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Product differentiation and welfare in a mixed duopoly with regulated prices: The case of a public and a private hospital*

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Abstract

Hospital markets are often characterised by price regulation and the existence of different ownership types. Using a Hotelling framework, this paper analyses the effect of different objectives of the hospitals on quality, profits, and overall welfare in a price regulated duopoly with symmetric locations. In contrast to other studies on mixed oligopolies, this paper shows that in a duopoly with regulated

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prices privatisation of the public hospital may increase overall welfare depending on the difference of the hospitals' marginal costs and the weight of the additional public hospital's motive.

KEYWORDS: mixed oligopoly, price regulation, quality, hospital competition.

JEL: L13, I18, H42

1 Introduction

As in other countries, public, non-profit and private (for-profit) hospitals compete with each other in Germany. Furthermore, an increasing number of public hospitals have been privatised over the last decade. Since the health care system is mainly publicly financed, regulatory authorities are interested in cost reducing and quality enhancing activities of the hospitals. This chapter analyses in a theoretical framework, whether and in which respect different objectives lead to different quality outcomes. Furthermore, given the assumed incentive structure, it shows whether and when a mixed duopoly would be preferred to a symmetric public or private duopoly from a welfare perspective.

A mixed oligopoly is in general defined as a market in which two or more firms with different objectives co-exist.¹ In their seminal paper on mixed oligopolies, Merrill and Schneider (1966) assume that the public firm maximises output facing a budget constraint. Often, the public firm is assumed to follow the public owner's interest and to maximise social surplus (for example De Fraja and Delbono, 1989; Cremer et al., 1991; Nishimori and Ogawa, 2002; Matsumura and Matsushima, 2004; Willner, 2006; Ishida and Matsushima, 2009). One issue inherent to that assumption lies in the multiple principal agent problems a hospital faces.² As Cutler (2000) notes, key considerations in the choice of organisational form for hospitals include

¹For surveys of the literature on mixed oligopolies compare De Fraja and Delbono (1990) and Nett (1993).

²In his comprehensive review Sloan (2000) classifies and evaluates the theoretical and empirical literature on non-profit hospitals' behaviour until 2000.

underlying concerns about agency problems and asymmetric information, the provision of public goods, and access to capital. At the same time, interests of major stakeholders, including administrators, staff, trustees, and communities may also play a role when choosing the ownership of a hospital.

To analyse the behaviour of firms in mixed oligopolies, mostly Cournot or Bertrand models are applied assuming that goods are homogeneous and prices can be set by the firms according to their objective functions. Although the assumptions about the firms' differences in costs and efficiency, number of firms, and timing may matter, it typically turns out that better allocations are achieved when public firms are present (e.g. Cremer et al., 1989) where in some cases the welfare-maximising first-best result can be attained. With endogenous costs for investments into efficiency gains, a public monopoly would be preferred to a mixed duopoly (Nishimori and Ogawa, 2002).

In this work, the goods (the treatments of the patients) are assumed to be differentiated. We follow an important approach to model product differentiation, spatial competition à la Hotelling (Hotelling, 1929). Cremer et al. (1991) apply a price-location game where the public firm pays higher wages and maximises social surplus under a non-negative profit constraint. They show that only for less than three and for more than five firms in the market, a mixed oligopoly with less than (n+1)/2 public firms leads to higher welfare than a private oligopoly. However, with endogenous production costs, privatisation of the public firm would improve welfare compared to a mixed duopoly because it would mitigate the loss arising from excessive cost-reducing investments of the private firm (Matsumura and Matsushima, 2004). In price regulated markets such as the hospital industry, firms rather compete in quality or location than in prices (Brekke, 2004; Brekke et al., 2006). Whilst prices and profits are easy to observe, it is difficult to measure a hospital's quality. The measurement of quality in studies of hospital competition has been in the focus of recent research (McClellan and Staiger, 1999b; Romano and Mutter, 2004; Gaynor, 2003). In Germany, quality regulation has been intensified significantly over the last ten years (introduction of minimum

 $^{^3}$ Gabszewicz et al. (2001) provide a comprehensive overview over location choice models.

quantities, external quality comparisons, and internal quality management as well as the obligation to publish quality reports). However, the evaluation of these means has only started recently and has not led yet to significant results with respect to quality differences between different hospital owners (Geraedts, 2006). Empirical studies of US hospitals find weak evidence that private hospitals may provide higher quality in some local or specific markets (McClellan and Staiger, 1999a; Santerre and Vernon, 2005). They model competition in location and quality between two profit maximising hospitals in a price regulated market. This approach is then applied on a mixed duopoly similar to Cremer et al. (1991).

We provide a first theoretical analysis of a mixed duopoly with regulated prices consisting of a public and a private (for-profit) hospital. However, due to the general setup, the results can be also applied to a duopoly of a private and a non-profit hospital or other price-regulated mixed oligopolies.⁵ As in other studies, the public hospital is assumed to maximise a linear combination of both profits and output. This assumption is considered to be more realistic and to mirror the interests of the different stakeholders better than the assumption of welfare-maximising behaviour of public hospitals.

The chapter proceeds as follows. In Section 2, preliminary assumptions will be shortly described. In Section 3, the quality choice of the two hospitals in the three scenarios (private profit-maximising duopoly, state-owned duopoly and mixed duopoly) will be analysed and the comparative statics characteristics of the quality choice in equilibrium will be shown. Finally, welfare-maximising prices will be derived in Section 4. The corresponding welfare levels, consumer rent, and profits in all three scenarios will be com-

⁴Gaynor and Vogt (2000) and Dranove and Satterthwaite (2000) review in detail the literature on antitrust and competition in mainly US health care markets, also considering differences across ownership types.

⁵While the literature on mixed oligopolies mostly deals with public firms, in the literature on health care markets, non-profit hospitals are assumed to also follow other incentives than to purely maximise profits such as to maximise output (Gaynor and Vogt, 2003; Lakdawalla and Philipson, 1998), quality (Newhouse, 1970; Ma and Burgess, 1993; Dranove and Satterthwaite, 2000), consumers' surplus (Steinberg and Weisbrod, 2005), physicians' interests (Pauly and Redisch, 1973) or other monetary or non-monetary incentives (e.g. charity in Dranove, 1988).

pared with each other and with the first-best scenario in Section 5 before Section 6 concludes.

2 The Structure of the Model

Assume that the two hospitals face a unit mass of patients, distributed uniformly on the line segment [0,1]. Locations x_i , i=1,2 are assumed to be exogenously fixed in the hospital sector. This assumption is realistic since locations are often regulated at least in rural regions. Hospitals cannot change their location in the short or medium term because of their size and infrastructural needs and local demand. Vertical differentiation may also be understood as specialisation versus diversification of the medical programs the hospitals offer. The only parameter hospitals can choose according to their respective maximisation problems is quality q_i given regulated price p. Marginal production costs c_i differ between the two hospitals and are constant with $p > c_i$, i = 1, 2. Let total marginal costs of production $C = c_1 + c_2$ and the cost difference $D = c_1 - c_2$ where c_1 and c_2 are exogenously given. Transportation costs, which the patients face, are quadratic in the distance between the patient's location z and the hospital i, i.e. $t(z - x_i)^2$.

Utility function and the indifferent patient

A patient located at z derives the utility from getting one unit of the service provided by hospital i located at x_i and providing the quality q_i

$$U(z, x_i, q_i) = v + q_i - t(z - x_i)^2 - p,$$
(1)

with price p > 0 and transportation costs t > 0. In this model, the patient pays the price per treatment either privately or for example as a co-payment to the health insurance. Furthermore, the constant valuation of consuming the good v is assumed to be sufficiently high such that the market is covered at any time. Due to the latter, a monopolistic hospital would always choose

⁶Linear transportation cost would lead to similar results.

zero quality as long as it is costly (unless otherwise regulated). A monopolist would earn non-negative profits as long as the regulated price exceeds average or marginal costs of production.⁷ The indifferent patient is located at

$$\bar{z} = \frac{1}{2}(x_1 + x_2) + \frac{q_1 - q_2}{2t(x_2 - x_1)}$$
 (2)

For such a location to exist, we need to assume in the following, that the distance between the hospitals x_2-x_1 is strictly positive. We concentrate our analysis on equilibria in pure strategies⁸ and assume throughout that $x_2 > x_1$, namely $x_1 \in [0, \frac{1}{2} - \bar{x}], x_2 \in [\frac{1}{2} + \bar{x}, 1]$, with $\bar{x} > 0$ small. We assume that the two hospitals are located symmetrically, i.e. $x_2 = 1 - x_1$. Then, $x_1 = \frac{1}{2}(1 - \Delta)$ and $x_2 = \frac{1}{2}(1 + \Delta)$ with distance $\Delta = x_2 - x_1$. That means that the marginal patient is located at

$$\bar{z} = \frac{1}{2} + \frac{q_1 - q_2}{2t\Delta} \tag{3}$$

Profit Functions

As in Brekke et al. (2006), the marginal production costs of one good and the costs of producing a certain quality can be linearly separated, where quality costs are the costs of investing into higher quality that are not related to the marginal cost of production. The cost of investing into higher quality is assumed to be $C_i^q(q_i) = \frac{1}{2}q_i^2$ throughout the analysis to ensure that the profit function is concave and a unique maximum exists. The profit of hospital i is defined as

$$\pi_i = (p - c_i)y_i - \frac{1}{2}q_i^2 \tag{4}$$

⁷Brekke et al. (2008) compare a monopolistic altruistic hospital with a market composed by two altruistic hospitals assuming that a fraction of patients may not be treated due e.g. high transportation costs and capacity constraints. They find that it depends on the hospital's valuation of consumer surplus as to which setting would be preferred by the regulator.

⁸Bester et al. (1996) show that the Hotelling location game with quadratic transportation costs and price competition possesses an infinity of mixed strategy Nash equilibria. In these equilibria coordination failure invalidates the principle of "maximum differentiation" discovered by d'Aspremont et al. (1977). For a similar finding, compare Wang and Yang (2001) showing the existence of mixed equilibria in a 2 stage price-quality game.

Under the assumptions that each patient consumes one unit of the service and that the market is covered, two hospitals' market shares are determined by the location of the indifferent patient \bar{z} , where $y_1 = \bar{z}$ and $y_2 = 1 - \bar{z}$ constitute the number of cases treated by the two respective hospitals. Furthermore, the framework is generalised by assuming that the two hospitals may differ with respect to their marginal costs c_i (compare Cremer et al., 1989). In Germany, private hospitals do not underlie the same regulatory restrictions as public or non-profit hospitals. Private hospitals are, in contrast to public hospitals, not obliged to pay the rather high public sector wages, for example. That is why, on average, private hospitals face lower personnel costs than public hospitals in Germany. Since personnel costs account for approximately 60% of total hospital costs, they are an important factor. Furthermore, private hospitals are less constrained in negotiating input prices with suppliers and to borrow capital while non-profit and public hospitals are not allowed to accumulate profits because of their legal forms. 10

The structure of the game is as follows: In stage 0, symmetric locations and marginal costs of production are exogenously fixed before prices are set by the regulatory authority in stage 1 and hospitals compete in quality in stage 2. The game will be solved by backward induction to identify a stable Nash-equilibrium.

The Three Scenarios

In general, a hospital's objective function is defined as $Z_i = \pi_i + \alpha_i y_i$. Here, as opposed to private profit-maximising hospitals, public hospitals are assumed to maximise their own profits plus a fraction of their market share, which depends positively on the hospital's quality. In the three possible scenarios the two hospitals behave as follows.

 $^{^960\%}$ of the private hospitals operate less than 100 beds. In this size category, costs per full-time equivalent employee sum up to $56,000 \in$ in public, $47,000 \in$ in non-profit and only $43,000 \in$ in private hospitals in 2007. However, for hospitals with more than 500 beds the costs per employee are similar across ownership types (between 52,000 and 54,000 €).

¹⁰Given those input prices and outputs, cost *inefficiency* will be shown to be highest in private hospitals in the years from 2000 to 2003 (see Chapter ??).

- 1. Scenario *PD* (profit-maximising duopoly): $\alpha_1 = \alpha_2 = 0$
 - As in Brekke et al. (2006), both hospitals behave as profit-maximising private hospitals and maximise their respective objective function $Z_i^p = \pi_i$, i = 1, 2.
- 2. Scenario SD (state-run duopoly): $\alpha_1 = \alpha_2 = \alpha > 0$ Both public hospitals follow the objective function $Z_i^s = \pi_i + \alpha y_i$.
- 3. Scenario MD (mixed duopoly): $\alpha_1 = \alpha > \alpha_2 = 0$ In this scenario, the mixed duopoly is analysed. It is assumed that hospital 2 is a profit-maximising private hospital, $Z_2^a = \pi_2$, while hospital 1 is a public hospital maximising the mixed objective function

3 Quality Choice in the Three Scenarios

3.1 Quality Choice

 $Z_1^a = \pi_1 + \alpha \bar{z}.$

In all three scenarios, the two hospitals choose their quality levels in equilibrium such that the first order conditions

$$\frac{dZ_i}{dq_i} = \frac{p - c_i + \alpha_i}{2t\Delta} - q_i = 0$$

are fulfilled. Thus, the hospital's quality level in the Nash-equilibrium can be derived to be

$$q_i = \frac{p - c_i + \alpha_i}{2t\Delta},\tag{5}$$

which is uniquely defined since $\frac{d^2Z_i}{dq_i^2} < 0$ and t > 0, $\Delta > 0$, $p > c_i$. The hospital i's quality level in equilibrium does not depend on the other hospital's quality. It only depends on the price mark-up, the patients' transportation costs and distance and the weight α_i . This equilibrium quality level is a dominant strategy for both hospitals. The first hospital provides higher quality $(q_1 > q_2)$ if $c_1 - c_2 < \alpha_1 - \alpha_2$, i.e. if the cost difference is smaller than the difference in the weights.

If $\alpha_i = 0$, i = 1, 2, the equilibrium collapses to a private profit-maximising duopoly (Scenario PD) in which the first hospital sets higher quality as long as $c_1 < c_2$ and vice versa, given symmetric locations. In Scenario SD the two public hospitals will produce higher quality than in Scenario PD, since they value market shares and thus patient's utility more than purely profit maximising hospitals.

The additional asymmetry of the mixed duopoly (*Scenario MD*) comes from the assumption that $\alpha_1 = \alpha > 0$ for the first hospital and $\alpha_2 = 0$ for the second (pure profit maximiser). Then,

$$q_1^a = \frac{p - c_1 + \alpha}{2t\Delta}$$

$$q_2^a = q_2^p = \frac{p - c_2}{2t\Delta}$$

and $q_1^a > q_2^a$ if $\alpha > c_1 - c_2 = D$. Put differently, depending on the underlying cost structure and on α it is possible that the private hospital produces at a higher quality level than the public hospital.

3.2 Comparative Statics with respect to Transportation Costs, Price Margin, and Distance

The comparative statics results hold similarly for all three scenarios, which only differ in their magnitudes of α_i and the levels of qualities and price.

Change in Location

The higher the distance Δ , the lower the two quality levels in equilibrium. This result complies with basic competition theory. When hospitals are close to each other (geographically or in the services they offer) competition becomes fierce and quality increases, especially if α_i , the valuation of the market share, is high.

$$\frac{dq_i}{d\Delta} = -\frac{(p - c_i + \alpha_i)}{2t\Delta^2} < 0 \tag{6}$$

For the two symmetric scenarios the only difference between the two hospitals' reactions is determined by the difference in cost of production.

Change in Patients' Transportation Costs

If the marginal transportation costs increase, switching to the other hospital will become more expensive, ceteris paribus. This softening of competition leads to a decrease in quality enhancing investments.

$$\frac{dq_i}{dt} = -\frac{p - c_i + \alpha_i}{2t^2 \Delta} < 0 \tag{7}$$

The reason is that the quality difference becomes less important for the location of the marginal patient when transportation costs increase (see (3)).

Change in Regulated Price and the Price-Cost Margin

Regarding the effect of the price cost margin on the hospital's quality, the comparative static result can easily be derived from the first order condition:

$$\frac{dq_i}{d(p-ci)} = \frac{1}{2t\Delta} > 0 \tag{8}$$

As expected, an increase in the margin will lead to higher quality levels for both hospitals. Patients will be compensated for the increase in prices by higher quality levels. This result holds independent of α_i . Note that the regulated prices and thus the magnitudes of change differ across the scenarios, though.

Change in Valuation of Market Shares

An increase in the weight of the quantity or market share α leads to an increase in quality provided.

$$\frac{dq_i}{d\alpha_i} = \frac{1}{2t\Delta} > 0 \tag{9}$$

In the mixed duopoly, the public hospital will increase its quality as shown in Equation (9) while the private hospital's quality does not depend on the public hospital's valuation α of the market share. It will thus not change.

While these results are clear, empirical studies analysing the effect of

competition on quality of hospitals have shown mixed results. Kessler and McClellan (2000) analyse the impact of competition on both costs and patient health outcomes in the US. They find that whilst the welfare effects of competition in the 1980s were ambiguous, post-1990 competition was welfare improving. Looking at mergers, Hamilton and Ho (1998) do not find any effect of mergers on mortality of either heart attack or stroke. Propper et al. (2004) show that more intense competition between hospitals is associated with higher death rates in the English NHS.

4 Regulating Prices

In the first step of the game the regulatory authority sets welfare-maximising prices in each of the three scenarios. The corresponding second-best results are compared to the first-best that will be derived first.

4.0.1 The Welfare Function and Consumer Surplus

In the following, total welfare is defined as $W = K + \pi_1 + \pi_2$, where in a duopoly with symmetric locations the consumer rent K is defined in general as

$$K = \int_0^{\bar{z}} (v + q_1 - t (x - x_1)^2 - p) dx + \int_{\bar{z}}^1 (v + q_2 - t (x - x_2)^2 - p) dx$$
$$= v - \frac{1}{12} t - p + \frac{1}{2} (q_1 + q_2) + \frac{1}{4} t \Delta (1 - \Delta) + \frac{1}{4\Delta t} (q_2 - q_1)^2$$
(10)

Given symmetric locations, the welfare function can be written as¹¹

$$W = v - \frac{1}{12}t + \frac{1}{4}t\Delta(1-\Delta) - \frac{1}{2}C + \frac{1}{2}(q_1+q_2) - \frac{1}{2}(q_1^2+q_2^2) + \frac{1}{4\Delta t}((q_1-q_2)^2 - 2(q_1-q_2)D),$$
(11)

¹¹Regulated prices as well as resulting quality, profits and consumer rent in equilibrium would not differ when adding the higher utility of the public hospital(s) to overall welfare $(W + \sum_i \alpha_i \bar{y}_i, i = 1, 2)$. However, total welfare would be higher than before when there are one or two public hospitals in the market (for a detailed analysis compare Section 5.1.2).

Note that overall welfare does not depend on the price chosen by the regulatory authority. Only the distribution of rents between consumers and producers differs with the price.

4.1 First-best Solution

To find the welfare-maximising first-best quality level of the two hospitals, the first derivatives of the welfare function with respect to q_1 and q_2 are set to zero. After rearranging, the welfare maximising quality levels are,

$$q_1^w = \frac{t\Delta - 1 - D}{2(t\Delta - 1)} \tag{12}$$

$$q_2^w = \frac{t\Delta - 1 + D}{2(t\Delta - 1)},\tag{13}$$

where $D=c_1-c_2$, $t\Delta>\frac{1}{2}$ for a local maximum to exist and $t\Delta>1+|D|$ or $t\Delta<1-|D|$ for both quality levels to be non-negative. The latter two restrictions ensure that a finite quality level exists $(t\Delta\neq 1)$. The difference between the quality levels is $q_1^w-q_2^w=-\frac{D}{t\Delta-1}$, which only depends on the hospitals' locations and their marginal costs. In the optimum, the public hospital's quality is higher than the private hospital's if $c_1 < c_2$ and $t\Delta > 1$ or vice versa. If marginal costs are equal for both hospitals, the welfare maximising quality levels are $q_1=q_2=\frac{1}{2}$ for both hospitals. Market shares are non-negative if $t\Delta>1+|D|$ (non-negative quality) and $t\Delta(t\Delta-1)>D>t\Delta(1-t\Delta)$. We assume that in equilibrium, hospitals should at least be able to produce at a non-negative profit level. In general,

$$\pi_1^w = (p - c_1) \frac{-D - t\Delta + t^2 \Delta^2}{2t\Delta (t\Delta - 1)} - \frac{(t\Delta - D - 1)^2}{8(t\Delta - 1)^2} \text{ and}$$
(14)

$$\pi_2^w = (p - c_2) \frac{D - t\Delta + t^2 \Delta^2}{2t\Delta (t\Delta - 1)} - \frac{(t\Delta + D - 1)^2}{8(t\Delta - 1)^2}.$$
(15)

Both profits are non-negative if the price mark-up is sufficiently high. That means that if the price is low (for example $p^s = t\Delta + \frac{1}{2}C - \alpha$ of Scenario SD derived below), this is only fulfilled if the restrictive non-zero-profit condition

$$t\Delta > t\widetilde{\Delta} = 1 + (c_1 + c_2) + \alpha$$
 holds.

Inserting first best q_1^w and q_2^w , the maximum welfare level will be

$$W^{w} = v - \frac{1}{12}t + \frac{1}{4} - \frac{1}{2}C + \frac{1}{4}t\Delta(1 - \Delta) + \frac{D^{2}}{4(t\Delta - 1)t\Delta}$$
 (16)

It can be easily shown that overall welfare in the first-best setting increases, the lower transportation costs, distance and marginal costs and the higher the cost-difference (if $t\Delta > 1$). The latter may result from different influences. If the cost difference increases, the quality of the disadvantaged hospital will decrease and it will thus attract fewer patients than the hospital with the lower cost of production. Furthermore, the assumption of symmetric locations may also play a role preventing the hospitals to move in different directions.

4.2 Price Regulation

In a second-best setting, hospitals behave in the second stage according to their objective functions and choose the quality levels derived in Section 3 as opposed to welfare-maximising quality of the first-best setting. This behaviour will be anticipated by the regulator in the first stage when setting prices. For stable solutions to exist in all scenarios, we need to assure concavity of the objective functions by assuming that distance and transportation costs are sufficiently high. In all scenarios as well as in the first-best case, a stable equilibrium exists with given welfare-maximising prices if the resulting profits, market shares, and quality levels are non-negative. An example for a sufficient condition ensuring a simultaneous equilibrium in all settings is given by $t\Delta > t\widetilde{\Delta} = 1 + (c_1 + c_2) + \alpha$. In the following, all equilibria are analysed under this assumption. However, except of in the first-best equilibrium it would be sufficient if $t\Delta > \frac{1}{2} + \frac{1}{2}(c_1 + c_2) + \alpha$. If the hospitals are close to each other or transportation costs are low, competition will be fierce leading the hospitals to overbid each other until one or both of the hospitals exit the market. In that case, no solution exists to the maximisation problem and we

¹²The private duopoly requests the least restrictive constraint with $t\Delta > \frac{1}{2}|D| + \frac{1}{4} \ge \frac{1}{4}$.

cannot identify a unique equilibrium.

4.2.1 Prices, Quality, Profits, and Welfare in the Private and the Public Duopoly

Welfare will be maximised by the price setting authority with respect to the quality choice of the hospitals of the second stage. Inserting the quality levels of Scenarios PD and SD into the welfare function, the second-best prices are given by

Scenario PD:
$$p^p = t\Delta + \frac{1}{2}C$$
 (17)

Scenario SD:
$$p^s = t\Delta + \frac{1}{2}C - \alpha$$
 (18)

where in Scenario SD both hospitals value their own output equally much $(\alpha_1 = \alpha_2 = \alpha)$. In the private profit-maximising duopoly (PD) the price is higher $(p^p > p^s)$ to induce the hospitals to produce at a higher quality level. The second derivative of the welfare function with respect to p is negative in both scenarios letting us conclude that the prices are in the respective local maxima of the welfare functions. The resulting quality levels correspond with each other in the two scenarios with $q_1^p = q_1^s = \frac{1}{2} - \frac{1}{4t\Delta}D$ and $q_2^p = q_2^s = \frac{1}{2} + \frac{1}{4t\Delta}D$. Thus, the higher price induces both profit-maximising hospitals to produce at the same quality level as if they were also considering output in their objective function. The first hospital's quality is lower than the quality of the second hospital if $c_1 > c_2$. Inserting the corresponding quality and price levels into the profit functions, we gain for the profit maximising duopoly (Scenario PD)

$$\pi_1^p = \pi_1^s + \frac{1}{2}\alpha(1 - \frac{1}{2(t\Delta)^2}D) \text{ and}$$
(19)

$$\pi_2^p = \pi_2^s + \frac{1}{2}\alpha(1 + \frac{1}{2(t\Delta)^2}D)$$
 (20)

and for the public duopoly (Scenario SD)

$$\pi_1^s = \frac{1}{32(t\Delta)^2} D\left(8\alpha - 4t\Delta + 3D\right) - \frac{1}{8} \left(4\alpha - 4t\Delta + 2D + 1\right) \text{ and } (21)$$

$$\pi_2^s = -\frac{1}{32(t\Delta)^2} D\left(8\alpha - 4t\Delta - 3D\right) - \frac{1}{8} \left(4\alpha - 4t\Delta - 2D + 1\right)$$
 (22)

A unique Nash equilibrium with non-negative quantities and profits exists if simultaneously $\pi_i \geq 0$ and $y_i \geq 0$ at equilibrium prices and quality levels.

Proposition 1 Let $p^s = t\Delta + \frac{1}{2}C - \alpha$ and $t\Delta > \alpha + \frac{1}{2}C + \frac{1}{2}$ with two public hospitals. Then, a unique Nash equilibrium with non-negative quantities and profits exists. In the private duopoly with $p^p = t\Delta + \frac{1}{2}C$, it suffices that $t\Delta > \frac{1}{2}|D| + \frac{1}{4}$ for a stable and unique Nash-equilibrium in pure strategies to exist.

Since the second best quality levels are equal across scenarios, welfare is independent of the price, and distance is exogenously fixed, welfare is equally high in both symmetric settings

$$W^{p} = W^{s}$$

$$= v - \frac{1}{12}t - \frac{1}{2}C + \frac{1}{4}t + \frac{1}{4}t\Delta(1-\Delta) + \frac{1}{16(t\Delta)^{3}}D^{2}(3t\Delta + 1).$$
(23)

The distribution of consumer rent and profits differs, though, since the price and profits are lower and the consumer rent is higher if both hospitals are state-run (SD).

Note that we restrict the regulatory authority to impose a single price for both hospitals. We would reach first best always if the government was able to perfectly discriminate between the hospitals and for example to account for the differences in marginal costs of production. However, in the hospital market we actually see that the hospitals receive the same price for the same treatment adjusted for case-mix severity (payments based on Diagnosis Related Groups).

4.2.2 Prices, Quality, Profits, and Welfare in the Mixed Duopoly (Scenario MD)

In the mixed duopoly, quality levels differ between the two hospitals. The welfare maximising price is

$$p^a = t\Delta + \frac{1}{2}C - \frac{1}{2}\alpha\tag{24}$$

with $p^s < p^a < p^p$. The price will always be higher in the mixed duopoly than in the symmetric public duopoly to induce the private hospital to produce at a higher quality level. For positive market shares of both hospitals, $2(t\Delta)^2 > D - \alpha$ and $2(t\Delta)^2 > -(D-\alpha)$ need to be assured which is given if $p > c_i \Leftrightarrow 2t\Delta > \alpha + |D|$ and $t\Delta > 1$. The corresponding quality levels $q_1^a = \frac{1}{2} - \frac{D-\alpha}{4t\Delta}$ and $q_2^a = \frac{1}{2} + \frac{D-\alpha}{4t\Delta}$ are higher and lower, respectively, than the levels in the two symmetric scenarios. Inserting the quality and price levels into the profit functions, we obtain

$$\pi_1^a = \frac{1}{32(t\Delta)^2} (4t\Delta - 5\alpha - 3D) (\alpha - D) + \frac{1}{4} \left(2t\Delta - \alpha - D - \frac{1}{2} \right)$$
 (25)

and

$$\pi_2^a = \frac{1}{32(t\Delta)^2} \left(4t\Delta - 3\alpha + 3D \right) \left(-\alpha + D \right) + \frac{1}{4} \left(2t\Delta - \alpha + D - \frac{1}{2} \right)$$
 (26)

Both hospitals stay in the market if profits $\pi_i > 0$ and market shares $y_i \geq 0$.

Proposition 2 Let $p^a = t\Delta + \frac{1}{2}C - \frac{1}{2}\alpha$ and $t\Delta > \alpha + \frac{1}{2}C + \frac{1}{2}$. Then, a unique Nash equilibrium with non-negative quantities and profits exists in the mixed duopoly. The public hospital's quality is higher than the private hospital's if $D < \alpha$, i.e. if the difference in marginal costs is lower than the valuation of the market share. The private hospital earns higher profits than the public hospital if $D < \alpha \frac{\alpha - t\Delta}{\alpha - t\Delta - 2t^2\Delta^2}$ which is possible even if $D > \alpha$.

The welfare level in the mixed duopoly is given by

$$W^{a} = v - \frac{1}{12}t - \frac{1}{2}C + \frac{1}{4} + \frac{1}{4}t\Delta(1 - \Delta)$$

$$+ \frac{1}{16(t\Delta)^{3}} \left((\alpha - D)^{2} - t\Delta(\alpha - D)(\alpha + 3D) \right)$$
(27)

5 Comparison of Welfare, Consumer Surplus, and Profits

Assume in the following that $t\Delta > \widetilde{t\Delta} = 1 + \frac{1}{2}C + \alpha$ to enable comparisons across all four scenarios (including the first-best scenario). Furthermore, let $c_i^p = c_i^s = c_i^a = c_i$, i = 1, 2. This assumption applies also when a hospital changes the ownership. That means that marginal costs of production do not alter after a switch from for example public to private ownership. In the following, all results are interpreted given this hypothetical setup.

5.1 Comparison of Welfare Levels

5.1.1 "Classic" Welfare Function

Given second best prices in the two symmetric scenarios, quality and welfare levels are of the same magnitude, no matter whether hospitals take into account market shares or only maximise profits. Furthermore, comparing (23) and (28) it can be shown that

$$W^a > W^p = W^s \Leftrightarrow D < -\alpha \frac{t\Delta - 1}{2(1 + t\Delta)}$$

with $D = c_1 - c_2$. Let D > 0. Then, the welfare level in the mixed duopoly is below the level in the two symmetric scenarios. In this case, a private duopoly would provide higher welfare than a mixed market due to its symmetric structure. Conversely, there is a difference in marginal costs D, for which a mixed duopoly increases welfare compared to two public or two private hospitals. The lower the valuation of the market share α (since $t\Delta > 1$) or the more intense the competition (low $t\Delta$), the more often a regulatory authority would implement a mixed duopoly compared to a symmetric setup as long as the public hospital has an advantage in marginal cost of production. That means, in a symmetric duopoly the hospital with lower costs of production should switch the ownership type.

Naturally, the first-best setting gives the highest welfare level since with Equations (23), (16) and (28) and $t\Delta > \widetilde{t\Delta} > 1$ the comparison shows

$$W^{s} - W^{w} = W^{p} - W^{w} = -\frac{D^{2} (t\Delta + 1)^{2}}{16(t\Delta)^{3} (t\Delta - 1)}$$
$$W^{a} - W^{w} = -\frac{(\alpha(1 - t\Delta) - D (t\Delta + 1))^{2}}{16(t\Delta)^{3} (t\Delta - 1)}$$

The first-best result can be reached in the symmetric Scenarios PD and SD if D=0 that means if marginal costs are equal across hospitals. Comparing the two symmetric settings, it is rather a political decision whether the public authority prefers to support producers by privatising both hospitals or to enlarge consumer rent. In the mixed duopoly, the first-best can only be reached if $t\Delta = \frac{\alpha - D}{\alpha + D} > \widetilde{t\Delta}$, thus if $c_1 \ll c_2$. In the case that the public hospital has a big cost advantage, a mixed setting would increase welfare compared to the symmetric settings. This may be due to the assumption that hospitals are nevertheless located symmetrically between 0 and 1 which decreases transportation costs compared to asymmetric locations. However, if the private hospital has the cost advantage, quality levels in the mixed duopoly would be that low that the first-best outcome cannot be reached even with regulated prices and symmetric locations.

5.1.2 Extension: Welfare with Additional Utility of Public Hospitals

Assume that the second part of the hospital's utility also increases overall welfare. That means in general that

$$W_j^u = W_j + \alpha_{1j}\bar{z}_j + \alpha_{2j}(1 - \bar{z}_j)$$
 (28)

where α_i differs according to the respective scenarios $j=\{\text{PD,SD,MD}\}$. Since the market shares are independent of the regulated prices, there is no effect on prices and thus also not on quantities, consumer surplus and profits. However, welfare in the public duopoly will always be higher than in a private duopoly $(W_p^u = W_s^u + \alpha)$. The threshold for which a mixed duopoly is still preferred to a private duopoly increases to $D_{PDMD}^u < \frac{1}{2}\alpha + \frac{4(t\Delta)^3}{1+3t\Delta}$, which is always positive and thus allows the public hospital to have higher marginal costs than the private hospital. The cost difference D can even be higher than α depending on $t\Delta$, which would result into lower quality of the public hospital. In contrast to the above results, this threshold increases in α and $t\Delta$.

A mixed duopoly leads to higher welfare than a duopoly of two staterun hospitals if $D^u_{SDMD} < \frac{1}{2}\alpha - \frac{4(t\Delta)^3}{1+3t\Delta} < D^u_{PDMD}$ where this threshold also increases in α but decreases in $t\Delta$ as it did in both symmetric scenarios in Section 5.1.1. Put differently, a mixed duopoly outperforms a duopoly of two public hospitals in terms of overall welfare not only if the public hospital has a cost advantage but also if competition between the two hospitals is fierce.

In the following, it is assumed that the additional utility from treating more patients does not enter overall welfare. This assumption only has an effect on the comparison of the scenarios with respect to welfare results since quality, prices, and quantities are not affected.

5.2 Comparison of Consumer Surplus

Since $\frac{\partial K}{\partial q_i} = \frac{q_i - q_j}{2t\Delta} + \frac{1}{2} > 0$ if $q_i - q_j > -t\Delta$, for at least one hospital $i \neq j$ the consumer surplus would be maximal if quality increased to infinity or distance is close to zero (leading to infinitely high quality via high competition between the hospitals). However, given the quality choice by the hospitals and inserting second best prices which are paid by the consumers, consumer

¹³The consumer surplus and the profits of the three scenarios are not compared with the first-best setting since in the latter any arbitrary price would lead to maximal welfare.

surplus in the case of two profit-maximising hospitals ($\alpha_i = 0$) is defined by

$$K^{p} = v - \frac{1}{12}t - \frac{1}{4}t\Delta(3+\Delta) + \frac{1}{2} - \frac{1}{2}C + \frac{1}{16t^{3}\Delta^{3}}D^{2}$$
 (29)

In Scenario SD ($\alpha > 0$ for both hospitals) the consumer surplus is higher, namely

$$K^s = K^p + \alpha \tag{30}$$

The consumer surplus in a public duopoly is higher due to higher quality and lower regulated prices. In the mixed duopoly it holds that

$$K^{a} = K^{p} + \frac{1}{2}\alpha + \frac{1}{16(t\Delta)^{3}}(\alpha - 2D)\alpha$$
 (31)

Proposition 3 Assume that a sub-game perfect Nash equilibrium exists where both hospitals are active in the market in all three scenarios, i.e. transportation costs and distance are sufficiently high with $t\Delta > \frac{1}{2}C + \alpha + \frac{1}{2}$. Then, $K^s > K^a > K^p$.

For an analysis of consumer rents with lower transportation costs, compare Appendix A.1.

5.3 Comparison of Profits

The profits of the first two scenarios are easy to compare with each other. Since welfare levels coincide but prices are higher in the duopoly with two profit-maximising hospitals than in the public duopoly, profits will be higher in the former duopoly than in the latter. From (19), (20), (21) and (22) it can be shown that $\pi_i^p > \pi_i^s$ if $|D| < 2(t\Delta)^2$. Compared to the mixed duopoly the following Proposition can be derived.

Proposition 4 Assume that a sub-game perfect Nash equilibrium exists where both hospitals are active in the market, i.e. transportation costs and distance are sufficiently high with $t\Delta > \frac{1}{2}C + \frac{1}{2} + \alpha$. Then, $\pi_i^p > \pi_i^a > \pi_i^s$ for i = 1, 2.

See Appendix A.2 for a comparison of the respective profit functions.

6 Conclusion

This analysis has shown that a mixed oligopoly can lead to the highest welfare and quality when compared to two public or two private hospitals and may come closest to the first-best solution. This result implies that it can be best to privatise one or several public hospitals when a public hospital is still present in the market.

Compared to the mixed duopoly, a private duopoly will be preferred if the public hospital that would be privatised faces relative marginal costs that exceed a certain threshold in the mixed duopoly. This threshold differs by the definition of the underlying welfare function and depends on the valuation of the market shares and the degree of competition.

When assuming a 'classic' welfare function, our result derived in a price regulated setting conflicts with the result by Cremer et al. (1991) who state that a mixed duopoly would be superior to a private duopoly in a price-location game although the public firm faces higher wages and thus higher marginal costs. Here, in the mixed duopoly, first-best can only be reached if the public hospital has a big cost advantage compared to the private (for-profit) hospital.

Further possible generalisations of this model include the introduction of endogenous costs, location choice, choice of slack, and the extension to more than two competitors. For future research on hospital privatisation, it is essential to identify the objectives of different ownership types empirically. Additionally, empirical studies of hospital competition should be conducted in which it is not only accounted for prices and costs, but also for quality.

References

- Bester, H., De Palma, A., Leininger, W. and Thomas, J. (1996). A noncooperative analysis of Hotelling's location game, *Games and Economic Behavior* **12**: 165–186.
- Brekke, K. (2004). Competition or coordination in hospital markets with unionised labour, International Journal of Health Care Finance and Economics 4: 65–89.
- Brekke, K., Nuscheler, R. and Straume, O. R. (2006). Quality and location choices under price regulation, *Journal of Economics & Management Strategy* **15**(1): 207–227.

- Brekke, K., Siciliani, L. and Straume, O. R. (2008). Hospital competition and quality with regulated prices, *Discussion Paper SAM 2*, Norwegian School of Business Administration.
- Cremer, H., Marchand, M. and Thisse, J.-F. (1989). The public firm as an instrument for regulating an oligopolistic market, *Oxford Economic Papers* **41**(2): 283–301.
- Cremer, H., Marchand, M. and Thisse, J.-F. (1991). Mixed oligopoly with differentiated products, *International Journal of Industrial Organization* **9**(1): 45–53.
- Cutler, D. M. (2000). Introduction, in D. M. Cutler (ed.), The changing hospital industry: comparing not-for-profit and for-profit institutions, A NBER conference report, Chicago, chapter 3, pp. 93–112.
- d'Aspremont, C., Gabszewicz, J. J. and Thisse, J.-F. (1977). On Hotelling's stability in competition, *Econometrica* 47(5): 1145–50.
- De Fraja, G. and Delbono, F. (1989). Alternative strategies of a public enterprise in oligopoly, *Oxford Economic Papers* 41: 302–311.
- De Fraja, G. and Delbono, F. (1990). Game theoretic models of mixed oligopoly, *Journal of Economic Surveys* 4(1): 1–17.
- Dranove, D. (1988). Demand inducement and the physician/patient relationship, *Economic Inquiry* **26**: 251–298.
- Dranove, D. and Satterthwaite, M. A. (2000). The industrial organization of health care markets, in A. J. Culyer and J. Newhouse (eds), *Handbook of Health Economics 1B*, Elsevier, Amsterdam, chapter 20, pp. 1093–1139.
- Gabszewicz, J. J., Thisse, J.-F., Fujita, M. and Schweizer, U. (2001). *Location Theory*, Routledge, New York.
- Gaynor, M. (2003). What do we know about competition and quality in health care markets?, Working Paper 12301, NBER.
- Gaynor, M. and Vogt, W. B. (2000). The concentration of medical spending-an update, in A. J. Culyer and J. Newhouse (eds), Handbook of Health Economics 1B, Elsevier, Amsterdam, chapter 27, pp. 1405–1487.
- Gaynor, M. and Vogt, W. B. (2003). Competition among hospitals, RAND Journal of Economics 34(4): 764–785.
- Geraedts, M. (2006). Auswirkungen von Qualitätsregulierungen auf das Angebot von Krankenhausleistungen, in K. J, R. B-P and S. H (eds), Krankenhausreport 2006–Schwerpunkt: Krankenhausmarkt im Umbruch, Schattauer, Stuttgart, pp. 187–203.
- Herr, A. (2008). Cost and technical efficiency of German hospitals: Does ownership matter?, *Health Economics* **17**: 1057–1071.
- Hotelling, H. (1929). Stability in competition, Economic Journal 39: 41-57.

- Ishida, J. and Matsushima, N. (2009). Should civil servants be restricted in wage bargaining? A mixed-duopoly approach, *Journal of Public Economics* forthcoming.
- Kessler, D. P. and McClellan, M. B. (2000). Is hospital competition socially wasteful?, Quarterly Journal of Economics pp. 577–615.
- Lakdawalla, D. and Philipson, T. (1998). Nonprofit production and competition, Working Paper 6377, NBER.
- Ma, C.-T. A. and Burgess, J. F. (1993). Quality competition, welfare, and regulation, Journal of Economics, Zeitschrift für Nationalökonomie **58**(2): 153–173.
- Matsumura, T. and Matsushima, N. (2004). Endogenous cost differentials between public and private enterprises: A mixed duopoly approach, *Economica* **71**(284): 671–688.
- McClellan, M. and Staiger, D. (1999a). Comparing hospital quality at for-profit and not-for-profit hospitals, *Working Paper w7324*, NBER.
- McClellan, M. and Staiger, D. (1999b). The quality of health care providers, Working Paper w7327, NBER.
- Merrill, W. and Schneider, N. (1966). Government firms in oligopoly industries: a short-run analysis, *Quarterly Journal of Economics* **80**: 400–412.
- Nett, L. (1993). Mixed oligopoly with homogeneous goods, Annals of Public and Cooperative Economics **64**(3): 317–322.
- Newhouse, J. P. (1970). Toward a theory of nonprofit institutions: An economic model of a hospital, *American Economic Review* **60**(1): 64–74.
- Nishimori, A. and Ogawa, H. (2002). Public monopoly, mixed oligopoly and productive efficiency, *Australian Economic Papers* **41**(2): 185–190.
- Pauly, M. V. and Redisch, M. (1973). The not-for-profit hospital as a physicians' cooperative., *American Economic Review* **63**: 87–99.
- Propper, C., Burgess, S. and Green, K. (2004). Does competition between hospitals improve the quality of care? Hospital death rates and the NHS internal market, *Journal of Public Economics* 88: 1247–1272.
- Romano, P. S. and Mutter, R. (2004). The evolving science of quality measurement for hospitals: Implications for studies of competition and consolidation, *International Journal of Health Care Finance and Economics* 4: 131–157.
- Santerre, R. E. and Vernon, J. A. (2005). Hospital ownership mix efficiency in the US: An exploratory study, *Working Paper W11192*, NBER.
- Sloan, F. A. (2000). Not-for-profit ownership and hospital behavior, in A. J. Culyer and J. Newhouse (eds), *Handbook of Health Economics 1B*, Elsevier, Amsterdam, chapter 21, pp. 1141–1174.

Steinberg, R. and Weisbrod, B. A. (2005). Nonprofits with distributional objectives: Price discrimination and corner solutions, *Journal of Public Economics* 89: 2205–2230.

Wang, X. H. and Yang, B. Z. (2001). Mixed-strategy equilibria in a quality differentiation model, *International Journal of Industrial Organization* 19(1-2): 213 – 226.

Willner, J. (2006). A mixed oligopoly where private firms survive welfare maximisation, J Ind Compet Trade 6: 235–251.

A Appendix

A.1 Consumer Surplus with high and low Transportation Costs

Comparing the consumer rents without obeying the necessary constraint on transportation costs and distance, we can identify three different orders of magnitude shown in the table below. In the case of high transportation costs (1 and 2), the order is clear, the symmetric public scenario is preferred by the patients with $K^s > K^a > K^p$. For low transportation costs, the asymmetric setting can lead to lowest (3) and highest (4) consumer surplus depending on the relative marginal costs of the two hospitals.

	$D > \frac{1}{2}\alpha$	$D < \frac{1}{2}\alpha$
$t\Delta > \frac{1}{2}\sqrt[3]{(-2D+\alpha)}$ if $D < \frac{1}{2}\alpha$	1	2
$t\Delta > \frac{1}{2}\sqrt[3]{(2D-\alpha)}$ if $D > \frac{1}{2}\alpha$	$K^s > K^a > K^p$	$\mathbf{K}_s > \mathbf{K}_a > \mathbf{K}_p$
$t\Delta < \frac{1}{2}\sqrt[3]{(-2D+\alpha)}$ if $D < \frac{1}{2}\alpha$	3	4
$t\Delta < \frac{1}{2}\sqrt[3]{(2D-\alpha)}$ if $D > \frac{1}{2}\alpha$	$K^s > K^p > K^a$	$K^a > K^s > K^p$

As expected, two profit maximising hospitals set quality levels such that the consumer surplus is always lowest across scenarios. Since it is assumed that $t\Delta > t\tilde{\Delta}$, only cases 1 and 2 will be observed in equilibrium.

A.2 Comparison of Profits

The hospital's profits in the mixed duopoly (25) and (26) are lower than the profits of the profit maximising hospitals in the private duopoly (19) and

(20) if

$$\pi_1^p - \pi_1^a > 0 \Leftrightarrow 8t^2\Delta^2 - 4t\Delta > -5\alpha + 2D$$

and

$$\pi_2^p - \pi_2^a > 0 \Leftrightarrow 8t^2\Delta^2 + 4t\Delta > 3(\alpha - 2D)$$

The profits of the first of the two public hospitals in the state-owned duopoly (21) are lower than the public hospital's profits of the mixed duopoly (25) if

$$\pi_1^s - \pi_1^a < 0 \Leftrightarrow 8t^2\Delta^2 + 4t\Delta > 5\alpha + 6D$$

The profits of the second public hospital (22) are lower than the private hospital's profits of the mixed duopoly (26) if

$$\pi_2^s - \pi_2^a < 0 \Leftrightarrow 8t^2 \Delta^2 - 4t\Delta > -3\alpha - 2D$$

In the stable Nash equilibrium it is assumed that transportation costs and distance are sufficiently high with $t\Delta > \frac{1}{2}C + \frac{1}{2} + \alpha$. Thus, the above inequalities are fulfilled in equilibrium and $\pi_i^p > \pi_i^a > \pi_i^s$.

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