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A Micro-Macro Linkage**

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Loan Supply and Bank Capital: A Micro-Macro Linkage*

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Abstract

In the presence of financial frictions, banks' capital position may constrain their ability to provide loans. The banking sector may thus have important feedback effects on the macroeconomy. To shed new light on this issue, we combine two approaches. First, we use microeconomic balance sheet data from Germany and estimate banks' loan supply response to capital changes. Second, we modify the model of Gertler and Karadi (2011) such that it can be calibrated to the estimated partial equilibrium elasticity of bank loan supply with respect to bank capital. Although the targeted elasticity is remarkably different from the one in the baseline model, banks continue to be an important originator and amplifier of macroeconomic shocks. Thus, combining microeconomic results with macroeconomic modeling provides evidence on the effects of the banking sector on the macroeconomy.

Keywords: DSGE, bank capital, loan supply, financial frictions

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1 Introduction

Although the banking sector was neglected in early dynamic stochastic general equilibrium models, it is very intuitive that it can generate important feedback effects on the macroeconomy.¹ First, the banking sector may amplify aggregate shocks, thereby causing larger business cycle fluctuations. Second, it may originate shocks (e.g. due to bank capital losses) that have important feedback effects on the macroeconomy. In both cases, financial frictions are the driving force for the transmission of the effects to the real economy. Gertler and Karadi (2011) (GK henceforth) have proposed a workhorse model which operates in these two dimensions. In the GK model, bankers can divert a fraction of assets and thereby only receive a certain amount of non-capital funding to prevent bankruptcy in equilibrium. This limits the loan volume that banks can lend to firms and generates an external finance premium between the interest rates on bank loans and the central bank interest rate. The ability to lend in the GK model is constrained by banks' capital position, which can only be increased by retained earnings. Thus, the connection between bank capital and loan supply is crucial for the effects of the banking sector on the macroeconomy.

Interestingly, so far there is no evidence whether the GK model can be calibrated to the actual microeconomic bank behavior. However, it is important to test whether the model suffers from potential micro-macro puzzles.² Suppose, banks' response to capital losses in the model is considerably larger than in the data. In this case, the model may play an excessively important role in the banking sector for the macroeconomy. Alternatively, it may equally be the other way around, in which case banks may be more vulnerable to capital changes in the data than in the model.

This paper shows how the estimated co-movement between bank capital and loan supply at the microeconomic level can be connected to the dynamic stochastic general equilibrium (DSGE) model developed by GK. We show that the original GK model has an implicit partial equilibrium elasticity of loan supply with respect to capital of one, i.e. an individual bank's capital loss of 1 percent is associated with a reduction in lending by this bank of 1 percent. We propose a tractable way of modifying the GK model in order to obtain partial equilibrium elasticities that are different from one. We estimate the partial equilibrium elasticity with microeconomic banking data. For this purpose, we

¹Financial frictions received great attention through the works of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1999). However, in these papers, banks' balance sheet structure does not matter. For a theoretical contribution on the effects of bank capital on the macroeconomy, see van den Heuvel (2002). For early applied microeconomic studies see, for example, Gambacorta and Mistrulli (2004) and Merkl and Stolz (2009). For an early contribution on the credit channel in Germany, see Hülsewig et al. (2006).

²Micro-macro puzzles are present in many other areas of macroeconomics. Aggregate models require, for example, a large Frisch elasticity of labor supply to obtain sufficiently large labor fluctuations, while microeconomic estimation delivers rather moderate labor supply elasticities (see e.g. Chetty et al., 2011). In another context, small-scale New Keynesian models require relatively rigid prices to be in line with aggregate price dynamics (Chari et al., 2000), while the microeconomic evidence points to smaller price durations (e.g. Bils and Klenow, 2004).

use supervisory microeconomic data provided by the Deutsche Bundesbank’s Borrowers’ Statistics as well as the prudential database BAKIS for all German banks and employ regional area fixed effects regressions and a local matching approach. We find that the actual elasticity of banks’ loan supply with respect to bank capital is around 0.25, i.e. considerably lower than in the original GK model. We calibrate the modified GK model to this elasticity and find that the banking system is still highly relevant for the macroeconomy. What is the intuition for this surprising result? The GK model has powerful general equilibrium (GE) effects. Assume there is an exogenous negative shock to banks’ capital in the model. In partial equilibrium (PE), whenever one specific bank is hit by a bank capital shock, it takes a long time until the bank returns to its steady state level of capital by retaining earnings. However, if all banks are hit by a bank capital shock, in general equilibrium market prices respond. The external finance premium (the interest rate banks charge relative to the riskless interest rate) increases due to a scarcity of aggregate loans. This raises banks’ expected income streams and thereby increases their scope for lending, relative to PE. Due to these powerful general equilibrium effects, the GK model does not suffer from a micro-macro puzzle. A different PE elasticity leaves the output and loan responses of the model largely unaffected. In addition, we are able to show that the relative importance of PE versus GE effects depends on the steady state leverage ratio. In a steady state with a lower leverage ratio, PE effects become more significant relative to GE effects. However, given the importance of GE effects, the external finance premium in the modified model becomes more volatile relative to the GK model and is thereby more in line with the aggregate data.

The methodological exercise in our paper aims at stimulating the interaction between macroeconomic financial models and applied microeconomic estimations. We look at this issue through the lens of the GK model and obtain the following insights. If an applied microeconomist finds a low PE elasticity, macroeconomic policy makers should not take this as a sign for the unimportance of the banking sector. However, if the estimated elasticity is not statistically significantly different from zero, this would represent an important piece of information for the applied macroeconomic modeler. In this case, banks’ loan supply behavior would not be constrained by the banks’ capital position. This would be a sign that the GK mechanism is not binding at all.

We use the GK model because it can be considered as a workhorse in this field. Obviously, there are other macroeconomic models where financial frictions in the banking system, and thus banks’ capital position, play an important role (e.g. Gerali et al., 2010). Given that our microeconomic estimations are not tailored to a particular model, this paper sets the stage for future research to analyze whether models other than that of GK lead to similar conclusions. Towards the end of our paper, we argue that our conclusions are robust when extending the GK model as proposed by Rannenberg (2016). He combines the mechanisms in the GK model with the financial accelerator mechanism developed by Bernanke et al. (1999).

Why do we perform our estimations with German data? Very importantly,

we have high-quality balance sheet information on an annual basis for the entire universe of German banks, which amounts to 1,700 institutions in the cross section in 2013. The majority of German banks has a business model which largely resembles that of the banks in the GK model. They have a regional business model that focuses on lending activities instead of fee income-driven activities. In addition, as in GK, these banks cannot issue capital, but have to absorb losses and grow by retaining earnings.³ The German banking system is not only very much in line with theory, but also allows us to identify the co-movement of bank capital and loan supply. Obviously, changes in lending may either be driven by demand or supply. In order to isolate supply effects, we use a regional fixed effects approach and additionally apply the matching method of Carlson et al. (2013). The regional principle of most German banks allows us to identify these effects appropriately.

Our paper provides a valuable contribution concerning the role of the German banking system for the macroeconomy. However, our results reach beyond the German case. Both our proposition of how to modify the GK model and the insights on the effects lower partial equilibrium elasticities have on aggregate outcomes are relevant for other economies as well.

The remainder of this paper is structured as follows. In Section 2, we describe the empirical strategy and provide empirical results. Section 3 contains the core banking part of the GK model, the analytical results and the modification of the model. In Section 4, we calibrate the modified model, show numerical results and demonstrate their implications. Section 6 concludes.

2 Empirical Analysis

2.1 The Effect of Bank Capital on Lending

Our empirical analysis aims at estimating an elasticity of loan supply with respect to changes in capital for German banks, i.e. the reaction of a single bank to capital changes. We then use this elasticity to calibrate the model in Section 4.

The existence of a relationship between bank capital and lending has been widely examined in empirical studies. The main issue when trying to estimate the effect of capital on lending is endogeneity.⁴ Factors that affect loan supply may also affect loan demand. Firms' demand for credit could therefore be driven not solely by the supply side of credit. It is thus necessary to separate supply from demand effects in order to estimate an exogenously driven change in the amount of loans resulting from a change in the capital position. The empirical

³The majority of German banks are either cooperative banks (which are owned by their customers) and their central cooperative banks (as money-center banks), or local savings banks (which are owned by local governments) and their state banks or "Landesbanken" (as money-center banks). Neither type of bank issues equity as an instrument to increase its capital stock on a usual basis. The third pillar consists of private commercial banks, which have access to the national and international markets for bank capital.

⁴A discussion of the endogeneity issue can be found in Peek and Rosengren (2000).

literature deals with this issue in various ways. Gambacorta and Mistrulli (2004) directly include control variables for credit demand in their regression. Jiménez et al. (2012) disentangle supply and demand by proxying credit demand by loan applications. Similarly, Bassett et al. (2014) use survey data to construct a credit supply indicator which is adjusted for bank specific and macroeconomic factors that affect loan demand. An earlier study by Hancock et al. (1995) uses a vector autoregressive methodology, which treats all variables as endogenous, to identify the effects of capital shocks on bank loans.

Another approach can be found in Carlson et al. (2013). The authors examine commercial banks in the United States. Their basic assumption is that banks operating in the same geographical area face the same economic environment and therefore have the same demand for credit. The authors create a set of matched banks based on the banks' geographical proximity and on the banks' business models.⁵ Then, each bank is compared to its matched set regarding its capital position. Changes in lending between each bank and its matched set can therefore be attributed to differences between the banks. The advantage of this approach is that the supply side of credit can be isolated, while differencing out the demand side of credit.

This advantage is even more evident for the German banking system, the focus of our analysis. German savings and cooperative banks, which constitute the majority of German banks, operate according to the regional principle ("Regionalprinzip"). The savings banks' statutes require them to conduct their day-to-day business primarily within a confined regional area. Moreover, as pointed out by Berger et al. (2016), the regional principle is also de facto enforced for cooperative banks. Stolz and Wedow (2011) add that the German economy is mainly dominated by small and medium-sized firms that borrow primarily from local savings and cooperative banks. Consequently, the assumption regarding the local environment is appropriate for Germany. The approach is thus particularly well suited for analyzing the relationship between capital and lending for German banks and, as stressed by Carlson et al. (2013), theoretically controls better for local demand conditions in contrast to proxy variables. We apply the matching algorithm to German data as a complementary method to a regional fixed effects approach.

Using German data also has other advantages. First, the Deutsche Bundesbank provides high quality bank-level data. Second, unlike the banks in many other (European) countries, most of the banks in the German three-pillar banking system have a business model which is dominated by regional lending and generating interest income, which is a prerequisite for properly estimating and clearly identifying the effects in the DSGE model. Third, Germany, as the largest economy in Europe, has an outstanding role for European financial markets and overall economic development. Therefore, it is of major interest to base our analyses on German data.

⁵A bank is either matched to another single bank (1-1 matched set) or to several banks (1-N matched set).

2.2 Methodology and Data

We estimate two empirical models in which we isolate the effect of bank capital on credit supply. First, a regional fixed effects model with interacted regional and time fixed effects to control for time-varying local demand effects, and second, the matching approach by Carlson et al. (2013) described above. To make the estimates comparable in both models, we use the same set of explanatory variables.

The regional fixed effects estimation equation for bank i and quarter t , including an interaction term of regional and time fixed effects, takes the following form:

$$l_{it} = \alpha + \beta cap_{it-1} + \sum_{j=1}^4 \delta_j l_{it-j} + \rho_{it} reg_i * year_t + \epsilon_{it}, \quad (1)$$

where l denotes the cyclical component of total domestic loans and cap is the cyclical component⁶ of total bank capital.⁷ The corresponding β is the coefficient of interest as it captures the effect of credit supply due to variation in the capital positions in banks' balance sheets. β is the on-impact elasticity of bank loan supply with respect to capital. In addition, we include four lags⁸ of the dependent variable to capture the extent to which loan supply can be explained by past values.

As described above, in order to control for local economic conditions, we include an interaction term consisting of reg , a regional district dummy⁹, and $year$, a time dummy. It captures the differential effect of economic conditions in the regional districts, including local demand for credit, for each year.¹⁰ The intercept α is the mean for the baseline categories of reg and $year$.¹¹

Next, we apply the two-step matching algorithm, proposed by Carlson et al. (2013), to our data. In a first step, banks are matched according to their geographic coordinates. The matching procedure is repeated for each bank in each quarter and solely relies on distance. In a second step, the business model of each bank is compared to the business models of the matched set

⁶Both series are deflated using the GDP deflator at the state level and, to allow for an elasticity interpretation of the coefficient in the regression model, the logarithm is taken. The series are detrended using the Hodrick-Prescott filter with a smoothing parameter of 1,600 since, in accordance with the model, we are interested in the cyclical component.

⁷Note that we choose total capital instead of regulatory capital in order to abstract from variation associated with changes in regulation. Also, regulatory capital differs from economic capital. Results (available on request) remain robust when using Tier 1 capital instead of total capital in the regression.

⁸We choose four lags in accordance with the quarterly frequency of our data. Selecting a shorter lag length yields similar results, which are available on request.

⁹There are 31 regional districts in Germany.

¹⁰Interacting regional district dummies and time dummies for every quarter of a year yields comparable results. The results are available upon request.

¹¹The results are robust to the inclusion of additional control variables that reflect the quality of banks' loans (such as non-performing loans or loan loss provisions). However, in order to make the estimates comparable to the theoretical model of Section 3, we do not include additional control variables in the regressions below.

of neighboring banks. For this purpose, we select a range of business model indicators and exclude those banks from the matched set whose business model greatly diverges from the reference bank's business model. The banks' business models are evaluated according to the banks' size (measured by total assets), their profit and loss accounts (measured by the net interest margin) and their balance sheet composition (measured by the ratios of corporate loans to total assets and deposits to total assets, respectively). Also, the role of non-interest income for banks' business models is compared across banks.¹²

Consequently, our matched sets include banks that have a similar balance sheet composition, size and profitability, being located near their reference banks. This is crucial for making the assumption that demand for credit is identical in the vicinity of the reference bank.

In a second step, we modify the estimation equation above by calculating for each variable the differences between the reference bank's observation and the average observations of its matched set. Since we eliminate the effect of economic conditions and loan demand via the averages of neighboring banks in the same year and quarter, interaction terms (*reg * year*) are no longer required in our regression model. The modified estimation equation for reference bank i , quarter t , and the average of the bank's matched set m has the following form:

$$l_{it} - l_{mt} = \beta (cap_{it-1} - cap_{mt-1}) + \sum_{j=1}^4 \delta_j (l_{it-j} - l_{mt-j}) + (\epsilon_{it} - \epsilon_{mt}). \quad (2)$$

The parameter β , although still an elasticity, now has a slightly different interpretation because it measures the effect on the domestic loan supply of bank i in comparison to the matched set of banks m .

However, both estimation techniques are equivalent in that they yield a coefficient β that shows the initial impact of capital changes on loans measured in terms of an elasticity (i.e. a 1 percent loss in total bank capital leads to a β percent reduction in total domestic loans). In the modified GK model presented in Section 3, there are no meaningful adjustment dynamics, i.e. the partial equilibrium short-run elasticity for a single bank would be equal to the bank's long-run elasticity. In the data, we explicitly control for dynamic adjustment. Thus, we adjust the coefficient for capital in order to get the total effect on loans over four quarters. We calculate the adjusted coefficient using following equation:

$$\beta_{adj} = \frac{\beta}{1 - \delta_1 - \delta_2 - \delta_3 - \delta_4}. \quad (3)$$

We obtain the standard errors for the adjusted elasticity using the delta method. Appendix A.4 contains a detailed derivation of the standard errors following Greene (2012). We use this adjusted elasticity as our benchmark for the calibration of the macro model.

¹²Details regarding the comparison of the banks' business models are provided in Appendix A.1.

Quarterly data at the individual bank level are provided for all German banks for the period from 1998 to 2013 by the Deutsche Bundesbank. Data sources, data preparation procedure and descriptive statistics are shown in Appendix A.1. We estimate equations (1) and (2) for the whole sample of all German banks (Table 1) and for a subset containing only regional banks (see Appendix A.2). Additionally, as dependent variable we use total domestic loans in the baseline regression (Table 1) and domestic corporate loans in the auxiliary regression presented in Appendix A.3).

2.3 Results

Table 1 shows that variation in banks' capital position significantly affects loan supply. Depending on the estimation procedure, the on-impact elasticity of total capital varies from 0.066 to 0.102. Therefore, an immediate increase in capital by 1 percent increases loan supply on average by 0.066 to 0.102 percent. The estimate in the regional fixed effects regression is higher than those in the matching regressions. This may be associated with the business model adjustment that we performed in the matching regressions, while no adjustment has been made in the regional fixed effects regressions.

Although equally statistically significant, the adjusted parameters are in general considerably larger than the on-impact parameters, due to the persistence of loans. Loans over the last four quarters help explain a considerable share of loans in the current quarter. The four lags of the dependent variable are jointly significant in all three specifications. We test this using a standard Wald test. The basis for the calibration of the macro model in Section 4 is the adjusted elasticity of loan supply with respect to capital changes, which is displayed in the last row of Table 1. The estimates range from 0.18 to 0.29.

Our numerical simulations in Section 5 are performed using an elasticity of 0.25. Therefore, if a bank's capital is 1 percent lower than its peer group's, its loan supply would be lower by 0.25 percent compared to its peer group.

Since our estimation method relies on local economic conditions and therefore regional lending being constant in the vicinity of a bank, we also perform the estimation for a subset of the data, including regional banks only. This procedure ensures that the results for the sample of all banks are not driven by (mostly large) private banks, Landesbanks and cooperative money center banks. The results are shown in Appendix A.2. They confirm that private banks are not the driving force in the computation of the elasticities. It is worth noting that banks in the model of Section 3 lend to firms only; households in the model do not borrow. In order to confirm that our empirical results are not driven by domestic retail loans, we estimate equations (1) and (2) by replacing total domestic loans with domestic corporate loans as the dependent variable. The results are shown in Appendix A.3. They indicate that banks' change in their capital position influences domestic corporate loans similarly to total domestic loans.

Summing up, a single bank's loan supply decreases on average by 0.25 percent when the bank faces a decline in its total capital of 1 percent. In the

following, we use this elasticity to investigate how the banking system as a whole behaves when it suffers a capital shortage, and which feedback effects any such capital shortage has on the real economy.

3 Banks' Response to Capital Shocks

After having estimated an elasticity of bank loan supply with respect to capital using German bank data, we will compute the corresponding partial equilibrium elasticity in the model of Gertler and Karadi (2011) and change it in order to match the estimated elasticity. In a first step, we use the GK model to back out the implicit partial equilibrium elasticity of bank loan supply with respect to bank capital. In a second step, we modify their model such that a different PE elasticity can be chosen for its calibration. For this purpose, we adjust the banking sector but leave the other sectors in the model unchanged. Hence, we will show a detailed derivation of the banking sector below, whereas all the remaining equations of the medium-scale model can be found in Appendix B.

3.1 The Baseline Banking Model

Banks in the Gertler and Karadi (2011) model borrow funds at the riskless rate R (set by the central bank) and lend to firms at rate R_k , where $R_k - R$ is defined as the external finance premium. Thus, the dynamic equation for the capital/net worth (N_j) of a particular bank j is

$$N_{jt+1} = (R_{kt} - R_t) Q_t S_{jt} + R_{t+1} N_{jt}, \quad (4)$$

where S_j is the quantity of loans and Q is the market price for loans. Note that bankers in GK hold the physical capital stock of the economy. Thus, the market price of loans is equal to the market price of physical capital.

Bankers maximize the discounted present value V_j by choosing an optimal quantity of loans. They take into account that they will survive with probability θ :

$$V_{jt} = \sum_{i=0}^{\infty} (1 - \theta) \theta^i \beta^{i+1} \Lambda_{t,t+i} [(R_{kt+1+i} - R_{t+1+i}) Q_{t+i} S_{jt+i} + R_{t+1+i} N_{jt+i}]. \quad (5)$$

As long as $R_k > R$, the bank would like to increase its loan volume indefinitely. Due to a moral hazard problem, banks are unable to do so. It is assumed that banks can divert a certain fraction of assets λ and households cannot recover this fraction in the event of default. According to the incentive compatibility constraint, households are only willing to supply funds to the bank if

$$V_{jt} \succeq \lambda Q_t S_{jt-1}, \quad (6)$$

Coefficients	Regional Fixed Effects	Matching 1:N	Matching 1:1
β	0.102*** (0.007)	0.098*** (0.003)	0.066*** (0.003)
δ_1	0.554*** (0.037)	0.575*** (0.003)	0.596*** (0.003)
δ_2	0.153*** (0.054)	0.091*** (0.004)	0.109*** (0.004)
δ_3	-0.085* (0.048)	-0.032*** (0.003)	-0.035*** (0.003)
δ_4	-0.021 (0.032)	-0.028*** (0.003)	-0.030*** (0.000)
α	0.003*** (0.000)	-	-
R^2	0.450	0.423	0.464
N	123,592	87,823	75,397
β_{adj}	0.285*** (0.025)	0.249*** (0.008)	0.184*** (0.009)

*Note: The table shows results for the regional fixed effects and the matching regressions for all banks. The dependent variable for the fixed effects regression is l_{it} and, for the matching regressions, $l_{it} - l_{mt}$ with l denoting the cyclical component total domestic loans. The regional fixed effects regression includes an interaction term of regional and year fixed effects, and standard errors are clustered at the regional district level. The standard errors for the adjusted elasticities are computed using the delta method. The parameters for the interaction terms are not reported for the regional fixed effects regression, but they are jointly significant when performing a regular Wald test. Note that we keep four lags of the dependent variable as explanatory variables since they are jointly significant for all three specifications. Standard errors are reported in parentheses. Coefficients with *, **, *** are significant at the 10 %, 5 %, and 1 % level respectively using standard t -distribution.*

Table 1: Results for the regional fixed effects and the matching regressions for all banks

i.e. the gain from diverting funds is smaller than the value of the bank. This will prevent diversion of funds in equilibrium.

Thus, in equilibrium bankers will lend up to the maximum possible amount:

$$V_{jt} = \lambda Q_t S_{jt-1}. \quad (7)$$

Substituting (7) into (5), we obtain:

$$(1 - \theta) \sum_{i=0}^{\infty} \theta^i \beta^{i+1} \Lambda_{t,t+i} [(R_{kt+1+i} - R_{t+1+i}) Q_{t+i} S_{jt+i} + R_{t+1+i} N_{jt+i}] = \lambda Q_t S_{jt-1}. \quad (8)$$

In order to express this equation in steady state, we have to take into account that assets and net worth of surviving banks grow at gross rate z in steady state. Intuitively, (surviving) banks retain their profits, which increases their capital and thereby their ability to borrow and lend money.

Thus, in steady state:

$$\lambda Q S_j = \frac{(1 - \theta) \beta [(R_k - R) Q S_j + R N_j]}{1 - \theta z \beta}. \quad (9)$$

Reformulating this term:

$$Q S_j = \frac{(1 - \theta) \beta R}{\lambda (1 - \theta z \beta) - (1 - \theta) \beta [(R_k - R)]} N_j. \quad (10)$$

This equation pins down the asset to capital ratio (leverage ratio) for bank j . Note that the leverage ratio only depends on aggregate variables (not bank-specific variables). This expression allows us to calculate a partial equilibrium elasticity of a particular bank's lending with respect to its net worth, i.e. the connection between these two variables at given market prices, as measured in the empirical analysis.

$$\frac{\partial \ln S_j}{\partial \ln N_j} = 1. \quad (11)$$

A 1% lower net worth (bank capital) is always associated with a 1% lower loan supply to firms in partial equilibrium. The intuition for the partial equilibrium elasticity of 1 is straightforward: The incentive compatibility constraint pins down a fixed leverage ratio, which does not depend on any bank-specific variables. Thus, in partial equilibrium (i.e. without adjustments of aggregate price variables such as R_k , R and Q), net worth and the balance sheet size always move together at this fixed ratio. Note that this is not a causal relationship but a correlation that applies irrespective of its cause, i.e. whether the change was triggered by the asset or the liability side. This is in line with our empirical identification strategy in Section 2.

Note that GK do not calculate this implicit PE elasticity because they follow a macroeconomic calibration strategy (i.e. targeting certain variables such as

the external finance premium). By contrast, we aim at calibrating the model to the elasticity of bank loan supply with respect to bank capital, based on the empirical estimate from Section 2.

3.2 The Modified Banking Model

In order to make the model more flexible and to be in line with the empirical elasticity, assume that diversion of assets is a function of the balance sheet size, which may either be concave or convex, i.e. $\lambda(S_j)$ with $\frac{\partial \lambda(S_j)}{\partial S_j} > < 0$. This means that it may be easier/harder for bankers to divert funds (e.g. due to a different corporate governance structure) if banks become larger over the business cycle and vice versa.

Replacing λ by $\lambda(S_j)$ in equation 7, we obtain in steady state:

$$\lambda(S_j) Q S_j = \frac{(1 - \theta) \beta [(R_k - R) Q S_j + R N_j]}{1 - \theta z \beta} \quad (12)$$

After some algebra (see Appendix B.2), we obtain the following partial equilibrium elasticity:

$$\frac{\partial \ln S_j}{\partial \ln N_j} = \frac{\lambda(S_j) (1 - \theta z \beta) - (1 - \theta) \beta (R_k - R)}{\lambda(S_j) (1 - \theta z \beta) - (1 - \theta) \beta (R_k - R) + \frac{\partial \lambda(S_j)}{\partial S_j} (1 - \theta z \beta) S_j}. \quad (13)$$

Note that for $\frac{\partial \lambda(S_j)}{\partial S_j} = 0$, we obtain the special case $\frac{\partial \ln S_j}{\partial \ln N_j} = 1$. For a convex function ($\frac{\partial \lambda(S_j)}{\partial S_j} > 0$), the elasticity becomes less than one. For a concave function ($\frac{\partial \lambda(S_j)}{\partial S_j} < 0$), it becomes greater than one.

In a nutshell, our simple model modification provides enough flexibility to calibrate the model according to microeconomic estimation results. Our empirical estimation has shown that the partial equilibrium elasticity is significantly less than one, and therefore we require $\frac{\partial \lambda(S_j)}{\partial S_j} > 0$. Intuitively, this means that for a bank that grows due to an expansionary business cycle shock, it is easier to divert a greater share of funds.¹³ As emphasized by GK, fund diversion should not be interpreted literally. Instead, we interpret the bankers as managers who grant extensive bonuses or implement inefficient organizational structures. Thus, it appears realistic for this type of inefficiency to become more severe when banks grow over time (e.g. due to higher organizational complexity).

In our numerical analysis, we use the following functional form, which is consistent with the convex shape:

$$\lambda(S_{jt}) = c S_{jt}^\Psi, \quad (14)$$

¹³It is important to note that there is no bank heterogeneity in the model. Due to our assumptions on bank entry (see Appendix B for further details), all banks have the same size. However, the business cycle affects the size and thus the ability of banks to divert funds.

where c and Ψ parameters and $c, \Psi > 0$.¹⁴ For further details regarding the model, see Appendix B.

Obviously, our model modification is not based on deep theoretical micro-foundations. However, it provides us with a very flexible way of recalibrating the GK model and checking whether we run into a micro-macro puzzle. As we will show in Section 5, the model's quantitative reactions to shocks are very similar when calibrating it to different PE elasticities. Due to powerful GE effects, we expect this result to be robust, even with different modeling assumptions.

4 Calibration

For our calibration, we attempt to stay as close as possible to GK for comparability reasons. At the same time, we adjust some parameter values due to the modified model structure and due to German specificities. Table 2 shows the choice of model parameters. As described by GK, 15 parameters are fairly standard in the DSGE literature. For comparability reasons, we do not change any of the parameters of households, intermediate goods firms, capital producing firms, retail firms and the government.

Only some of the parameters/functions for financial intermediaries are different. We pick the same survival rate of bankers (θ) as GK. While the fraction of assets that can be diverted (λ) is exogenous in GK, it is replaced by the function $\lambda(S_{jt}) = cS_{jt}^\Psi$ in our model. We choose the remaining three parameters (c, Ψ, ω) to hit three targets, namely, the steady state leverage ratio (ϕ), the steady state spread (external finance premium) between banks' loans and the riskless interest rate ($R_k - R$) and the partial equilibrium elasticity of bank loans with respect to bank capital changes. We are interested in assessing how the economy behaves when the partial equilibrium elasticity of loan supply with respect to changes in a bank's capital position is calibrated to the estimated elasticity. Given that the implied elasticity in the original GK model is one, we take the estimate of 0.25 for our numerical simulation.

We target the same external finance premium as GK, namely 100 basis points on an annual basis. However, we choose a steady state leverage ratio of 7 (instead of 4). Similarly to the balance sheet composition of banks in the model, the leverage ratio of 7 is calculated by the ratio of bank loans to capital.¹⁵

How do we determine the partial equilibrium elasticity of bank loans with respect to bank capital in the model? It is important that this elasticity corresponds to our empirical analysis, which measures the loan supply effects of bank capital changes at the bank level. To replicate this in our model, we switch off all general equilibrium effects. In particular, we simulate the partial equilibrium banking model where market prices such as asset prices are not affected

¹⁴See Appendix C.2 for the assumptions that we require in order to impose stationarity on the model.

¹⁵We use aggregate data provided by the Deutsche Bundesbank that corresponds to the definition in our empirical analysis. Therefore, we take total bank loans to domestic firms and households and total bank capital. Results for a leverage ratio of 4 are available upon request. The key findings of the paper are not affected by the choice of the leverage ratio of 7.

Households		
β	0.99	Discount rate
h	0.815	Habit parameter
χ	3.409	Utility weight
φ	0.276	Labor supply parameter
Financial intermediaries		
c	$7 * 10^{-4}$	Scaling parameter in diversion function
Ψ	3.600	Convexity in diversion function
ω	0.002	Transfer to entering bankers
θ	0.972	Survival rate of bankers
Intermediate goods firms		
α	0.330	Effective capital share
U	1.000	Steady state utilization rate
$\delta(U)$	0.025	Steady state depreciation rate
ς	7.200	Elasticity of depreciation rate wrt utilization
Capital producing firm		
η_i	1.728	Inverse elasticity of net investment to capital price
Retail firms		
ε	4.167	Elasticity of substitution on the goods market
γ	0.779	Calvo parameter (fixed prices)
γ_p	0.241	Price indexation
Government		
κ_π	1.500	Weight on inflation in Taylor rule
κ_y	0.125	Weight on output gap in Taylor rule
ρ	0.8	Smoothing parameter
G/Y	0.2	Steady state share of government spending

Table 2: Parametrization of the Model

by a policy shock. In addition, no adjustments are made to the firms' physical capital, aggregate employment or monetary policy. Intuitively, if a bank is atomistic, a bank-specific capital shock does not affect any of the economy-wide variables. Thus, the bank has to adjust by its own means (e.g. by having a different leverage ratio according to the incentive constraint and by retaining earnings).

We solve our model with a log-linearization (i.e. with a first-order Taylor approximation). The bank leverage ϕ (in log-linearized form) is

$$\eta\hat{\phi}_t = \phi v\hat{v}_t + \eta\hat{\eta}_t - \Psi^2\phi c^2 S^{2\Psi-1}\hat{s}_t, \quad (15)$$

where v is the expected discounted marginal gain of a bank of increasing assets and η is the ceteris paribus gain from having an extra unit of net worth (see Appendix B and GK).

In the GK model, the last term on the right hand side is not present. This term allows us to calibrate the partial equilibrium elasticity to 0.25. For this purpose, we require $\Psi = 3.6$. While this implies a rather strong convexity of $\lambda = cS^\Psi$ (in terms of the underlying nonlinear function), it has to be kept in mind that we identify the partial equilibrium elasticity based on a log-linearization, where the effects are reasonable because they are based on the estimated empirical estimations. Ignoring partial equilibrium adjustments of \hat{v}_t and $\hat{\eta}_t$, the direct reaction of the leverage ratio to changes in the balance sheet size would be $\Psi^2\phi c^2 S^{2\Psi-1}/\eta = 0.85$. Ceteris paribus, a 1% reduction in assets would allow the bank to increase its leverage ratio by roughly 0.85%.

5 Simulation Results

5.1 Modified Model Reactions

Figures 1 to 4 show the model economy's response to a net worth shock, an aggregate productivity shock, an interest rate shock and a capital quality shock, respectively, for our modified model, for the baseline Gertler and Karadi (2011) model, and for a frictionless economy without a banking sector. The persistence of all shocks is set as in GK.¹⁶ The shock size is normalized to 1 percent for aggregate productivity, the interest rate shock and the capital shock. For the net worth shock, the exogenous shock size is 25 percent. Figure 1 shows that the endogenous fall in net worth is even larger due to an endogenous decline in market prices and, hence, a depreciation of assets.

In all four cases, our model economy, which is calibrated to the estimated microeconomic elasticity, generates a somewhat milder recession than the GK baseline model (see the responses of GDP, investment and net worth in the first row of the figures). The intuition is straightforward. Banks face a decline in their capital due to the negative aggregate shocks. They reduce their loan

¹⁶The net worth shock has no past dependence, the productivity shock has an autocorrelation of 0.9, the interest rate shock an autocorrelation of 0.8 and the asset quality shock an autocorrelation of 0.66.

supply due to a reduction in the quantity of deposits they are able to attract (resulting from their asset diversion constraint). With our modified asset diversion constraint, the fraction of assets that can be diverted increases as loan supply falls. Therefore, banks can leverage themselves more than in the GK model immediately after the respective aggregate shock hits the economy. Thus, the loan supply decreases by less in our model than in the GK model. Given that banks have to deleverage to a greater extent in our model when the shock decays, the external finance premium increases by more after the shock. This allows faster recapitalization for banks in our model, which accelerates the reversion to the steady state.

While aggregate variables react qualitatively by less in our model than in the GK economy, in three out of four exercises (net worth, aggregate productivity, and the interest rate shock) the quantitative reaction is fairly similar. In other words: Although the microeconomic elasticity is substantially smaller in our model than in GK, the macroeconomic differences remain very modest for three shocks. Why is this the case? Under these three shocks, the leverage ratio in the GK model shows a strong response while the loan volume movements are relatively moderate. As a consequence, the extra effects due to a relaxed asset diversion constraint are moderate. This can be understood by looking at the relevant log-linearized equation. If $\hat{\phi}_t$ already moves a lot due to changes in \hat{v}_t and $\hat{\eta}_t$, the extra effects due to changes in the balance sheet volume are small:

$$\eta\hat{\phi}_t = \phi v\hat{v}_t + \eta\hat{\eta}_t - \Psi^2\phi c^2 S^{2\Psi-1}\hat{s}_t. \quad (16)$$

The net worth shock (Figure 1) is well suited to illustrate the differences between the reaction in partial and general equilibrium. The exogenous net worth shock is set to 25 percent.¹⁷ In PE¹⁸, the loan volume declines by 6.25 percent due to the calibrated elasticity of 0.25 and the absence of GE effects. By contrast, the loan volume drops only by around 0.2 percent in GE (see lower right panel of Figure 1). One of the key reasons is the strong rise in the external finance premium in GE, which increases banks' current and future expected profits. A strong increase in profits loosens their incentive compatibility constraint. Banks are able to collect more deposits than without this external finance premium increase. Thus, the decline in loans in response to a bank capital reduction is much less severe in GE than in PE. Due to powerful general equilibrium effects, the PE elasticity is not a key driver for the GK model for most shocks. In other words, the GK model does not run into a micro-macro puzzle, as other DSGE models often do. This does not mean that the estimated microeconomic elasticity is completely unimportant. If it is not statistically different from zero, this would be a sign that the GK mechanism is not binding in the first place.

The only shock where our model modification makes a substantial quantitative difference is the capital quality shock. The reason can be seen in Figure 4.

¹⁷Note that the endogenous net worth declines more substantially in GE due to a fall in the market price of loans.

¹⁸Simulations are available upon request.

In contrast to the net worth shock, the capital quality shock has a stronger effect on the loan volume.

Intuitively, a capital quality shock acts both as a negative loan supply and negative loan demand shock. Similar to a net worth shock, it affects the loan supply. Given that the capital becomes less productive, assets/loans lose value and banks' net worth is reduced. But at the same time, the decline in the marginal product of capital leads to a decline in the demand for productive capital and thus in loans. Both mechanisms lead to a fall in the loan volume. This reduces banks' ability to divert assets due to the moral hazard problem. At the same time, their ability to attract external funding falls by less. This explains why we obtain a substantial difference between our model and the GK model for the capital quality shock.

It is also interesting to compare the IRFs in our model to the frictionless economy (i.e. without a banking sector and therefore with an external finance premium of zero). Interest rate shocks have a greater effect under financial frictions than in the frictionless economy. By contrast, the differences for productivity shocks are smaller because productivity shocks affect the leverage ratio to a lesser extent than interest rate shocks.

For the capital quality shock, our model comes closer to the frictionless economy. However, in the short run, the drop in investment is twice as large as in the frictionless economy. In the medium run, the value of assets returns to its old value, implying a positive loan supply shock (which is absent in the frictionless model), and investment recovers more quickly.

Overall, the banking sector remains a substantial source of disturbance and amplification for the real economy. Despite reducing the microeconomic elasticity by 75%, the feedback effects on the macroeconomy remain strong. The main reason is that general equilibrium effects are very powerful in the model. In a partial equilibrium framework, the capital drop of an atomistic bank does not lead to any price adjustments. In the full general equilibrium model, there are various adjustment mechanisms that lead to a larger difference between the partial equilibrium and the general equilibrium elasticity. This can most easily be illustrated for the net worth shock. If an individual bank is hit by a negative 1% capital shock, its lending will go down by 0.25%. However, in the general equilibrium model (see Figure 1), the co-movement between net worth and the capital shock is quantitatively much smaller, due mainly to the endogenous adjustment of the external finance premium. If all banks are hit by a net worth shock, the reduced supply for loans increases their price R_k . This higher return on loans leads to larger (expected) future profits and thereby allows banks to collect more deposits (due to a relaxed asset diversion constraint).

Although the reaction of loan supply and output remains similar for most aggregate shocks, there is one stark difference. Given the importance of the general equilibrium effect, the standard deviation of the external finance premium becomes more volatile in our modified model than in the GK model. Why is this important? Rannenberg (2016) shows that the standard deviation of the EFP is only half as large in the GK model as in the data. Thus, our modified

model helps to bridge this gap and bring the model closer to the data.¹⁹

5.2 Different Steady State Leverage Ratio

In our baseline calibration, we have targeted a steady state leverage ratio of 7, consistent with German aggregate data. This number corresponds to the ratio of total bank loans to domestic firms and households relative to total bank capital for the observation period from 1998 to 2013.

After the Great Recession in 2008/09, banks have increased their capital positions, which led to a decline in leverage ratios. Leverage ratios were trending downwards during the last years and reached levels of 5 in 2016. Therefore, it is interesting to see how our conclusions from the previous section depend on the steady state leverage ratio.

In order to analyze the influence of a varying leverage ratio, we recalibrate our model to a steady state leverage ratio of 3 and 5 (instead of 7) and redo the quantitative exercises from above.²⁰ We calculate impulse response functions for the case when the economy is hit by a net worth shock.

To illustrate the differences of variations in the leverage ratio, we calculate cumulative loan and output responses.²¹ These cumulate responses indicate the overall loan and output loss due to the negative net worth shock. Table 3 shows the additional cumulative output and loan losses in the GK model relative to our modified model.

Steady State Leverage Ratio	7	5	3
% Difference: Cumulative Output Loss	18.3%	29.2%	50.2%
% Difference: Cumulative Loan Loss	20.1%	31.9%	54.4%

The table shows for different steady state leverage ratios the cumulative increase in percent of the output and loan loss for the modified model relative to the GK model when the economy is hit by a net worth shock.

Table 3: Cumulated Output and Loan Loss for Variations in the Leverage Ratio

Obviously, in all cases the cumulative loan/output losses are larger in the GK model than in the modified model. Our main conclusion that the banking system is an important originator of shocks for the macroeconomy (in our modified model) remains unaffected (in a frictionless model, the concept of a net worth shock does not even exist). However, the differences between the GK model and our modified model become larger with a lower steady state leverage ratio.

¹⁹Rannenberg (2016) uses business cycle statistics on the external finance premium (EFP) for the United States. However, the volatility of the EFP is much larger in Germany than in the United States (results are available on request).

²⁰We leave all other targets unchanged. To match the PE elasticities, we increase Ψ .

²¹We use 100 quarters.

What is the underlying reason? Our quantitative exercise from section 5 shows that general equilibrium effects are the driving forces for the amplification of shocks, leaving only a minor role to the PE elasticity. These general equilibrium effects are a lot more powerful in a highly leveraged banking system than in a lowly leverage banking system. The underlying compatibility constraint illustrates that banks' ability to collect funds is limited by their intertemporal present value in the GK model:

$$V_{jt} = \sum_{i=0}^{\infty} (1 - \theta) \theta^i \beta^{i+1} \Lambda_{t,t+i} [(R_{kt+1+i} - R_{t+1+i}) Q_{t+i} S_{jt+i} + R_{t+1+i} N_{jt+i}]. \quad (17)$$

When the external finance premium $R_k - R$ increases, this raises the bank's value V (relatively) more for highly leveraged banks compared to lowly leverage banks due to higher return on equity stemming from the leverage effect. Therefore, the ability to collect additional funds increases by more for the highly leveraged banking system (due to external finance premium increases). In different words, a highly leveraged banking system in the GK model generates more powerful general equilibrium effects.

This exercise provides an interesting insight regarding the importance of the PE elasticity (when estimated based on microeconomic data). Partial and general equilibrium elasticities of loan supply with respect to bank capital losses are closer to one another in an economy with low steady state leverage. By contrast, in a highly leveraged economy, the PE elasticity is a rather poor proxy to assess the financial (in)stability of the banking system (observed through the lens of the GK model).

5.3 Extensions

Rannenberg (2016) criticizes that the GK model generates a countercyclical leverage ratio. Given that banks own the capital stock of firms in the GK model, a recession leads to an immediate decline in net worth and, consequently to an increase in the leverage ratio. However, for US data the leverage ratio shows a positive correlation with GDP. Therefore, Rannenberg (2016) proposes to extend the GK model by including the Bernanke et al. (1999) financial accelerator mechanism. If banks do not own the capital stock but lend to firms, they only have to bear a small share of losses in recessions, net worth declines considerably less, and the leverage ratio becomes procyclical, as in US data.

Regarding our paper, two comments apply: First, the cyclical patterns of capital and the leverage ratio (defined as total assets divided by total capital) are very different in Germany (which we have taken as a reference point) compared to the United States. The cyclical component of bank capital has a correlation with the cyclical component of GDP of 0.33 (observation period from 1998:1 to 2013:4, Hodrick-Prescott filter with smoothing parameter $\lambda = 1600$).

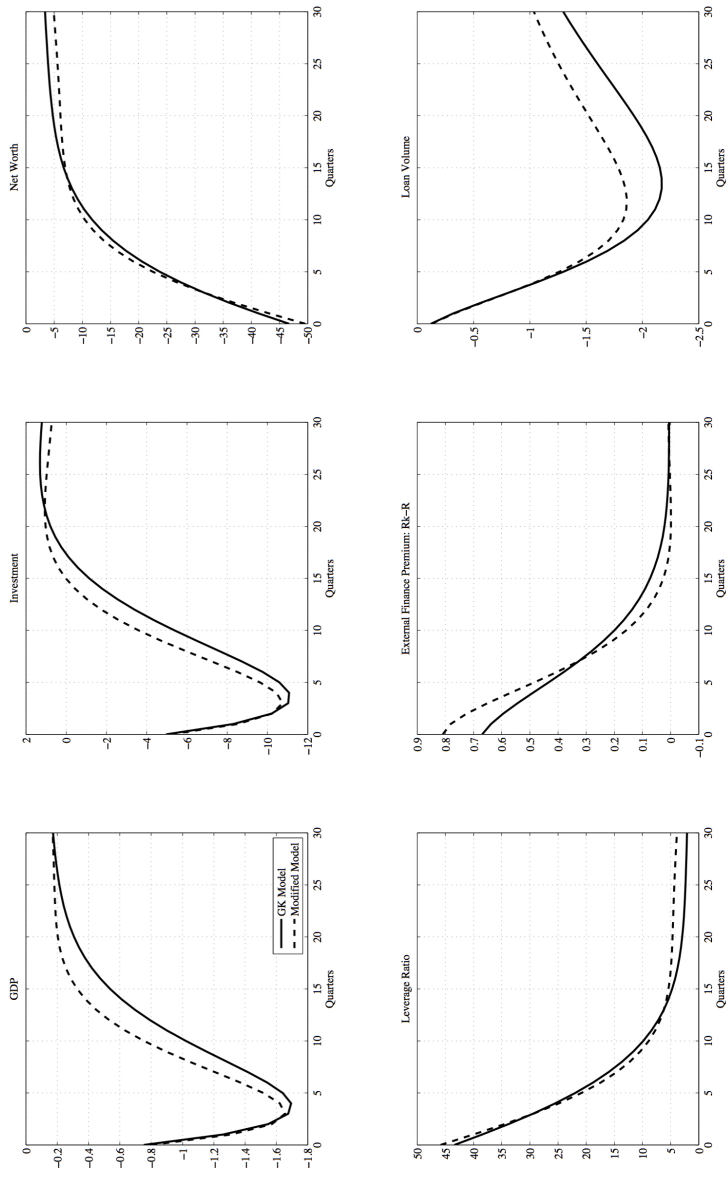


Figure 1: Net Worth Shock

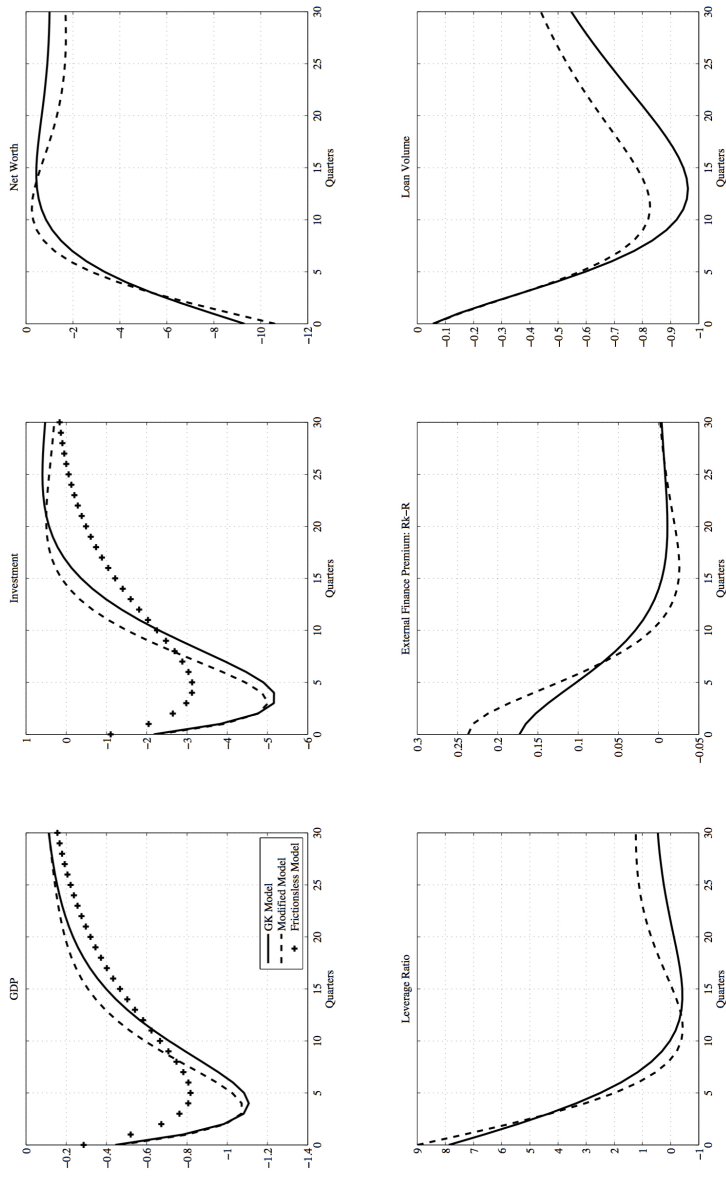


Figure 2: Aggregate Productivity Shock

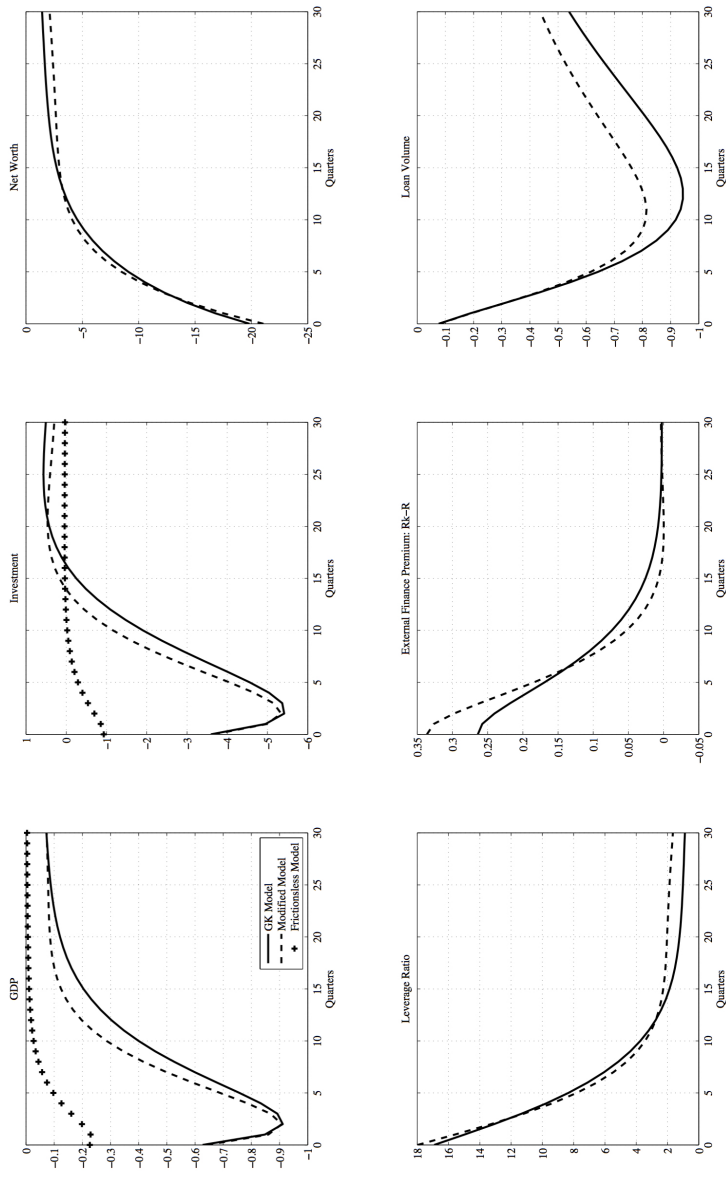


Figure 3: Interest Rate Shock

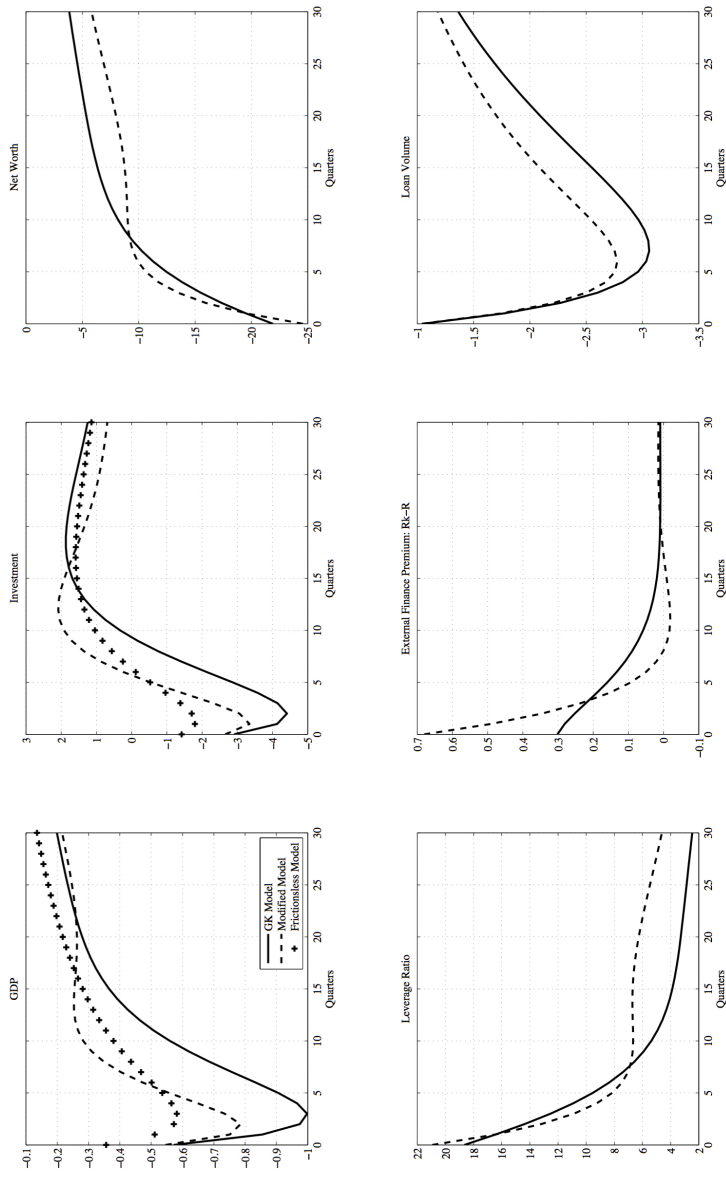


Figure 4: Capital Quality Shock

The leverage ratio is acyclical (correlation of 0.002) and the ratio of loans to bank capital is countercyclical (correlation of -0.35), as predicted by GK. Interestingly, the loan to capital ratio becomes more countercyclical in the Great Recession (correlation of -0.46).

Second, we have also applied our mechanism to the Rannenberg (2016) model and our main conclusions remain unaffected, despite the different cyclicity of the leverage ratio. As in the GK model, a lower PE elasticity does not affect the aggregate effects of various aggregate shocks considerably. This strengthens our conclusions that general equilibrium effects are of major importance in models with capital constrained banks in the vein of GK.²²

6 Conclusion

This paper shows an application of a methodology for connecting microeconomic behavior of the banking sector to macroeconomic modeling. Specifically, we estimate an empirical elasticity of bank loan supply with respect to capital changes of 0.25 using supervisory data provided by the Deutsche Bundesbank. Given that the implied partial equilibrium elasticity in the Gertler and Karadi (2011) model equals 1, we modify their model in order to be able to calibrate it to the estimated elasticity. By simulating different aggregate shocks, we assess the model's behavior in light of its ability to generate amplification. The simulated results suggest that the lower microeconomic elasticity generally has a dampening effect on the shocks. Nevertheless, the banking sector is still of great importance as a source and amplifier of business cycle fluctuations. We attribute this to the powerful general equilibrium effects in the model, i.e. the much lower partial equilibrium elasticity is not translated into a proportionally lower general equilibrium elasticity.

Although the outcomes of our modification approach are certainly model dependent, we regard our study as an important contribution to the financial frictions literature. To our knowledge, this study is the first attempting to link microeconomic evidence to a dynamic general equilibrium (DSGE) model with a banking sector. We leave it to future research whether other financial DSGE models are similarly robust with respect to different microeconomic elasticities.

Our results are relevant both for applied macroeconomic modelers and applied microeconometricians. We have shown that the GK model has an implicit PE elasticity of one. Interestingly, a modification of this elasticity does not affect the model's ability to generate strong macroeconomic amplification. This outcome is highly relevant for applied macroeconomic modelers. To our knowledge, we are the first to show that general equilibrium effects are very important in the GK model. In contrast to several other macroeconomic models, there is no micro-macro puzzle.

This is also an important message for applied microeconometricians with a focus on the banking system. A low estimated PE elasticity of loan supply

²²For space reasons, we abstain from showing the modified Rannenberg (2016) model equations and results. The impulse response functions are available upon request.

with respect to capital may be favorable regarding the resilience of the banking system. However, through the lens of the GK model, a banking system with a smaller PE elasticity does not necessarily generate much better macro outcomes (in terms of smaller fluctuations for a given set of aggregate shocks). Nevertheless, the estimated microeconomic elasticities have an important meaning. If the estimated elasticity were negative or not statistically significantly different from zero, this would be a sign that the GK mechanisms are either non-binding or inoperative. However, our estimations provide no evidence pointing in that direction. Overall, with this paper we seek to make a first step towards stimulating the fruitful interaction between both applied macroeconomics and microeconometrics in the financial frictions literature.

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A Empirical Model

A.1 Data, Data Processing and Descriptive Statistics

All supervisory micro data are provided by the Deutsche Bundesbank. In particular, we use portfolio-level data from the Bundesbank's Borrowers' Statistics, as well as bank balance sheet data from the Bundesbank's prudential database BAKIS (including the auditors' reports, "Sonderdatenkatalog", with information on the profit and loss accounts). The data set is available from 1998 to 2013. Our dependent variable, loans l , represents total loans to domestic households and enterprises including mortgage loans. Capital cap is total bank capital. Both loans and capital have been deflated using the GDP deflator on the state level. In the case of bank mergers, we artificially create a third bank from the year of the merger in the dataset. The merger treatment procedure increases the total number of banks in the data set (i.e. the merger treatment causes the number of banks to exceed the maximum number of banks in a given year).

For the matching regressions, we match banks based on their geographic coordinates and business model characteristics. Therefore, we convert street addresses of banks to longitude and latitude coordinates using "Google geocoding".²³ Then, for each bank, we perform distance matching by finding 10 banks that have minimum distance to the reference bank.²⁴ Of these 10 banks, only those that have a similar business model as the reference bank are kept in the matched set, similarity being evaluated with balance sheet, off-balance sheet and profit and loss data. First, and similar to Carlson et al. (2013), we retain only those banks in the matched set, whose total assets are within the range of one third to three times the total assets of the reference bank. Second, we compare several ratios defining a bank's business model: the ratio of corporate loans to total loans, the share of fee income to the sum of interest income and fee income, the ratio of total off-balance sheet activities to total assets, the ratio of deposits to total assets, the net interest margin to total assets, and the ratio of interbank liabilities to total interest-bearing liabilities. We then compute the sum of the standardized squared differences between the ratios of each bank and the reference bank, standardizing the variance of each ratio to 0.01. Hence, the sum of squared differences indicates the discrepancy between the reference bank's and the matched bank's business models. Due to the normalization of the ratios' variance, the threshold for the sum of standardized squared differences in order for the bank to be kept in the matched set is 0.06. The reference bank is compared either to a single bank (1:1-matching), i.e. the bank's best match, or to a group of banks, i.e. those banks that remain in the matched set (1:N-matching). Since the matching procedure is performed for each quarter and each bank, the number of banks N in the match varies over time and between banks. Summary statistics for the regional fixed effects and the matching regressions are presented in Table 4.

²³The geocoding is performed in Stata using the command *geocode3*.

²⁴We use Roy Wada's *distmatch* command in Stata.

Variable	Units	Regional Fixed Effects			Matching 1:N			Matching 1:1		
		Mean	Std. Dev.	Median	Mean	Std. Dev.	Median	Mean	Std. Dev.	Median
Number of matched banks	-	-	-	-	1.66	0.85	1	1	0	0
Distance between reference bank and matched banks	km	-	-	-	15.48	10.23	14.06	16.29	11.08	14.81
Total loans	million euros	766	3,410	1,870	601	2,820	192	528	2,140	194
Total capital	million euros	135	922	26.5	102	790	27.1	88.3	668	27.3
Total assets	million euros	1,220	3,780	320	953	2890	354	895	2,660	352
Ratio of corporate loans to total loans	%	47.14	16.42	46.37	46.59	12.80	45.83	46.43	12.55	45.74
Ratio of deposits to total assets	%	40.13	11.93	38.95	39.89	9.41	39.18	40.05	9.47	39.36
Ratio of fee income to interest income and fee income	%	13.54	8.81	12.28	13.34	5.99	12.77	13.41	5.83	12.90
Ratio of off-balance sheet activities to total assets	%	5.71	4.33	4.66	5.24	3.02	4.62	5.16	2.99	4.54
Net interest margin to total assets	%	2.52	0.62	2.53	2.53	0.46	2.54	2.52	0.44	2.53
Number of observations			123,592			87,823			75,397	

Table 4: Summary Statistics

A.2 Estimation Results for the Subset of Regional Banks

We estimate equations 1 and 2 for the subset of regional banks, which is confined to cooperative and savings banks, while excluding (mostly large) private banks, Landesbanks and cooperative money center banks from the sample. In contrast to the whole sample, banks only match with other regional banks.

Table 5 shows the regression results for the regional fixed effects and the matching regressions.

Coefficients	Regional Fixed Effects	Matching 1:N	Matching 1:1
β	0.084*** (0.006)	0.085*** (0.002)	0.034*** (0.002)
δ_1	0.695*** (0.067)	0.726*** (0.003)	0.817*** (0.003)
δ_2	0.070 (0.082)	-0.030*** (0.004)	-0.015*** (0.003)
δ_3	-0.052* (0.048)	0.010*** (0.002)	-0.007*** (0.002)
δ_4	-0.015 (0.023)	-0.027*** (0.002)	-0.025*** (0.002)
α	0.001*** (0.000)	-	-
R^2	0.679	0.639	0.738
N	112,604	80,542	69,256
β_{adj}	0.308*** (0.011)	0.265*** (0.005)	0.147*** (0.006)

*Note: The table shows results for the regional fixed effects and the matching regressions for regional banks. The dependent variable for the fixed effects regression is l_{it} and for the matching regressions $l_{it} - l_{mt}$ with l denoting the cyclical component of total domestic loans. The regional fixed effects regression includes an interaction term of regional and year fixed effects, and standard errors are clustered at the regional district level. The standard errors for the adjusted elasticities are computed using the delta method. The parameters for the interaction terms are not reported for the regional fixed effects regression, but they are jointly significant when performing a regular Wald test. Note that we keep four lags of the dependent variable as explanatory variables since they are jointly significant for all three specifications. Standard errors are reported in parentheses. Coefficients with *, **, *** are significant at the 10 %, 5 %, and 1 % level respectively using standard t -distribution.*

Table 5: Results for the fixed effects and the matching regressions for regional banks

A.3 Estimation Results for all Banks with Corporate Loans as the Dependent Variable

We estimate equations 1 and 2 for all banks using corporate loans as the dependent variable.

Table 6 shows the regression results for the regional fixed effects and the matching regressions.

Coefficients	Regional Fixed Effects	Matching 1:N	Matching 1:1
β	0.086*** (0.007)	0.094*** (0.004)	0.065*** (0.004)
δ_1	0.594*** (0.036)	0.632*** (0.003)	0.642*** (0.004)
δ_2	0.097** (0.040)	0.030*** (0.004)	0.049*** (0.004)
δ_3	-0.051 (0.041)	-0.011*** (0.003)	0.002 (0.004)
δ_4	-0.003 (0.024)	-0.048*** (0.003)	-0.047*** (0.003)
α	0.015*** (0.001)	-	-
R^2	0.450	0.447	0.469
N	123, 141	87, 651	75, 247
β_{adj}	0.236*** (0.025)	0.250*** (0.011)	0.183*** (0.012)

*Note: The table shows results for the regional fixed effects and the matching regressions for all banks. The dependent variable for the fixed effects regression is l_{it} and for the matching regressions $l_{it} - l_{mt}$ with l denoting the cyclical component of domestic corporate loans. The regional fixed effects regression includes an interaction term of regional and year fixed effects, and standard errors are clustered at the regional district level. The standard errors for the adjusted elasticities are computed using the delta method. The parameters for the interaction terms are not reported for the regional fixed effects regression, but they are jointly significant when performing a regular Wald test. Note that we keep four lags of the dependent variable as explanatory variables since they are jointly significant for all three specifications. Standard errors are reported in parentheses. Coefficients with *, **, *** are significant at then 10 %, 5 %, and 1 % level respectively using standard t -distribution.*

Table 6: Results for the regional fixed effects and the matching regressions for all banks

A.4 Delta Method

If a sequence of $K \times 1$ random vectors \mathbf{z}_n is root-n asymptotically normally distributed with

$$\sqrt{n}(\bar{\mathbf{z}}_n - \boldsymbol{\mu}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma}), \quad (18)$$

where n is the total number of observations, $\boldsymbol{\mu}$ is a vector of population means and $\boldsymbol{\Sigma}$ is the variance-covariance matrix, and if $\mathbf{g}(\mathbf{z}_n)$ is a set of continuous differentiable functions, then $\mathbf{g}(\mathbf{z}_n)$ is root-n asymptotically normally distributed with

$$\sqrt{n}[\mathbf{g}(\bar{\mathbf{z}}_n) - \mathbf{g}(\boldsymbol{\mu})] \xrightarrow{d} N[\mathbf{0}, \mathbf{G}(\boldsymbol{\mu}) \boldsymbol{\Sigma} \mathbf{G}(\boldsymbol{\mu})'], \quad (19)$$

where $\mathbf{G}(\boldsymbol{\mu})$ is the matrix of partial derivatives $\frac{\partial \mathbf{g}(\boldsymbol{\mu})}{\partial \boldsymbol{\mu}'}$ (Greene, 2012). In order to compute the standard errors for the long-run coefficients in our regressions, we take the partial derivatives of the estimated long-run coefficient

$$\hat{\beta}_{SS} = \frac{\hat{\beta}}{1 - \hat{\delta}_1 - \hat{\delta}_2} \quad (20)$$

with respect to the parameters of the regression equation. The derivatives are combined in the vector \mathbf{g} . For the fixed effects regression, the vector of partial derivatives is

$$\mathbf{g}' = \frac{\partial \hat{\beta}_{SS}}{\partial \mathbf{b}'} = \left[0, \frac{1}{1 - \hat{\delta}_1 - \hat{\delta}_2}, 0, \frac{\hat{\beta}}{(1 - \hat{\delta}_1 - \hat{\delta}_2)^2}, \frac{\hat{\beta}}{(1 - \hat{\delta}_1 - \hat{\delta}_2)^2}, 0, 0, \mathbf{0} \right] \quad (21)$$

where $\mathbf{b} = [\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}_1, \hat{\delta}_1, \hat{\theta}, \hat{\nu}, \hat{\rho}_{it}]$. For the matching regressions, the vector of partial derivatives is

$$\mathbf{g}' = \frac{\partial \hat{\beta}_{SS}}{\partial \mathbf{b}'} = \left[\frac{1}{1 - \hat{\delta}_1 - \hat{\delta}_2}, 0, \frac{\hat{\beta}}{(1 - \hat{\delta}_1 - \hat{\delta}_2)^2}, \frac{\hat{\beta}}{(1 - \hat{\delta}_1 - \hat{\delta}_2)^2}, 0, 0 \right] \quad (22)$$

where $\mathbf{b} = [\hat{\beta}, \hat{\gamma}, \hat{\delta}_1, \hat{\delta}_1, \hat{\theta}, \hat{\nu}]$. Using (19), we can compute the asymptotic variance for the estimated long-run coefficient as

$$\mathbf{g}' \left[s^2 (\mathbf{X}' \mathbf{X})^{-1} \right] \mathbf{g}, \quad (23)$$

where scalar s^2 is the estimated error variance, $\mathbf{X}_{(nT \times K)}$ is the data matrix, nT is the number of observations and K is the number of variables in the regression. s^2 is computed as follows:

$$s^2 = \frac{\mathbf{e}' \mathbf{e}}{n - K},$$

where $\mathbf{e}_{(nT \times 1)}$ is the vector of least squares residuals from equation $\mathbf{e} = \mathbf{y} - \mathbf{X}\mathbf{b}$. Therefore, the dimension of the asymptotic variance estimator of the long-run coefficient is a scalar.

B Theoretical Model

We modify the model by Gertler and Karadi (2011) to make it flexible enough to integrate the partial equilibrium elasticity from the microeconomic estimations.

B.1 Households

Households maximize intertemporal utility subject to their budget constraint. As in GK, they face habit formation. Thus, the optimal labor supply equation is

$$\varrho_t W_t = \chi L_t^\varphi, \quad (24)$$

where W_t is the real wage, L_t is the labor input, χ is a weight in the utility function and ϱ_t is defined as follows:

$$\varrho_t = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1}. \quad (25)$$

The Euler consumption equation is

$$E_t \beta \Lambda_{t,t+1} R_{t+1} = 1. \quad (26)$$

with the stochastic discount factor

$$\Lambda_{t,t+1} = \frac{\varrho_{t+1}}{\varrho_t}. \quad (27)$$

B.2 Financial Intermediaries

We modify the financial intermediary's problem in order to be flexible enough for our calibration. Gertler and Karadi's baseline model is nested. Banker j 's net worth is

$$N_{jt+1} = (R_{kt+1} - R_{t+1}) Q_t S_{jt} + R_{t+1} N_{jt} \quad (28)$$

$$= [(R_{kt+1} - R_{t+1}) \phi_{jt} + R_{t+1}] N_{jt} \quad (29)$$

$$\phi_{jt-1} = \frac{\eta_t}{\lambda(S_{jt}) - v_t}. \quad (30)$$

In contrast to GK, we assume that the fraction of assets that a banker can divert ($\lambda(S_{jt})$) is a function of its balance sheet size. GK's model is nested by setting $\lambda'(S_{jt}) = 0$.

As in Gertler and Karadi (2011), we define

$$v_t = E_t [(1 - \theta) \beta \Lambda_{t,t+1} (R_{kt} - R_t) + \beta \Lambda_{t,t+1} \beta \theta \varkappa_{t,t+1} v_{t+1}], \quad (31)$$

and

$$z_{t,t+1} = \frac{\phi_{t+1}}{\phi_t} z_{t,t+1}. \quad (32)$$

Further, we define

$$\eta_t = E_t [(1 - \theta) + \beta \Lambda_{t,t+1} \beta \theta z_{t,t+1} \eta_{t+1}], \quad (33)$$

with

$$z_{t,t+1} = (R_{kt} - R_t) \phi_t + R_{t+1}. \quad (34)$$

As GK, we assume that new bankers enter the market. Their net worth is

$$N_{nt} = \omega Q_t S_{t-1}. \quad (35)$$

In GK, the size of banks is irrelevant because the leverage ratio is the same and independent of bank size. Thus, GK do not have to keep track of the size distribution. However, equation (30) shows that the leverage ratio is a function of bank size if $\lambda'(S_{jt}) \neq 0$. In order to prevent a complex heterogenous agents model (with a distribution of banks with different sizes), which would be difficult to compare to the baseline GK model, we use two assumptions. First, we assume that all banks in the market start with an equal size. As equation (34) shows, surviving banks' assets grow at rate z . To ensure stationarity of the model, we assume that the asset diversion function $\lambda(S_j)$ grows along this balanced growth path, namely $\lambda_t(S_j) = z^t \lambda_0(S_j)$ (see separate Appendix C.2 for details). However, given that we log-linearize around the steady state in period 0, we omit the subscript 0 in the λ -function for expositional convenience. Thus, in the absence of aggregate shocks, banks' assets will grow at a constant rate. Second, in order to prevent heterogeneity from being created by new banks, we assume that banks who enter the market have the same size as existing banks.

The overall aggregate net worth in the economy is

$$N_t = (N_{et} + N_{nt}) \exp^{\varepsilon_t^N}, \quad (36)$$

where N_{et} is the existing net worth, N_{nt} is the newly injected net worth and ε_t^N is an i.i.d. shock to net worth.

B.3 Intermediate Goods Producing Firms

Intermediate goods producing firms use capital (K_t) and labor L_t to produce goods (Y_t). In addition, they choose an optimal capital utilization rate (U_t):

$$Y_t = A_t (U_t \xi_t K_t)^\alpha L_t^{1-\alpha}, \quad (37)$$

where A_t is total factor productivity and ξ_t is the capital quality shock. Note that aggregate productivity is subject to aggregate shocks:

$$A_t = A_{t-1}^{\rho^a} \exp^{\varepsilon_t^a}, \quad (38)$$

where ε_t^a is an i.i.d. shock.

The aggregate production function is subject to two types of aggregate shocks. First, aggregate total factor productivity (A_t) may vary. Second, there is a capital quality shock ξ_t .

$$\xi_t = \xi_{t-1}^{\rho^\xi} \exp \varepsilon_t^\xi, \quad (39)$$

where ε_t^ξ is an i.i.d. shock.

Firms' profit maximization yields the following optimal utilization rate:

$$P_{mt} \alpha \frac{Y_t}{U_t} = \delta' (U_t) \xi_t K_t \quad (40)$$

where P_{mt+1} is the price of the intermediate good and $\delta' (U_t)$ is the first derivative of the depreciation rate of capital with respect to the intensity of capital utilization.

The optimal labor demand is

$$P_{mt} (1 - \alpha) \frac{Y_t}{L_t} = W_t, \quad (41)$$

where the marginal product of labor is equal to the wage.

The optimal capital demand is

$$R_{kt+1} = \frac{\left[P_{mt+1} \alpha \frac{Y_{t+1}}{\xi_{t+1} K_{t+1}} + Q_{t+1} - \delta U_{t+1} \right] \xi_{t+1}}{Q_t}, \quad (42)$$

where the rental price of capital R_{kt+1} is equal to the return on capital.

B.4 Capital Producing Firms

GK assume flow adjustment costs of investment, which depend on the net investment flow. The capital price is

$$Q_t = 1 + f(\cdot) + \frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} f'(\cdot) - E_t \beta \Lambda_{t,t+1} \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} \right)^2 f'(\cdot), \quad (43)$$

with $f(1) = f'(1) = 0$ and $f''(1) > 0$.

B.5 Retail Firms

Retail firms pick an optimal price level P_t^* subject to the Calvo mechanism and indexation. The first order condition is

$$\sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^i (1 + \pi_{t+k-1})^{\gamma_p} - \mu P_{mt+1} \right] Y_{ft+i} = 0, \quad (44)$$

where γ is the Calvo probability that prices cannot be adjusted, γ_p is the degree of indexation and $\mu = \varepsilon / (\varepsilon - 1)$ is the mark-up.

Aggregate prices can then be expressed as

$$P_t = \left[(1 - \gamma) (P_t^*)^{1-\varepsilon} + \gamma (\pi_{t-1}^{\gamma_p} P_{t-1})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (45)$$

B.6 Resource Constraints and Policy

The aggregate resource constraint is

$$Y_t = C_t + I_t + f \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} \right) (I_{nt} + I_{ss}) + G_t, \quad (46)$$

i.e. aggregate output consists of consumption, investment and investment adjustment costs.

Capital is the remaining past capital plus the new investment. The past capital is multiplied by one minus the depreciation rate and the capital quality shock.

$$K_{t+1} = (1 - \delta(U_t)) \xi_t K_t + I_t \quad (47)$$

The government finances its spending by lump-sum taxation.

Monetary policy follows a Taylor rule with interest rate smoothing:

$$\left(\frac{1 + i_t}{1 + \bar{i}} \right) = \left(\frac{\pi_t}{\pi_t^*} \right)^{\kappa_\pi (1-\rho)} \left(\frac{Y_t}{Y_t^*} \right)^{\kappa_y (1-\rho)} i_{t-1}^\rho \exp^{\varepsilon_t^i}, \quad (48)$$

where κ_π is the weight on inflation in the Taylor rule, κ_y is the weight on output, ρ is the smoothing parameter, π_t^* is the natural level of inflation and Y_t^* is the flex-price level of output and ε_t^i is the interest rate shock.

In contrast to GK, we do not model any unconventional monetary policy.

Real and nominal interest rates are linked via the Fisher equation:

$$1 + i_t = R_{t+1} E_t \pi_{t+1} \quad (49)$$

C Detailed Derivations

C.1 Partial Equilibrium Elasticity in the Full Model

In the full model, we obtain following equilibrium equation:

$$\lambda(S_j) Q S_j = \frac{(1 - \theta) \beta [(R_k - R) Q S_j + R N_j]}{1 - \theta z \beta}. \quad (50)$$

We define an implicit function

$$f := (\lambda(S_j) (1 - \theta z \beta) - (1 - \theta) \beta (R_k - R)) Q S_j - (1 - \theta) \beta R N_j. \quad (51)$$

and use the implicit functions theorem:

$$\begin{aligned} \frac{\partial S_j}{\partial N_j} &= -\frac{\frac{\partial f}{\partial N_j}}{\frac{\partial f}{\partial S_j}} \\ &= \frac{(1-\theta)\beta R}{(\lambda(S_j)(1-\theta z\beta) - (1-\theta)\beta(R_k - R))Q + \frac{\partial \lambda(S_j)}{\partial S_j}(1-\theta z\beta)QS_j} \end{aligned} \quad (52)$$

This yields the following partial equilibrium elasticity that is displayed in the main text:

$$\frac{\partial \ln S_j}{\partial \ln N_j} = \frac{\lambda(S_j)(1-\theta z\beta) - (1-\theta)\beta(R_k - R)}{\lambda(S_j)(1-\theta z\beta) - (1-\theta)\beta(R_k - R) + \frac{\partial \lambda(S_j)}{\partial S_j}(1-\theta z\beta)S_j}. \quad (53)$$

C.2 Balanced Growth Path

In the model, assets and liabilities at a given bank grow at gross rate z in steady state. Therefore, in steady state profits also grow with rate z . To ensure stationarity of the model, we assume that the asset diversion function $\lambda(S_j)$ grows along this balanced growth path, namely $\lambda_t(S_j) = z^t \lambda_0(S_j)$. This can be illustrated best in terms of the steady state. From period 0 perspective, the equilibrium incentive compatibility condition is

$$(1-\theta)\beta \sum_{i=0}^{\infty} (\theta z\beta)^i [(R_k - R)QS_j + RN_j] = \lambda_0(S_j)QS_j, \quad (54)$$

where the left hand side represents the discounted steady state profits (i.e. the value of the firm) and the right hand side represent the amount of funds a bank can divert.

From period t perspective, the equilibrium asset diversion condition changes as follows:

$$(1-\theta)\beta \sum_{i=0}^{\infty} (\theta z\beta)^i [(R_k - R)QS_j z^t + RN_j z^t] = \lambda_t(S_j)QS_j \quad (55)$$

because banks start with a higher asset and net worth level.

With our assumption from above, the asset diversion function also trends over time and we obtain:

$$z^t (1-\theta)\beta \sum_{i=0}^{\infty} (\theta z\beta)^i [(R_k - R)QS_j + RN_j] = z^t \lambda_0(S_j)QS_j. \quad (56)$$

Canceling the z^t -terms, this yields exactly the same outcome as equation (54). Thus, our assumption of the time trend on the asset diversion function yields a stationary model. Given that we log-linearize around the steady state in period 0, we omit the subscript $_0$ in the λ -function for expositional convenience.

C.3 The Log-linearized Leverage Equation

We assume that the diversion of funds depends on the asset size:

$$\lambda(S_{jt}) = cS_{jt}^\Psi \quad (57)$$

Thus, the new leverage equation is

$$Q_t S_{jt} = \frac{\eta_t}{\lambda(S_{jt}) - v_t} N_{jt}, \quad (58)$$

or

$$\phi_t = \frac{\eta_t}{\lambda(S_{jt}) - v_t}. \quad (59)$$

Log-linearizing the last equation, we obtain:

$$\hat{\phi}_t = \hat{\eta}_t - \frac{\lambda'(S) \lambda(S)}{\lambda(S) - v} \hat{\lambda}(s_{jt}) + \frac{v}{\lambda(S) - v} \hat{v}_t. \quad (60)$$

For the specific functional form shown in equation (57) we log-linearize and obtain following equation:

$$\hat{\lambda}(S_{jt}) = \Psi \hat{s}_{jt}. \quad (61)$$