we may assume that the frequency f_c is exogeneousely given and determined by the priest in collaboration with the local church bodies.

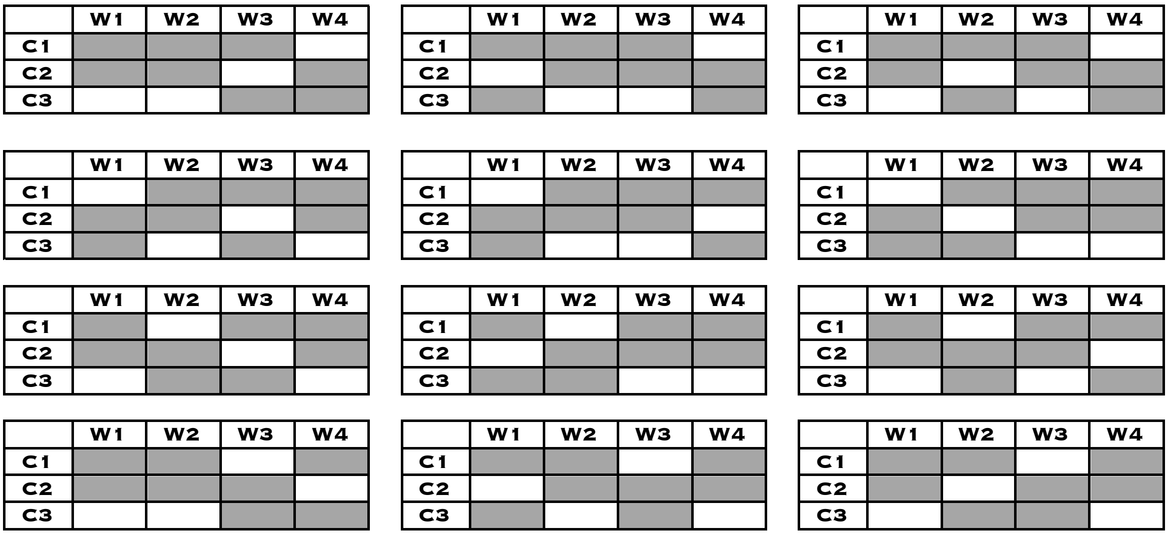
Priests and Eucharistic Mass Supply. Obviousely, a priest cannot celebrate any number of masses a week. In this section we will assume that our priest celebrates a given number $\bar{x} \in \mathbb{Z}$ of masses each week:

$$\sum_{c \in C} x_{cw} = \bar{x}, \ \forall w \in W \tag{3.2}$$

This *mass supply* will be met by deciding at which churches $c_w^1, c_w^2, \ldots, c_w^{\bar{x}}$ the offered masses will be celebrated in each of the $|W|$ weeks (spacial mass allocation). For later reference let us introduce $M = \{1, \ldots, \bar{x}\}$ as the set of the $\bar{x}$ masses to be allocated each week. The next section will discuss realistic values for $|M| = \bar{x}$ based on the Code of Canon Law and relax the equality constraint in (3.2) allowing for a variable mass supply.

Mass Plans. Now we can introduce the definition of a *mass plan*. An allocation $x \in \{0,1\}^{|C \times W|}$ is called a *mass plan*, if constraints (3.1) and (3.2) are satisfied. In this case mass demand is met by mass supply in every week $w \in W$ and in every church $c \in C$. The problem of finding a mass plan

FIGURE 4. A First Mass Planning Problem: 3 Churches and 4 Weeks



will be denoted as the *mass planning problem*, which is obviousely a feasibility problem. Let us consider a first example that illustrates our concepts:

Example 3.1. We will consider a planning horizon of 4 weeks with $W = \{1, 2, 3, 4\}$. Assume we are given a parish with three different churches $C = \{c_1, c_2, c_3\}$ and mass frequencies of $3, 3$ and 2, respectively. The responsible priest celebrates 2 masses every week. Figure 4 depicts the 12 feasible mass plans that satisfy all mass demand and mass supply constraints.

3.2. **Equivalent Network Formulation.** In this subsection we will introduce an equivalent network formulation for the above mass planning problem. This network formulation will give us some insights into the complexity of our first mass planning model.

Let us formally introduce the concept of a graph. A graph G is a pair (N, E) with N denoting the set of network nodes and E being the set of edges. In the present mass planning setting our graph is a time-expanded network with super nodes that connect the subgraphs corresponding to the different planning weeks (time periods). To distinguish between different node types of the node set N, we will partition N in three different node groups $\{\bar{N}, \tilde{N}, \hat{N}\}$ that are described below. Later we will see that a feasible mass plan corresponds to an assignment of the supplied masses at the nodes $\bar{N}$ through the intermediate nodes $\tilde{N}$ to the end nodes $\hat{N}$. Analogous to the node set partition the edge set E will be introduced as the union of the two edge sets $\hat{E}$ and $\bar{E}$. To construct our time-expanded mass planning graph $G = (N, E)$ we will proceed as follows: In every week $w \in W$ we will introduce for each mass $m \in M$ a node $n_m^w \in \hat{N}$. We will denote the nodes in $\hat{N}$ as destinations. Analogous, for every week $w \in W$ we will define for each church $c \in C$ a node $n_c^w \in \tilde{N}$. These nodes will be refert to as intermediate network nodes. Then, $\hat{E}$ describes the edges that join node pairs $(n_c^w, n_m^w) \in \tilde{N} \times \hat{N}$. Obviousely, the introduction of these edges yields $|W|$ bipartite subgraphs (see also Figure 5a for an example). To connect these subgraphs, let us now define for each church $c \in C$ a super node $n_c \in \bar{N}$. The nodes in $\bar{N}$ will also be refert to as origins. The corresponding edges $e \in \bar{E}$ are defined as node pairs (n_c, n_c^w), with $n_c \in \bar{N}$ and $n_c^w \in \tilde{N}$. As an immediate consequence of the introduction of $\bar{E}$, every super node $n_c \in \bar{N}$ is connected to each of the $|W|$ bipartite subgraphs exactly once. Figure 5a depicts the resulting time-expanded graph using the network representation of Example 3.1.

Now, let us define for each origin in $\bar{N}$ a supply of f_c and for each destination in $\hat{N}$ a demand of 1. Obviousely, a feasible mass plan corresponds to an assignment of the supplied masses at the origins through the intermediate nodes $\tilde{N}$ to the destinations. We will call such an assignment a *(feasible) network flow* in our mass planning graph. A network flow that corresponds to a solution to our mass planning problem in Example 3.1 is depicted in Figure 5b in red.

Introducing the edge variables y_e for all $e \in E$ in addition to the original mass allocation variables x, we obtain the following network formulation for our mass planning problem:

$$\sum_{e=(n_c,n_c^w)\in\bar{E}} y_e = f_c, \qquad \forall n_c \in \bar{N} \tag{3.3}$$

$$-y_{e=(n_c,n_c^w)} + \sum_{e=(n_c^w,n_m^w)\in\hat{E}} y_e = 0, \qquad \forall n_c^w \in \tilde{N} \tag{3.4}$$

$$\sum_{e=(n_c^w,n_m^w)\in\hat{E}} y_e = 1, \qquad \forall n_m^w \in \hat{N} \tag{3.5}$$

$$y_e \in \{0,1\}, \qquad \forall e \in E \tag{3.6}$$

Constraints (3.3) ensure that a flow of f_c leaves each origin $n_c \in \bar{N}$. Analogous, Constraints (3.5) guarantee that a flow of 1 arrives at each destination $n_m^w \in \hat{N}$. Constraints (3.4) and (3.6) ensure flow balance at every intermediate node $n_c^w \in \tilde{N}$ and integrality of each flow variable y_e. The next lemma shows that this network representation is indeed equivalent to our original mass planning model:

Lemma 3.2. *The original and the network based model of the mass planning problem are equivalent.*

Proof. Let us be given a solution x^*_{cw} to the original model formulation. Let C^w describe for every week $w \in W$ the churches $c \in C$ for which $x^*_{cw} = 1$ holds. Without loss of generality we will sort the different churches in C^w in ascending order. Obviousely, we can set $y_{(n_c^w,n_m^w)} = 1$ if church $c \in C^w$ is the m^{th} element in C^w. This satisfies for each destination $n_m^w \in \hat{N}$ its demand of 1. Given the churches in C^w we can also set $y(n_c, n_c^w) = 1$ if $x^*_{cw} = 1$. This step ensures flow balance of the intermediate nodes $n_c^w \in \tilde{N}$ and satisfies the supply of f_c at all origins $n_c \in \bar{N}$.

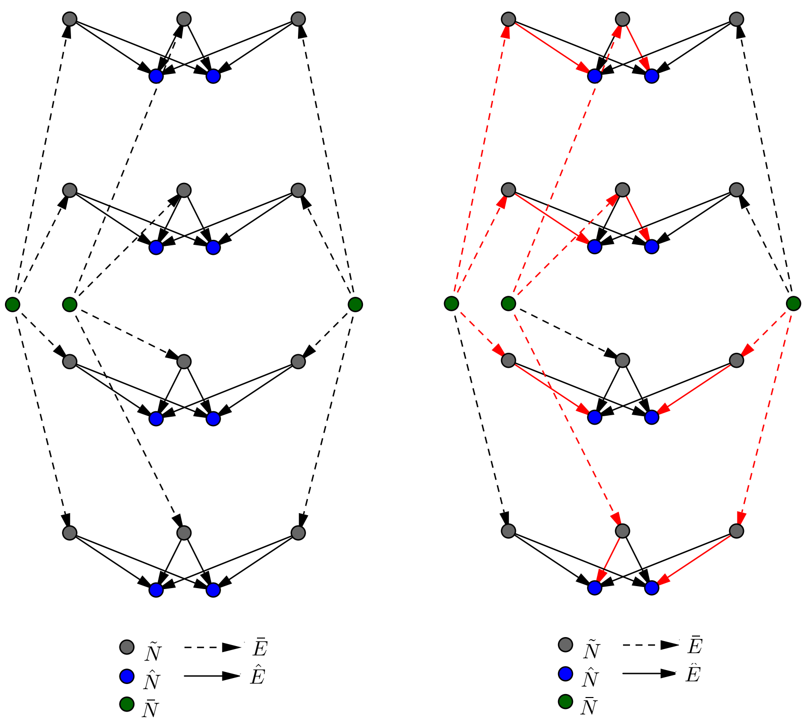
Now let us be given a solution y^*_e to the network formulated model. Obviousely, if $y^*_{(n_c,n_c^w)} = 1$, then we can set $x_{cw} = 1$. Given that in every feasible solution to the network model demand and supply will be met, we immediately arrive at a feasible solution to the original mass planning model. □

Obviousely, the above network based model is formulated as a transshipment problem (see for instance Orden (1956) or Chvátal (2002)). An immediate consequence of the total dual integrality of the transshipment problem is that our mass planning problem can be solved in polynomial time:

Theorem 3.3. *The mass planning problem can be solved in polynomial time.*

Proof. The network formulation has $|W| \cdot |C| \cdot (1 + |M|)$ binary variables. Apart from that it has $|W| \cdot (|C| + |M|) + |C|$ constraints. Given that the network based representation of the mass planning problem is formulated as a transshipment problem, its polynomiality carries over to the original mass planning formulation. □

FIGURE 5. Construction of a Mass Planning Graph (Example 3.1)



(A) MASS PLANNING GRAPH (B) A FEASIBLE FLOW (RED)

Obviousely, our mass planning problem resembles traditional mass planning where priests used to structure and plan their ecclesial services separately for their own parish. The polynomiality of the mass planning problem and its network formulated version confirm the intuition that traditional mass planning in the past was much easier than the present mass allocation in the context of parish clusters introduced below. As another important aspect our network formulation also shows that the mass planning problem is feasible if and only if total mass supply is equal to total mass demand, as otherwise no feasible network flow exists:

$$\sum_{c \in C} f_c = \bar{x} \cdot |W| \tag{3.7}$$

We note that in the case of a general binary programming problem such a characterization is rather unlikely to be found.

4. PLANNING OF EUCHARISTIC MASSES: A MULTI-PARISH, MULTI-PRIEST MODEL

In this section we will introduce a more realistic parish cluster model assuming different priests, assistant priests as well as lay persons. Given this personnel of a parish cluster, we want to find an assignment to the masses (Eucharistic masses and Liturgies of the Word) celebrated throughout the planning period that minimizes or maximizes given optimization criteria. Again demand and supply decisions concern both temporal and spacial aspects. We will formulate this *optimal mass planning problem* as an all integer vector optimization problem.

4.1. **Priests and Eucharistic Mass Supply.** Assume we are given a parish cluster. Let P be the set of priests and C the set of churches of the parish cluster. $C_p^* \subset C$ describes for each priest $p \in P$ the set of churches located in his parish(es). Analogous, $C_p^{**} \subset C_p^*$ denotes priest p's parish church(es). Let us now introduce the following binary variable x_{pcw}:

$$x_{pcw} = \begin{cases} 1, & \text{if priest } p \text{ celebrates an Eucharistic mass at church } c \text{ in week } w \\ 0, & \text{otherwise} \end{cases}$$

We will assume that every $k \in \mathbb{Z}$ weeks a priest must have a mass-free weekend for excursions with (youth) groups, baptism weekends etc. Introducing the variable y_{pw} with

$$y_{pw} = \begin{cases} 1, & \text{if priester } p \text{ does not have a mass-free weekend in } w \\ 0, & \text{otherwise} \end{cases}$$

we have

$$\sum_{w=a}^{a+k-1} y_{pw} = k-1, \ \forall p \in P, \forall a \in W \setminus \{|W|-k+1, \dots, |W|\}. \tag{4.1}$$

We note that Constraint (4.1) symmetrically distributes the mass-free weekends over the planning period ensuring that the resulting mass plan will have a simple structure being easy to remember. Apart from that Constraint (4.1) also guarantees that a priest does not have several mass-free weekends in consecutive weeks.

In each non mass-free week priests celebrate between $\overline{x}_p \in \mathbb{Z}$ and $\underline{x}_p \in \mathbb{Z}$ Eucharistic masses that can be seen as given upper and lower mass supply bounds. We note that we explicitly allow the priests $p \in P$ to have different bounds. This seems useful if for instance ill or older priests cannot celebrate the same number of masses as young and fit priests (fairness):

$$\underline{x}_p y_{pw} \leq \sum_{c \in C} x_{pcw} \leq \overline{x}_p y_{pw}, \ \forall p \in P, \forall w \in W \tag{4.2}$$

According to the Code of Canon Law, in 'normal' situations priests should not celebrate more than once a day (see Can. 905 §1). However, exclusionary rules apply for instance in the case of pastoral necessity or a priest shortage allowing a priest to celebrate up to 3 masses on Sundays (see Can. 905 §2). Therefore, a realistic upper bound will lie around 3 masses per week.

Furthermore, we will assume that during any interval of $l \in \mathbb{Z}$ consecutive weeks a priest must hold at least $g \in \mathbb{Z}$ Eucharistic masses in churches of his own parish:

$$\sum_{c \in C_p^*} \sum_{w=a}^{a+l-1} x_{pcw} \geq g, \ \forall p \in P, \forall a \in W \setminus \{|W|-l+1, \dots, |W|\}. \tag{4.3}$$

Analogous, we will assume that during any period of $m \in \mathbb{Z}$ consecutive weeks a priest must hold at least $n \in \mathbb{Z}$ Eucharistic masses in his parish churches:

$$\sum_{c \in C_p^{**}} \sum_{w=a}^{a+m-1} x_{pcw} \geq n, \ \forall p \in P, \forall a \in W \setminus \{|W|-m+1, \dots, |W|\}. \tag{4.4}$$

4.2. **Assistant Priests and Mass Supply.** Now, let be given a set S of assistant priests. We will introduce the following binary variable:

$$z_{scw} = \begin{cases} 1, & \text{if assistant priest } s \text{ celebrates a mass at church } c \text{ in week } w \\ 0, & \text{otherwise} \end{cases}$$

We will assume that in each week w an assistant priest celebrates between $\overline{z}_s \in \mathbb{Z}$ and $\underline{z}_s \in \mathbb{Z}$ Eucharistic masses:

$$\underline{z}_s \leq \sum_{c \in C} z_{scw} \leq \overline{z}_s, \ \forall s \in S, \forall w \in W \tag{4.5}$$

We note that we do not explicitly model mass-free weekends for assistant priest which could easily be integrated. The reason behind this simplification is that asisstant priests will in general offer less masses and provide them on a regular basis.

4.3. **Lay Persons and Supply of Liturgies of the Word.** Let be given a set L describing members of the laity. We will assume that each lay person $l \in L$ can celebrate at most $\overline{v}_l \in \mathbb{Z}$ and at least $\underline{v}_l \in \mathbb{Z}$ Liturgies of the Word in each week $w \in W$. Introducing the binary variable

$$v_{lcw} = \begin{cases} 1, & \text{if member } l \text{ of the laity celebrates a Liturgy of the Word at church } c \text{ in week } w \\ 0, & \text{otherwise} \end{cases}$$

we arrive at the following constraint:

$$\underline{v}_l \leq \sum_{c \in C_l^*} v_{lcw} \leq \overline{v}_l, \ \forall l \in L, \forall w \in W \tag{4.6}$$

Assuming that a lay person will only offer a Liturgy of the Word at predetermined churches (for instance at churches located in his/her parish), in Equation (4.6) C_l^* denotes the churches lay person l is willing to celebrate a Liturgy of the Word at.

As Liturgies of the Word are offered on a purely voluntary basis, a lay person's total, intertemporal mass supply will also be limited. Therefore we will assume that a lay person can celebrate a maximum of $\overline{\overline{v}}_l$ Liturgies of the Word throughout the entire planning period:

$$\sum_{w \in W} \sum_{c \in C_l^*} v_{lcw} \leq \overline{\overline{v}}_l, \ \forall l \in L \tag{4.7}$$

From a theological point of view, if an Eucharistic mass is celebrated at church $c \in C$ in week $w \in W$, then at this church there must not be a Liturgy of the Word. This rule underlines the difference between an Eucharistic mass celebration and a Litugy of the Word and explicitly highlights the uniqueness and importance of the Eucharist (DBK, 2006). Therefore, our integrated planning of Liturgies of the Word must not negatively affect or replace Eucharistic mass celebrations:

$$\left(\sum_{p \in P} x_{pcw}\right) + \left(\sum_{s \in S} z_{scw}\right) + \left(\sum_{l \in L} v_{lcw}\right) \leq 1, \ \forall c \in C, \forall w \in W \tag{4.8}$$

Similar to assistant priests we do not model endogenous mass-free weekends for the laity. However, we will assume that the different lay persons are only willing to offer a Liturgy of the Word in predetermined weeks $\tilde{W}_l$, for each $l \in L$. We can think of the weeks $W \setminus \tilde{W}_l$ as weekends where lay person l is engaged in other (leisure) activities. We also note that similar to Eucharistic masses, Liturgies of the Word should be symetrically allocated over the planning horizon (see below).

4.4. **Churches and Eucharistic Mass Demand.** Let us again denote the frequency of mass celebrations of church $c \in C$ by f_c. We will assume that the masses celebrated in a given church $c \in C$ should symmetrically be distributed over the planing horizon W. This requirement ensures that the resulting mass plan has a simple structure that is easy to remember. Obviousely, in a symmetric allocation there should be an Eucharistic mass every $|W|/f_c$ weeks. If this number is integral, then the following set of constraints ensures symmetry of mass celebrations:

$$\left(\sum_{p\in P}\sum_{w=a}^{a-1+|W|/f_c} x_{pcw}\right)+\left(\sum_{s\in S}\sum_{w=a}^{a-1+|W|/f_c} z_{scw}\right)=1,\ \forall c\in C:\frac{|W|}{f_c}\in\mathbb{Z}, a\in W\setminus\left\{|W|+1-\frac{|W|}{f_c},\dots,|W|\right\} \tag{4.9}$$

In our analysis we will implicitly assume that given the planning horizon W, the fraction $|W|/f_c$ is integer for all $c \in C$. However, we note that in any other case one possibility to deal with a fractional value would be to enlarge the planning horizon restoring integrality. Presenting an alternative approach, let us consider the following example: Assume a planning horizon of six weeks and consider a church with a frequency of four. Obviousely, there cannot be an Eucharistic mass celebration every 1.5 weeks. However, in such a case there should be between 1 and 2 Eucharistic masses every 2 subsequent weeks. In the general case, let us define $\overline{w} = \lceil |W|/f \rceil$. Then, we arrive at the following symmetry condition:

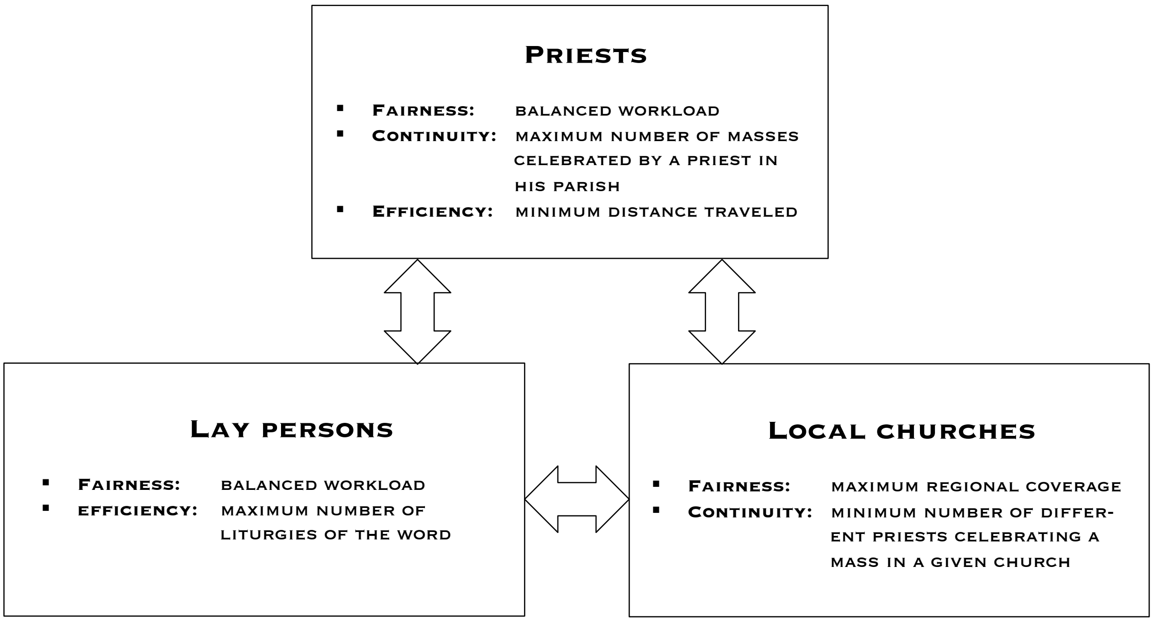
$$1\le\left(\sum_{p\in P}\sum_{w=a}^{a-1+\overline{w}} x_{pcw}\right)+\left(\sum_{s\in S}\sum_{w=a}^{a-1+\overline{w}} z_{scw}\right)\le 2,\ \forall c\in C,\forall a\in W\setminus\{|W|+1-\overline{w},\dots,|W|\} \tag{4.10}$$

4.5. **Optimization Criteria.** Accounting for the different economic players of priests (including assistant priests), lay persons and catholics - in our model represented by the local churches - there might be very different objectives that are pursued by these groups at the same time. Depending on the structures of the respective parish cluster, some of these objectives have a complementary relationship, others rather conflict each other. In the latter case, an efficient trade-off between these objectives must be found. Figure 6 depicts possible objectives of the three groups of interest: Obviousely, a mass plan should be fair in the sense that respecting the priests' *fitness*, their work (mass celebrations) should adequately be shared by the different priests. While the same is true for lay persons, fairness for the different churches means that in a given week mass celebrations should not be concentrated at a certain regional area but rather maximize regional coverage. This spacial criterion improves the possibility of attending an Eucharistic mass by reducing the regional distance between the place of the Eucharistic mass celebration and the homes of the respective catholics (parish). Turning again to the priests and their mass supply, priests are obviousely related to their parish(es). Especially in view of a continuity aspect, the special relationship between a priest and his parish(es) aims at maximizing the number of masses celebrated by a priest in his parish(es). At the same time continuity also aims at minimizing the number of different priests that celebrate an Eucharistic mass in a given church. Another objective might concern the economic efficiency of a mass allocation. Obviousely, a mass plan should minimize the distance that is traveled by the priests to arrive at the churches where they celebrate an Eucharistic mass at. With regard to the decreasing number of Eucharistic masses available and the increasing interval between two subsequent Eucharistic mass celabrations at a given church, a maximization of the number of Liturgies of the World that can adequately be integrated into a mass plan might also become an important aspect. This criterion tries to make use of the masses that are provided on a voluntary basis and explicitly encourages voluntary work.

We will now discuss some of the above criteria more in detail:

Workload Differences and Fairness. Obviousely, a priest assignment should minimize overall workload differences of priest pairs. In the presence of integer variables a direct problem arises when 2 or more priests have different upper or lower mass supply bounds. To give a first example, let us consider 2 priests with lower bounds of 0 and 2. We will assume corresponding upper

FIGURE 6. Optimization Criteria and Possible Conflicts of Interests



bounds of 2 and 3, respectively. Even though a situation where both priests celebrate 2 masses seems to be fair at first glance, in this scenario the mass supply of the fitter priest 2 lies at its lower bound whereas the mass supply of the less fitter priest 1 lies at its upper bound. This introductory example shows that a fair allocation must obviousely account for differences in lower and upper mass bounds. To formalize the concept of fairness used in our model, we will proceed as follows (to keep explanations as simple as possible, in the following we will ignore mass-free weeks for priests):
Using the accumulated lower bounds and the accumulated differences between the upper and lower bound of a priest's mass supply, we can define the following normalized mass supply of a priest $p \in P$:

$$\frac{\left(\sum_{c\in C}\sum_{w\in W} x_{pcw}\right) - |W|\cdot \underline{x}_p}{|W|\cdot\left(\overline{x}_p - \underline{x}_p\right)}, \ \forall p \in P : \overline{x}_p \neq \underline{x}_p \tag{4.11}$$

In the case $\overline{x}_p = \underline{x}_p$, priest p has to celebrate in each week $w \in W$ the given number of masses $\underline{x}_p$. Therefore, he can obviousely be excluded from our fairness analysis. Now, let us sort the different priests according to their fitness in ascending order:

$$\frac{\left(\sum_{c\in C}\sum_{w\in W} x_{1cw}\right) - |W|\cdot \underline{x}_1}{|W|\cdot\left(\overline{x}_1 - \underline{x}_1\right)} \leq \ldots \leq \frac{\left(\sum_{c\in C}\sum_{w\in W} x_{|P|cw}\right) - |W|\cdot \underline{x}_{|P|}}{|W|\cdot\left(\overline{x}_{|P|} - \underline{x}_{|P|}\right)} \tag{4.12}$$

TABLE 3. Fairness Among Priests

Individual Mass supply		Normalized Mass Supply		Feasible Allocation?
priest 1	priest 2	priest 1	priest 2	
0		0		YES
1	2	0.5	0	NO
2		1		NO
0		0		YES
1	3	0.5	1	YES
2		1		YES

Then, in order to minimize overall workload differences we can just use the following normalized mass supply difference between the fittest priest $|P|$ and the least fittest priest 1:

$$\min f_1(x) = \frac{\left(\sum_{c\in C}\sum_{w\in W} x_{|P|cw}\right) - |W|\cdot \underline{x}_{|P|}}{|W|\cdot\left(\overline{x}_{|P|} - \underline{x}_{|P|}\right)} - \frac{\left(\sum_{c\in C}\sum_{w\in W} x_{1cw}\right) - |W|\cdot \underline{x}_{1}}{|W|\cdot\left(\overline{x}_{1} - \underline{x}_{1}\right)} \tag{4.13}$$

Obviousely, in the case where all priests have identical lower and upper bounds, the above objective simplifies to:

$$\min f_1'(x) = \sum_{c\in C}\sum_{w\in W}\left(x_{|P|cw} - x_{1cw}\right) \tag{4.14}$$

We note that the above criteria can directly be expanded to the other two groups of assistant priests and lay persons.

Table 3 illustrates our fairness concept for the introductory example with 2 priests. As can be seen, the two allocations $(1,2)$ and $(2,2)$ are not feasible as they violate the normalized mass supply condition (4.12).

Continuity and Minimal number of priests celebrating a mass in a given church. Let us introduce for all priests $p \in P$ and all churches $c \in C$ a binary variable κ_{pc} indicating whether priest p holds at least one Eucharistic mass at church c throughout the entire planning period:

$$\sum_{w\in W} x_{pcw} \le f_c \cdot \kappa_{pc}, \ \forall c \in C, p \in P \tag{4.15}$$

$$\sum_{w\in W} x_{pcw} \ge \kappa_{pc}, \ \forall c \in C, p \in P \tag{4.16}$$

Constraint (4.15) guarantees that in the case $\sum_{w\in W} x_{pcw} \ge 1$, we have $\kappa_{pc} = 1$. Analogous, Constraint (4.16) ensures $\kappa_{pc} = 0$ if $\sum_{w\in W} x_{pcw} = 0$. Obviousely, the minimization of the number of priests celebrating at least one mass in a given church can be expressed as:

$$\min f_2(x) = \sum_{p\in P}\sum_{c\in C}\kappa_{pc} \tag{4.17}$$

Number of masses not celebrated by a priest in his parishes. The minimization of the number of Eucharistic masses hold in churches not belonging to some priest's own parishes can be expressed as:

$$\min f_{31}(x) = \sum_{p \in P} \sum_{c \in C \setminus C_p^*} \sum_{w \in W} x_{pcw} \tag{4.18}$$

This minimization is obviousely equivalent to a maximization of the number of Eucharistic masses celebrated in churches belonging to a priest's own parishes:

$$\max f_{32}(x) = \sum_{p \in P} \sum_{C_p^*} \sum_{w \in W} x_{pcw} \tag{4.19}$$

Number of Litugies of the Word celebrated throughout the planning horizon. Maximizing the total number of Liturgies of the Word celebrated in the parish cluster, we can use the sum of all Liturgies of the Word as an objective function:

$$\max f_4(x) = \sum_{l \in L} \sum_{c \in C} \sum_{w \in W} v_{lcw} \tag{4.20}$$

4.6. **Solution Approach.** The main difficulty in solving our mass planning problem lies in the simultaneous optimization of several (possibly contrary) objectives. Such a multi objective approach is commonly refert to as a *vector optimization problem* of the form

$$\min_{x \in X} f(x) = \left(f_1(x), f_2(x), \ldots\right)^T, \tag{4.21}$$

where X describes the feasible region of the problem (see for instance Chen, Huang and Yang (2005), Ehrgott (2005), Jahn (2004) or Luc (1989)). In the case of a single objective $f(x) \in \mathbb{R}$, given any pair of feasible solutions $(x_1, x_2) \in X \times X$, we can always decide whether $f(x_1) \geq f(x_2)$ or $f(x_1) < f(x_2)$ holds. Obviousely, such a comparison is not possible in the case of a vector-valued objective which yields the problem that in general an optimal point cannot be computed. Therefore, we will introduce the concept of a pareto optimal point. We call a point $x^* \in X$ pareto optimal, if there does not exist a point $x' \in X$ satisfying $f_i(x') \leq f_i(x^*)$ for all $i \in \{1, 2, \ldots\}$, and $f_i(x') < f_i(x^*)$ for at least one $i \in \{1, 2, \ldots\}$. Intuitively, a point is pareto optimal if there exists no other feasible point for which at least one entry of the vector-valued objective function is strictly smaller while the other entries are not larger.

A common strategy to compute pareto optimal solutions to a given vector optimization problem is to introduce weights $g_i \geq 0$ for the different objectives and to construct a single scalar objective function $f'(x)$. This approach is refert to as the weighted-sum or *scalarization method*:

$$\min f'(x) = g_1 \cdot f_1 + g_2 \cdot f_2 + \ldots \tag{4.22}$$

As the scalarization method only needs a ranking of the different objectives, this solution strategy seems directly applicable for the use in economic practice. For this reason our case study in the next section will build on this solution approach.

5. Case Study

Our case study is based on a German parish cluster with around 9.000 catholics. Using this medium-sized parish cluster, we will illustrate the scope of our approach. We implemented our mass planning model in ZIMPL and used SCIP to solve our problem instances. All experiments were performed on a 12 core computer equipped with two AMD Opteron(tm) 2435 Processors, with 2x6 MB cache and 64 GB DDR2-RAM, running Linux (in 64 bit mode).

Let us now briefly review the relevant problem data. The considered parish cluster consists of 6 parishes with 3 different priests. Each of the 3 priests is responsible for 2 parishes and celebrates between two and three masses per weekend. Let us consider a planning period of 4 months (16 weeks). We will assume the two objectives of i) fairness maximization among priests and ii)

TABLE 4. Input Data: Churches

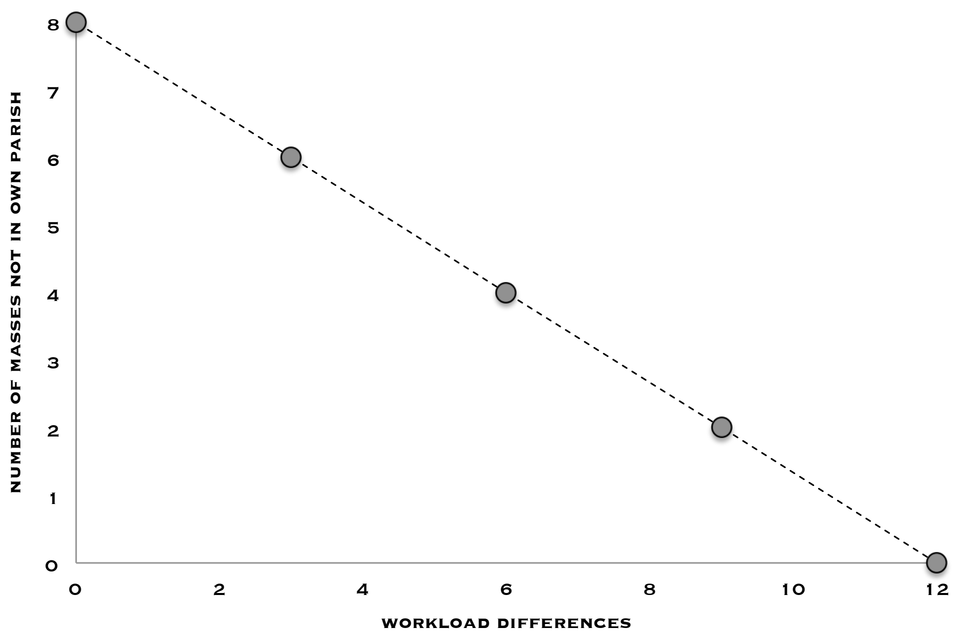
	Church	Parish Church	Frequency a	Frequency b
Parish 1	Church 1	✓	16	16
	Church 2	-	4	8
Parish 2	Church 3	✓	16	16
Parish 3	Church 4	✓	16	16
	Church 5	-	8	8
	Church 6	-	2	8
	Church 7	-	2	4
Parish 4	Church 8	✓	16	16
	Church 9	-	2	8
	Church 10	-	2	4
Parish 5	Church 11	✓	16	16
	Church 12	-	4	8
	Church 13	-	4	8
	Church 14	-	2	4
Parish 6	Church 15	✓	16	16
	Church 16	-	2	16
	Church 17	-	4	4

TABLE 5. Input Data: Lay Persons

	Lay person 1	Lay person 2	Lay person 3	Lay person 4
W01	✓	✓	-	✓
W02	-	✓	-	✓
W03	-	✓	✓	✓
W04	-	✓	✓	✓
W05	✓	✓	✓	✓
W06	✓	✓	-	✓
W07	✓	✓	✓	✓
W08	✓	✓	✓	✓
W09	✓	✓	✓	✓
W10	✓	✓	✓	✓
W11	✓	✓	✓	✓
W12	✓	✓	✓	✓
W13	✓	✓	✓	✓
W14	✓	-	✓	✓
W15	✓	✓	✓	✓
W16	✓	✓	✓	✓

maximization of the number of masses celebrated by each priest in churches of his own parish. Table 4 shows the standard mass frequencies (a) for the 17 different churches as well as a second fequency variant (b). In 2 model extensions we will assume an assistant priest as well as 4 lay persons that offer Liturgies of the Word in church 7, 10, 14 and 17, respectively. Each lay person is willing to offer 1 Liturgy of the Word every 4 weeks. Table 5 shows for each lay person the different weeks where he/she is available.

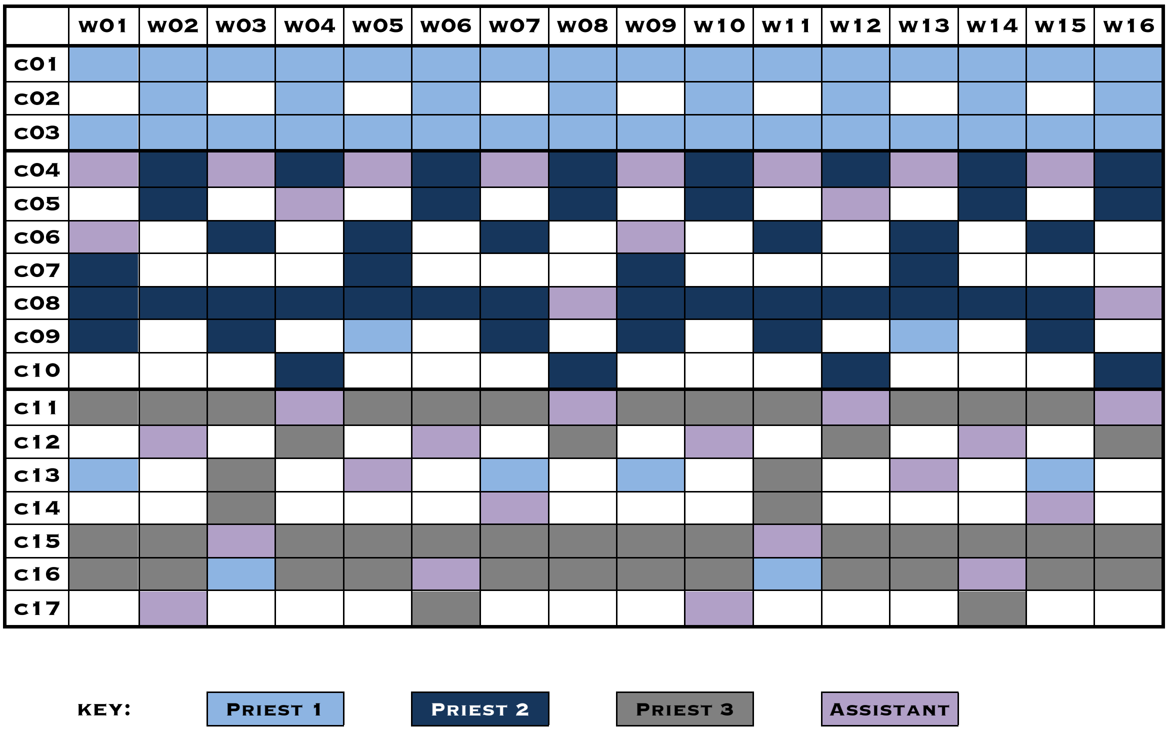
FIGURE 7. Trade-off Between Contiguity and Fairness: A Linear Pareto Curve



5.1. **Planning of Eucharistic Masses: Reference Scenario with 3 Priests.** In this subsection we will consider the basic mass frequencies in Table 4a. Obviousely, the parishes of priest 1 have less churches than the parishes of priest 2 and priest 3. This imbalance immediately results in a trade-off between the two objectives of fairness and contiguity: Given a feasible mass allocation with an identical workload for all priests, priest 1 will have to celebrate some of his masses in the parishes of priest 2 or priest 3. Analogous, given a feasible mass allocation where each priest celebrates all of his masses in churches of his own parishes, priest 1 will celebrate less masses than priest 2 and priest 3. Figure 7 depitcs this trade-off as a pareto curve which follows in the present example a linear relationship. It can be seen that increasing the number of masses not celebated in churches of a priest's parishes by 2, minimal workload differences between priest 3 and priest 1 decrease by 3. In the case where all masses are celebrated in the priests' own parishes, there will be a minimal workload difference of 12. Analogous, assuming a situation with no workload differences, we arrive at 8 masses that cannot be celebrated in churches of a priest's own parishes. Obviousely, these results help to identify existing conflicts of interests together with alternative allocations as efficient solutions to these conflicts, which is an important issue not only in economic theory but also in economic practice.

5.2. **Deployment of an Assistant Priest.** Now, let us assume that there is an assistant priest celebrating between 1 and 2 masses each week. Given this mass supply increase, we will assume that the priests decided to increase mass frequencies according to frequency variant (b). We will consider the case where both optimization criteria have identical weights of 0.5. The resulting mass plan is depicted in Table 6. As can be seen, all priests celebrate 48 masses throughout the entire planning period, which is obviousely a fair mass allocation. However, as an immediate consequence, priest 1 celebrates 8 masses in churches not belonging to his own parishes. The

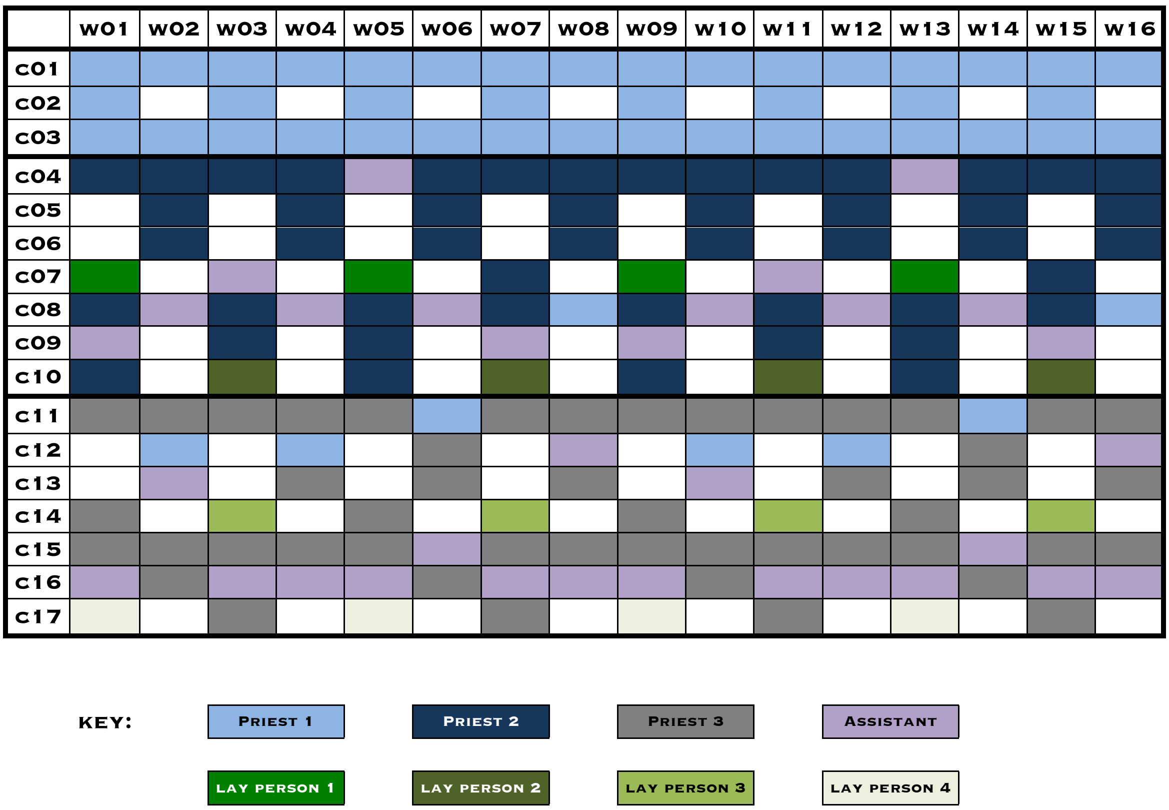
TABLE 6. Optimal Mass Plan Assuming 3 Priests and 1 Assistant Priest



deployed assistant priest supports the priests by celebrating 2 Eucharistic masses each week in the parishes of priest 2 or priest 3. Even though both the assistant priest and priest 1 are deployed in a flexible way, the comprehensible and systematic planning can positively influence the understanding and acceptance of the mass plan and in a consequence the priests' job satisfaction.

5.3. **Integrated Planning of Liturgies of the Word.** Let us now consider a situation where 4 lay persons can offer Liturgies of the Word (see also Table 5). Obviousely, the current mass plan in Table 6 does not allow for the celebration of all 16 Liturgies of the Word. However, a reallocation of the Eucharistic masses gives an integrated mass plan which has an identical objective function value compared to the initial allocation in Table 6 but allows for all 16 Liturgy of the Word celebrations. Table 7 depicts this integrated planning solution. Obviousely, in this solution Eucharistic masses and Liturgies of the Word are planned on an alternating basis to ensure an efficient use of time between 2 subsequent Eucharistic mass celebrations in a given church. Let us also point out that in general an integrated planning of Eucharistic masses and Liturgies of the Word actively includes lay persons in the planning process enhancing the status of their work. Ultimately, this can positively influence voluntary work in the parish cluster.

TABLE 7. Optimal Mass Plan Assuming 3 Priests, 1 Assistant Priest and 4 Lay Persons



6. SUMMARY

In this paper we introduced an integrated planning model for the celebration of Eucharistic masses and Liturgies of the Word in the context of parish clusters. Parish clusters are associations of parishes forming a new organisational unit that is responsible for diverse ecclesial services (like the planning of mass sites and times). The presented mass planning model can be seen as a first attempt to use methods from vector optimization for an efficient allocation of resources related to the church. Our results show that even though in the single parish, single priest case the planning of (Eucharistic) masses can be done in polynomial time, a more realistic model assuming parish clusters results in a much more complex problem. In a case study we illustrate our concepts using a parish cluster with 17 churches, 3 priests, 1 assistant priest and 4 lay persons. The results show that innovative team work arrangements in the church can help to ensure fairness and quality of mass plans accounting for the resource scarcity coming along with the shortage of priests.

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