The Convergence of the Gender Pay Gap
– An Alternative Estimation Approach –*

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Abstract

So far, little work has been done on directly estimating differences of earning gaps. Studies estimating pay differentials generally compare them across different subsamples or rely on the Juhn-Murphy-Pierce decomposition. Both methods contain serious drawbacks that we overcome by proposing an extension of the Oaxaca-Blinder-type decomposition. Our method solves both the index number and the indeterminacy problem of standard Oaxaca-Blinder type decompositions. We present two empirical applications to illustrate the methodology.

Keywords: Pay Differential; Statistical Significance; Decomposition; Index-Number Problem.

JEL - Classification: J7, J13, J310

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1 Introduction

Gender differentials in the labor market have obtained much attention from policy makers and researchers leading to the implementation of equal pay legislation and the promotion of equal opportunities. However, differences in pay between men and women persist (see for example Blau and Kahn, 1992, 2003,0,0; Goldin, 2014). Gender-related wage gaps are found in various subsamples; across sectors and occupations as well as over time and countries. According to Eurostat, the average Gender Pay Gap (GPG) in the European Union (EU) is at about 16 percent for the last five years. Yet, there are tremendous differences across countries: while Italy and Portugal have average GPGs well below 10 percent, the differential in Germany, Great Britain, Estonia and Slovakia, for example, is above 20 percent.

A general finding in the literature is that the difference in pay by gender cannot be entirely explained by differences in human capital, job or firm characteristics, but that the unexplained part of the gap is considerably large. Diverse policies (anti-discrimination laws, female board quotas and family-friendly policies) were implemented in order to fight differences in pay by gender. By decomposing the wage gap of interest for different sub-samples, the literature identified various causes of the difference in pay between men and women.

Different GPGs are found across time. In particular, declining GPGs are observed with slower convergence in recent decades (see Blau and Kahn, 2006; England, 2006). The main reasons for this decline are found to be the catching-up of women in terms of education and labor market experience (Goldin, 2006), technical development (Black and Spitz-Oener, 2010), changes in attitudes towards women in the labor market, less occupational segregation (Cotter, 2004; England, 2006; Mandel and Semyonov, 2014) and anti-discrimination laws (Fortin, 2015). In particular, research has shown that the unexplained part – also known as the coefficient effect – of the GPG has been reduced subsequently over time (Mandel and Semyonov, 2014).

Differences in pay are revealed also across sectors and in particular in the public and private sectors. Moreover, the Public-Private Sector Wage Gap (PPWG) differs for men and women (Arulampalam *et al.*, 2007; Lucifora and Meurs, 2006; Melly, 2005). In particular, the difference in pay by gender is generally smaller in the public compared to the private sector and, regardless of gender, levels in the public sector differ (Lucifora and Meurs, 2006). The public sector is generally the preferred sector of women due to its fairer recruitment and selection criteria as well as remuneration schemes and better implementation of anti-discrimination laws (Gornick and Jacobs, 1998; Grimshaw, 2000; Castagnetti and Giorgetti, 2018).

In general, studies examining changes in the wage gap over time or between groups/sectors make use either of the Juhn *et al.* (1991) method (Blau and Kahn, 1997, 2006) or of the double Oaxaca-Blinder (OB) decomposition (Smith and Welch, 1989). Both methods rely on estimations obtained on different subsamples. However, the conclusions about the drivers of the change of wage gaps between groups are generally different when estimated directly or when based on the comparison of results obtained on different subsamples. It is evident that in the second case - estimation obtained on different subsamples - it is not possible to draw inference on the difference of the components of interest. Moreover, the standard method, i.e. ex-post comparison of the decomposition results, does not allow to catch time - (or sector-) and gender-specific effects that may exist simultaneously, i.e. interactions across gender and time or sector and gender.

In this paper we propose an extension of the OB decomposition (Blinder, 1973a; Oaxaca, 1973) for estimating the difference between two wage gaps. Our approach allows to perform both the inference on the changes of the wage gap by groups across sub-samples and the comparison of the different components across time or sectors. For instance, it can be tested if there has been a significant change in the explained or unexplained part of the decomposition of interest. Even though most applications of OB can be found in the labor market and discrimination literature (see Stanley and Jarrell (1998) and Weichselbaumer and Winter-Ebmer (2005) for meta studies), our method, as for the standard OB decomposition, can be employed to study (the evolution of) group differences in any (continuous and unbounded) outcome variable.

However, the OB decomposition has several drawbacks. The most cited is the *index number problem*: the decomposition is not unique to the choice of the non-discriminatory wage structure. Solutions in the literature consist in estimating a pooled wage structure (Neumark, 1988; Oaxaca and Ransom, 1994) or assigning different weights to the two groups (Reimers, 1983; Cotton, 1988). The intercept-shift approach (Fortin, 2008) generalizes the approach of Neumark (1988) and Oaxaca and Ransom (1994) allowing different intercepts in the pooled sample. Fortin (2008) re-writes the decomposition of the GPG in terms of advantages of men and disadvantages of women by including the group indicator and parameter restrictions. Thereby, the decomposition does no longer depend on the choice of the non-discriminatory wage structure. For a recent application, see for example Elder and Haider (2010) and Magnani and Zhu (2012). However, as Lee (2015) stressed out, the intercept-shift approach of Fortin (2008) set the reference parameter for the OB decomposition, i.e. the parameter that would prevail in a world under no discrimination, on the variance difference among categories instead of on the level difference. Moreover, the reference intercept in Fortin (2008) is arbitrary: the same OB decomposition holds with different reference intercepts.

A second problem of the decomposition is known as the *omitted group problem*: in case of categorical variables the decomposition depends on the choice of the omitted group in the regression model (see Jones, 1983; Oaxaca and Ransom, 1994).

We show that our decomposition overcomes both the index number and the omitted group problem. Moreover, our decomposition does not suffer of arbitrary reference intercepts and relies on the level difference. We illustrate our method by presenting two empirical applications. First, we examine the evolution of the GPG in Italy from 2005 to 2016. Second, we analyze the PPWG between women and men in 2016 in Italy. For each application, we compare the standard Oaxaca-Blinder decomposition to our proposed approach. We expect to find a statistically significant change in differences in endowments by gender over time as well as a statistically significant change in differences in remuneration between men and women over time. This may indicate the effectiveness of anti-discrimination policies. For the analysis of the PPWG we expect, in line with the literature, to find larger pay gaps for women between the public and the private sector than for men. Additionally, we expect to find a larger effect of the unexplained component in the PPWG for women; while differences in human capital may be the main driver of the pay differential for men, they may not explain the difference in the PPWG for women.

For the first case, the findings of the study reveal interesting differences in results when applying our proposed estimation methodology compared to the *standard* approach.[1] Human capital and individual characteristics are found to be the only statistically significant driving force of the convergence of the GPG in the last decade in Italy. On the contrary, by comparing

[1]That is the OB decomposition and ex-post comparison of the decomposition results.

the different components of the GPGs by OB, differences in returns to observable wage characteristics, often referred to as the 'unexplained' part of the GPG, seem to play a role in closing the gap over the last decade in Italy. For the second case, we can confirm the conclusions drawn from the comparison

2 Changes in the Gender Pay Gap: Methods in Use in the Literature

An often used methodology to study labor-market outcomes by groups (sex, race, and so on) is to decompose mean differences in log wages based on linear regression models in a counterfactual manner. The procedure is known in the literature as the Oaxaca-Blinder decomposition. It estimates the wage separately for the two groups and then decomposes the wage differential in two components; endowments and coefficients.

$$
\begin{aligned}
\bar{y}_M - \bar{y}_F &= \bar{x}_M^{'}\hat{\beta}_M - \bar{x}_F^{'}\hat{\beta}_F \\
&= (\bar{x}_M^{'} - \bar{x}_F^{'})\hat{\beta}_M + \bar{x}_F^{'}(\hat{\beta}_M - \hat{\beta}_F) \qquad (1)
\end{aligned}
$$

where $\bar{y}_G$ is the dependent varaibel (e.g. the log of hourly wages) of group $G = (M, F)$ evaluated at the mean and $\bar{x}_G$ and $\hat{\beta}_G$ are $K \times 1$ vectors of average characteristics and estimated coefficients for group G, respectively.[2]

The first term is the effect due to differences in observable characteristics, such as education or work experience. As different observed characteristics are expected to have different effects on earnings, the difference in observed characteristics is also referred to as the *explained component* or the *quantity effect*. The second term is the effect due to differences in returns on observable

[2] An alternative is represented by the three-fold decomposition that decomposes the wage differential in endowments, coefficients and interactions:

$$
\begin{aligned}
\bar{y}_M - \bar{y}_F &= \bar{x}_M^{'}\hat{\beta}_M - \bar{x}_F^{'}\hat{\beta}_F \\
&= (\bar{x}_M^{'} - \bar{x}_F^{'})\hat{\beta}_F + \bar{x}_F^{'}(\hat{\beta}_M - \hat{\beta}_F) + (\bar{x}_M^{'} - \bar{x}_F^{'})(\hat{\beta}_M - \hat{\beta}_F)
\end{aligned}
$$

where the last term (*interaction term*) accounts for differences in endowments and coefficients that may exist simultaneously between groups.

wage characteristics. This component is generally referred to as the *unexplained part* or the *price effect* of the GPG. The unexplained part is often used as a measure for discrimination, but it also incorporates the effect of group differences in unobserved predictors.[3]

The existence and degree of discrimination has been a controversial issue. One of the main sources of the controversy is that the wage equation cannot include all relevant variables due to unobservability of skills and individual productivity. Therefore, observationally equivalent people based on the characteristics in the wage equation may not be truly equivalent (so-called omitted variable problem). In this case the OB decomposition would over-estimate the degree of discrimination, as the price effect is now the sum of discrimination and differences in unobserved characteristics. Based on the OB decomposition, different studies examining changes in the wage gap over time (or between groups/sectors) make use of the double OB decomposition as proposed in Smith and Welch (1989):

$$\begin{aligned}\Delta(\bar{y}_{MT} - \bar{y}_{FT}) &= [(\bar{x}'_{FT} - \bar{x}'_{MT}) - (\bar{x}'_{Ft} - \bar{x}'_{Mt})]\hat{\beta}_{tM} \\ &+ (\bar{x}'_{FT} - \bar{x}'_{Ft})(\hat{\beta}_{tF} - \hat{\beta}_{tM}) \\ &+ (\bar{x}'_{FT} - \bar{x}'_{MT})'(\hat{\beta}_{TM} - \hat{\beta}_{tM}) \\ &+ \bar{x}'_{FT}[(\hat{\beta}_{TF} - \hat{\beta}_{TM}) - (\hat{\beta}_{tF} - \hat{\beta}_{tM})] \qquad (2)\end{aligned}$$

where the subscripts T and t refer to current-year and base-year, respectively, and $\Delta(\bar{y}_{MT} - \bar{y}_{FT}) = (\bar{y}_{MT} - \bar{y}_{FT}) - (\bar{y}_{Mt} - \bar{y}_{Ft})$. The first term in (2) measures the predicted change in group $M - F$ wages that occurs because of differences in observed characteristics over time $T - t$ that are valued at base-year group M parameter values. The second term measures group interactions. If individuals in group F are paid less than those in group M for a given characteristic, $(\hat{\beta}_{tF} - \hat{\beta}_{tM}) < 0$, then individuals in group F will lose relative to group M in the case of increasing average sets of endowments over time and gender. The third term measures year interaction and the fourth term measures group-year interaction. The decomposition in (2) is conducted by comparing parameter estimates on different samples and periods; i.e. inference cannot be drawn on the single components of the decomposition.

[3]The unexplained portion of the GPG may include effects of unobserved characteristics such as individual productivity, motivation or educational quality (Blau and Kahn, 2006).

Juhn *et al.* (1991) (JMP) proposed a decomposition equation for changes in wage differentials that explicitly considers the effect of unobserved skills on the gender wage gap. Evaluated at the sample mean, the wage equation for group $G = (M, F)$ may be written as:

$$\bar{y}_G = \bar{x}_G^{'}\hat{\beta} + \hat{\sigma}\hat{\theta}_G \tag{3}$$

where $\hat{\beta}$ is the estimated parameter vector from a pooled wage regression,[4] $\hat{\theta}_G$ is the mean standardized residual in group G, and $\hat{\sigma}$ is the mean standard error estimate, i.e. the average percentile rank. While $\hat{\beta}$ represents an estimate of the vector of observed prices, $\hat{\sigma}$ is an estimate of wage dispersion, which is often interpreted as an estimate of unobserved prices (see Blau and Kahn, 1997 and Gupta *et al.*, 2006) and $\hat{\theta}_G$ represents some measure of generally (unobserved) labor market ability. Given (3), the wage differential between group M and F is:

$$\bar{y}_M - \bar{y}_F = (\bar{x}_M^{'} - \bar{x}_F^{'})\hat{\beta} + (\hat{\theta}_M - \hat{\theta}_F)\hat{\sigma} \tag{4}$$

where the first component represents the explained part of the wage gap, *predicted gap*, and the second component represents the residual or unexplained part of the wage gap, *residual gap*. From (4), the change in the wage gap between years T and t is:

$$\begin{aligned}\Delta(\bar{y}_{MT} - \bar{y}_{FT}) &= \hat{\beta}_T\Delta(\bar{x}_{MT}^{'} - \bar{x}_{FT}^{'}) + (\bar{x}_{MT}^{'} - \bar{x}_{FT}^{'})\Delta\hat{\beta}_T + \\ &\quad \hat{\delta}_T\Delta(\hat{\theta}_{FT} - \hat{\theta_{MT}}) + (\hat{\theta}_{Ft} - \hat{\theta}_{Mt})\Delta\hat{\delta}_T \end{aligned} \tag{5}$$

where the first term represents the difference in mean endowments and the second term represents difference in returns to endowments. The last two terms correspond to the change in the residual wage gap. In particular, the first term of the residual wage gap has been termed

[4]The JMP decomposition considers the estimation of one group equation only, namely the non-discriminated group, assuming that the discriminated group is affected by the same economic forces that influence the wage distribution of the non-discriminated group. Thus, the estimated prices of measured characteristics are assumed to affect both groups in the same way, and the residuals are decomposed into a portion reflecting the prices of unmeasured skills and a portion reflecting the quantities of unmeasured skills, with the former affecting both groups similarly (Yun, 2009). Therefore, the JMP decomposition relies on two strong assumptions; first, OLS estimations of one group are unbiased while the OLS estimations of the other group are biased and, second, the level of discrimination is constant over time. For these issues and for addressing the index number problem, we present the JMP decomposition in the case of the pooled wage regression.

the *ranking effect* and can be split between the effect of movements of group F in the wage distribution after adjusting for changes in human capital characteristics, $\hat{\delta}_T \Delta(\hat{\theta}_{FT})$, and the term $-\hat{\delta}_T \Delta(\hat{\theta}_{MT})$ which is the effect of movements of group M in the wage distribution at time T after controlling for changes in human capital characteristics. The last term is interpreted as the *dispersion* or unobserved prices effect (see Gupta *et al.* 2006).

However, unlike Oaxaca type decomposition analysis of wage differentials, the JMP method provides coefficients and characteristics effects only at an aggregate level. Due to this shortcoming, the JMP method cannot be used for the detailed decomposition of the variation in the GPG. More importantly, as stressed by Suen (1997), the JMP decomposition of wage residuals into standard deviation (the price of unobserved skills) and percentile ranks (the level of unobserved skill) is unbiased only when the two measures are independent.

Moreover, Juhn *et al.* (1991) and Juhn *et al.* (1993) do not derive the statistic distribution of decomposition components. Inference on the different components can be conducted by using the approach in Gupta *et al.* (2006) where the standard errors are derived. However, the standard errors are derived under two strong assumptions: (i) the standard deviation and percentile ranks are independents; (ii) the covariance between estimators in different time periods is approximately zero.

3 Proposed Decomposition

The method we propose starts from the OB decomposition proposed by Gelbach (2016) that divides cross-specification differences in OLS estimates of the female coefficient in a path-independent way. Following Gelbach (2016), we rely on the **omitted-variable-bias (OVB) formula**, to consistently estimate the decomposition conditional on all covariates. As in the standard OB framework, sequencing problem do not occur when using the OVB formula for decompositions.[5]

[5]Indeed, when starting from a base specification and sequentially adding regressors, the order of addition influences the coefficient estimates.

The Role of the Intercepts

We start by focusing on the role of the intercept terms that are generally attributed to the second term in (1): $(\hat{\alpha}_M - \hat{\alpha}_F)$. The difference in the intercepts is often commented as difference in the starting points. Indeed Blinder (1973b) called this part the *unexplained* part of discrimination. However, the interpretation of the difference in the intercepts may not be straightforward: the intercept coefficients is influenced by the reference group(s) used for the indicator variable(s) in the model. Moreover, the intercept is influenced by the choice of scale for continuous variables in the model. Jones (1983) suggests that the problem is so critical that interpretation of the intercept is arbitrary and concludes that the intercept is uninterpretable.

Relying on the OVB formula sequential decomposition of the wage gap, we propose a different interpretation of the difference in the intercepts. Consider the following linear model for the wage regression on the sample composed by both groups of interest. We consider as groups those composed by males and female, $G = (F, M)$:

$$y = \iota\alpha + F\alpha_1 + X\beta_1 + XF\beta_2 + \epsilon_1 \tag{6}$$

where ι is a vector of ones, F is a vector of a dummy variable equal to 1 if the individual is a female and zero otherwise, X is the matrix of regressors, XF is the interaction effect, and ϵ_1 is the vector of error terms. The specification in (6) represents, by using the terminology of Gelbach (2016), the *full* model. Observe that the least squares estimate of $\hat{\alpha}_1$ is equal to: $\hat{\beta}_F - \hat{\beta}_M$ where

$$y = \iota\beta_F + X\beta_{1F} + \epsilon_F \text{ for females}$$

and

$$y = \iota\beta_M + X\beta_{1M} + \epsilon_M \text{ for males}$$

are the two wage regressions for the female and male group, respectively. Observe also that the difference in the average observed y between the two groups, i.e. $\bar{y}_F - \bar{y}_M$, is given by $\hat{\gamma}_F$ in the following regression:

$$y = \iota\gamma + F\gamma_F + \epsilon_2 \text{ for the whole sample} \tag{7}$$

where F is a vector of a dummy variable for being a female. The model in (7) represents the *base* model. The difference between the base and full model reads as:

$$\hat{\gamma}_F - \hat{\alpha}_1 = \alpha_1^{base} - \alpha_1^{full}$$

represents the part of the gap explained by the regressors (X, XF) and can be decomposed as the sum of two components by means of the OVB formula as shown in the next Section. If α_1^{full} would be equal to zero, this would imply that the model explains all the observed GPG. α_1^{full} is the part of the GPG that cannot be explained by the quantity and the price effect. Therefore, instead of attributing the difference in the intercepts to the price component without a clear interpretation of its source, we focus the analysis on the components that can be truly attributed to either part of the decomposition, i.e. to differences in endowments (the *explained component*) or part of differences in remuneration (the *unexplained component*).

Changes in Wages over Time

We focus on the estimation of the difference in the GPG over time in order to be able to draw inference on the difference of the respective pay gaps, i.e. gather information on the differences of the GPG across years. Moreover, we can investigate what are the main contributors to the convergence of the GPG over time: gender differences in educational attainment, in labor market presence or institutional settings such as equal-pay legislation.

The method proposed can be applied to various distinct cases of group differences in outcome variables over time, sector ...

Consider to estimate the wage equation separately by G (gender) and J (year):

$$y_{GJ} = \iota\alpha_{GJ} + X_{GJ}\beta_{GJ} + \epsilon_{GJ} \tag{8}$$

with $G = F, M$ (for F = female and M = male), $J = t, T$ (for t = starting period and

T = ending period); and where y_{GJ} is the $N \times 1$ vector of logarithmic wages of G in year J, α_{GJ} is the intercept, ι is the $N \times 1$ vector of constants, and X is a $N \times K$ matrix of exogenous regressors. β_{FJ} is the corresponding $K \times 1$ vector of coefficients and ϵ_{FJ} is a $N \times 1$ vector containing the error terms. The estimation provides four sets of parameter estimates of the same dimension, given the assumption that the set of regressors is the same for the four combinations considered. Evaluating the estimation at the mean, given the OLS property that OLS estimates must go through the mean of the data, equation (8) becomes:

$$\widehat{\bar{y}_{GJ}} = \bar{y}_{GJ} = \hat{\alpha}_{GJ} + \bar{x}'_{GJ}\hat{\beta}_{GJ} \tag{9}$$

where $\hat{\alpha}_{GJ}$ is the intercept estimate, $\bar{x}_{GJ}$ is the $K \times 1$ column vector of sample means of observable characteristics in X:

$$\bar{x}'_{GJ} = \left[\bar{x}_{1,GJ}, \bar{x}_{2,GJ}, \ldots, \bar{x}_{K,GJ}\right]$$

and $\hat{\beta}_{GJ}$ is the corresponding $K \times 1$ vector of parameter estimates.

Now, consider estimating the joint model instead. As in Gelbach (2016), we distinguish between two sets of regressors, X_1 and X_2, where X_1, represents the regressors of the *base* specification containing only a constant, an interaction term between the gender and year dummies as well as the group and time dummies themselves:

$$X_1 = \left[1, FJ, F, J\right]$$

where

$$F = \begin{cases} 1 & \text{if } female \\ 0 & \text{if } male \end{cases} \qquad J = \begin{cases} 1 & \text{if } year = t \\ 0 & \text{if } year = T \end{cases}$$

The base model is defined as follows:

$$y = \alpha_0^{base} + FJ\alpha_1^{base} + F\alpha_2^{base} + J\alpha_3^{base} + \epsilon^{base} \tag{10}$$

The second set of regressors, X_2, of dimension $(N \times 4K)$, contains the matrix of characteristics X and the interactions of the gender and year dummies with X:

$$X_2 = [X, FX, JX, FJX] \tag{11}$$

where FX and JX are the interaction variables between the regressors X, the female dummy, F and the starting period dummy, J, respectively. Analogously, FJX represents the interaction variable among regressors X, The *full* model is then defined as follows:

$$y = \alpha_0^{full} + FJ\alpha_1^{full} + F\alpha_2^{full} + J\alpha_3^{full} + X\beta_1 + FX\beta_2 + JX\beta_3 + FJX\beta_4 + \epsilon^{full} \tag{12}$$

The link between the parameters of the *full* model and the four equations represented in (8) follows straightforward.[6]

Now consider the set of regressors X_2 as omitted variables. By means of the OVB formula we have:

$$\hat{\alpha}^{base} = \hat{\alpha}^{full} + (X_1'X_1)^{-1}X_1'X_2\hat{\beta}^{full} \tag{13}$$

where the vector of parameter estimates from the base model (10) is:

$$\hat{\alpha}^{base\prime} = (\hat{\alpha}_0^{base}\ \hat{\alpha}_1^{base}\ \hat{\alpha}_2^{base}\ \hat{\alpha}_3^{base}) \tag{14}$$

and $\hat{\alpha}^{full}$ is the 4×1 vector containing the coefficient estimates of X_1 from the full model (12). $(X_1'X_1)^{-1}X_1'X_2$ is the linear projection of X_2 on X_1 and

$$\hat{\beta}^{full\prime} = (\hat{\beta}_1\ \hat{\beta}_2\ \hat{\beta}_3\ \hat{\beta}_4) \tag{15}$$

is the $(1 \times 4K)$ vector of coefficients from the full model (12). Model (13) can be decomposed as follows:

$$\hat{\alpha}^{base} = \hat{\alpha}^{full} + \hat{\delta} \tag{16}$$

[6]Appendix A reports the relationship between the two sets of estimation results.

where $\hat{\delta} \equiv \hat{\alpha}^{base} - \hat{\alpha}^{full} = (X_1'X_1)^{-1} X_1'X_2\hat{\beta}^{full}$ and

$$\hat{\delta} = \hat{\delta}_X + \hat{\delta}_{FX} + \hat{\delta}_{JX} + \hat{\delta}_{FJX} \tag{17}$$

where $\hat{\delta}_S = \hat{\Gamma}^S\hat{\beta}_S^{full}$, with $\hat{\Gamma}^S = (X_1'X_1)^{-1} X_1'S$ of dimension $(4 \times K)$ and S is the portion of the matrix (11) that corresponds to the set of regressors S, for $S = X, \ldots, FJX$ in (11).[7]

The Decomposition

Our interest relies in the estimation and decomposition of the GPG across two periods, t and T:

$$\Delta_T - \Delta_t = \left(\bar{y}_{MT} - \bar{y}_{FT}\right) - \left(\bar{y}_{MT} - \bar{y}_{Ft}\right)$$

with Δ_T being the GPG in T.

It can be easily shown that:

$$\begin{aligned}\Delta_T &= \left(\bar{y}_{MT} - \bar{y}_{FT}\right)\\ &= -\hat{\alpha}_2^{base}\end{aligned}$$

and

$$\begin{aligned}\Delta_t &= \left(\bar{y}_{Mt} - \bar{y}_{Ft}\right)\\ &= -\hat{\alpha}_1^{base} - \hat{\alpha}_2^{base}\end{aligned}$$

Therefore:

$$\Delta_T - \Delta_t = \hat{\alpha}_1^{base}$$

Therefore, given (14), we are interested in the second row of $\hat{\alpha}^{base}$, i.e. $\hat{\alpha}_1^{base}$ in order to ob-

[7]Accordingly, $\hat{\delta}_X = \hat{\Gamma}^X\hat{\beta}_X^{full}$, with $\hat{\Gamma}^X = (X_1'X_1)^{-1} X_1'X$ of dimension $(4 \times K)$ and $\hat{\beta}_X^{full}$ is the $(K \times 1)$ vector $\hat{\beta}_1$ in (15).

tain the differences of the respective wage gaps; $\Delta_T - \Delta_t$. Following (13) and (16)-(17), we decompose $\hat{\alpha}_1^{base}$ accordingly. In particular, for decomposing $\hat{\alpha}_1^{base}$ we refer to the second row of $(X_1'X_1)^{-1}X_1'X_2$:

$$(X_1'X_1)^{-1}X_1'X = \begin{bmatrix} / \\ (\bar{x}'_{MT} - \bar{x}'_{FT}) - (\bar{x}'_{Mt} - \bar{x}'_{Ft}) \\ / \\ / \end{bmatrix}$$

$$(X_1'X_1)^{-1}X_1'FX = \begin{bmatrix} / \\ (\bar{x}'_{Ft} - \bar{x}'_{FT}) \\ / \\ / \end{bmatrix}$$

$$(X_1'X_1)^{-1}X_1'JX = \begin{bmatrix} / \\ (\bar{x}'_{Ft} - \bar{x}'_{Mt}) \\ / \\ / \end{bmatrix}$$

and

$$(X_1'X_1)^{-1}X_1'FJX = \begin{bmatrix} / \\ \bar{x}'_{Ft} \\ / \\ / \end{bmatrix}$$

The second row of equation (13), i.e. the delta wage gap evaluated at the mean, is thus:

$$
\begin{aligned}
\hat{\alpha_1}^{base} &= \underbrace{(\hat{\alpha}_{MT} - \hat{\alpha}_{FT}) - (\hat{\alpha}_{Mt} - \hat{\alpha}_{Ft})}_{\hat{\alpha}_1^{full}} + [(\bar{x}'_{MT} - \bar{x}'_{FT}) - (\bar{x}'_{Mt} - \bar{x}'_{Ft})]\underbrace{\hat{\beta}_{MT}}_{\hat{\beta}_1} \\
&+ (\bar{x}'_{Ft} - \bar{x}'_{FT})\underbrace{(\hat{\beta}_{FT} - \hat{\beta}_{MT})}_{\hat{\beta}_2} \\
&+ (\bar{x}'_{Ft} - \bar{x}'_{Mt})\underbrace{(\hat{\beta}_{Mt} - \hat{\beta}_{MT})}_{\hat{\beta}_3} \\
&+ \bar{x}'_{Ft}\underbrace{[(\hat{\beta}_{MT} - \hat{\beta}_{FT}) - (\hat{\beta}_{Mt} - \hat{\beta}_{Ft})]}_{\hat{\beta}_4} \\
&= \Delta_T - \Delta_t
\end{aligned}
$$

or, equivalently:

$$\hat{\alpha_1}^{base} = \hat{\alpha_1}^{full} + \hat{\delta}_1 + \hat{\delta}_2 + \hat{\delta}_3 + \hat{\delta}_4$$

where:

$$
\begin{aligned}
\hat{\delta}_1 &= (\bar{x}_{Mt} - \bar{x}_{Ft})\hat{\beta}_{Mt} - (\bar{x}_{MT} - \bar{x}_{FT})\hat{\beta}_{Mt} \\
\hat{\delta}_2 &= \bar{x}_{Ft}(\hat{\beta}_{Mt} - \hat{\beta}_{Ft}) + \bar{x}_{FT}(\hat{\beta}_{Ft} - \hat{\beta}_{Mt}) \\
\hat{\delta}_3 &= (\bar{x}_{FT} - \bar{x}_{MT})\hat{\beta}_{MT} - (\bar{x}_{FT} - \bar{x}_{MT})\hat{\beta}_{Mt} \\
\hat{\delta}_4 &= \bar{x}_{FT}(\hat{\beta}_{Mt} - \hat{\beta}_{Ft}) - \bar{x}_{FT}(\hat{\beta}_{MT} - \hat{\beta}_{FT})
\end{aligned}
$$

The above expression can be re-written as a *double* Oaxaca-Blinder decomposition:

$$\hat{\alpha_1}^{base} - \hat{\alpha_1}^{full} = \underbrace{(\hat{Q}_t + K)}_{\hat{\delta}_1} + \underbrace{(\hat{P}_t + W)}_{\hat{\delta}_2} + \underbrace{(-\hat{Q}_T - K)}_{\hat{\delta}_3} + \underbrace{(-\hat{P}_T - W)}_{\hat{\delta}_4}$$

where $\hat{Q}_t = (\bar{x}_{Mt} - \bar{x}_{Ft})\hat{\beta}_{Mt}$, is the estimated *quantity effect* and $\hat{P}_t = \bar{x}_{Ft}(\hat{\beta}_{Mt} - \hat{\beta}_{Ft})$, the estimated *price effect* in period t, and $\hat{Q}_T = (\bar{x}_{MT} - \bar{x}_{FT})\hat{\beta}_{MT}$, and $\hat{P}_T = \bar{x}_{FT}(\hat{\beta}_{MT} - \hat{\beta}_{FT})$, the estimated *quantity* and *price* effect in period T, respectively.

4 Inference

The asymptotic distribution of $\sqrt{N}(\hat{\delta} - \delta)$, with $\hat{\delta} = (\hat{\delta}_1, \ldots, \hat{\delta}_4)$ has been derived in Gelbach (2016) and it is summarized in the Appendix **XX**. Given the distribution of the parameters $\hat{\delta}$ the decomposition proposed allows to carry out inference on the dynamic of the single components of the GPG.

If the interest relies, for instance, in investigating the convergence of the GPG, it can be analyzed whether the convergence can be explained either by the convergence of human capital endowments (explained or quantity components) or by the effect of policy intervention (unexplained or price components).

The hypothesis that the convergence has been driven by the catching up of the education level can be tested by:

$$H_0 : \delta_1 + \delta_3 = 0$$

that is equivalent to test for

$$H_0 : Q_t = Q_T$$

For the effectiveness of gender anti discrimination laws, it can be tested if the components of the price effects have been constant over time:

$$H_0 : \delta_2 + \delta_4 = 0$$

that is equivalent of testing:

$$H_0 : P_t = P_T$$

i.e. the H_0 is that there was no change in characteristics between M and F over time. Moreover, each $\hat{\delta}$ can be decomposed in its single components. For instance, the contribution to the GPG of labor market experience to the quantity component in period t can be extracted from $\hat{\delta}_1$ and similarly for the other components. Indeed each $\hat{\delta}$ is given by

$$\hat{\delta}_i = \sum_{g=1}^{G} \hat{\delta}_{ig} = \sum_{g=1}^{G} \hat{\Gamma}_{ig} \hat{\beta}_g \text{ for i=1, ..., 4} \tag{18}$$

5 The Index Number Problem

As is well known in the literature, the Oaxaca-Blinder decomposition is not unique. Therefore, the choice of the non-discriminatory wage structure (men or women) matters and leads to different results (Oaxaca and Ransom, 1994; Cotton, 1988; Fortin *et al.*, 2011a). Several approaches have been proposed to circumvent this problem (Reimers, 1983; Cotton, 1988; Neumark, 1988; Oaxaca and Ransom, 1994; Fortin, 2008).

We propose an extension of our method in order to have a wage decomposition invariant to the reference category adopted. Considering the standard case of the GPG, Fortin (2008) includes gender intercept shifts along with an identification restriction, in the regression of females and males pooled together:

$$y_i = \gamma_0 + \gamma_{0F} F_i + \gamma_{0M} M_i + X_i \gamma + \epsilon_i$$

subject to:

$$\gamma_{0F} + \gamma_{0M} = 0$$

where F_i (M_i) is equal to one if the individual is female (male) and zero otherwise. The identification restriction, $\gamma_{0F} + \gamma_{0M} = 0$, imposes that the pooled wage equation truly represents a non-discriminatory wage structure, i.e. a wage structure where the advantage of men is equal to the disadvantage of women:

$$\bar{y}_M - \bar{y}_F = (\bar{X}_M - \bar{X}_F)\hat{\gamma} + (\hat{\gamma}_{0M} - \hat{\gamma}_{0F}) \tag{19}$$

The first component on the RHS, $(\bar{X}_M - \bar{X}_F)\hat{\gamma}$, is the explained part, while $\hat{\gamma}_{0M}$ and $\hat{\gamma}_{0F}$ are the *advantage of men* and the *disadvantage of women*, respectively. In particular:

$$\begin{aligned} \hat{\gamma}_{0M} &= \bar{X}_M(\hat{\beta}_M - \hat{\gamma}) + (\hat{\alpha}_M - \hat{\gamma}_0) \qquad \textit{advantage of men} \\ \hat{\gamma}_{0F} &= \bar{X}_F(\hat{\beta}_F - \hat{\gamma}) + (\hat{\beta}_{0F} - \hat{\gamma}_0) \qquad \textit{disadvantage of women.} \end{aligned}$$

where $\hat{\alpha}_M, \hat{\alpha}_F, \hat{\beta}_M, \hat{\beta}_F$ are the estimated coefficients of the wage equations for men and women, respectively:

$$y_{iM} = \alpha_M + X_M \beta_M + \epsilon_{iM} \tag{20}$$

$$y_{iF} = \alpha_F + X_F \beta_F + \epsilon_{iF} \tag{21}$$

Solution for Differences in Wages over Time

The extension of the decomposition described above to the case of the estimation of the difference of wage gaps follows straightforward. The set of regressors considered in Section 3 becomes:

$$\begin{aligned} X_1 &= [1, (F-M)J, (F-M), J] \\ X_2 &= [X, (F-M)X, JX, (F-M)JX] \end{aligned}$$

The base model is:

$$y_i = \gamma_0^{base} + (F_i - M_i)J_i\gamma_{FJ}^{base} + (F_i - M_i)\gamma_F^{base} + J_i\gamma_J^{base} + \epsilon_i^{base} \tag{22}$$

while the full model is defined as follows:

$$\begin{aligned} y_i &= \gamma_0^{full} + (F_i - M_i)J_i\gamma_{FJ}^{full} + (F_i - M_i)\gamma_F^{full} + J_i\gamma_J^{full} + \\ &+ X_i\gamma + (F_i - M_i)X_i\gamma_{XF} + J_iX_i\gamma_{XJ} + (F_i - M_i)J_iX_i\gamma_{XJF} + \epsilon_i^{full} \end{aligned} \tag{23}$$

$(\gamma_0^{base}\ \gamma_{FJ}^{base}\ \gamma_F^{base}\ \gamma_J^{base})$ is the vector of coefficients estimates of X_1 from the base model (22), and $(\gamma_0^{full}\ \gamma_{FJ}^{full}\ \gamma_F^{full}\ \gamma_J^{full})$ is the vector containing the coefficient estimates of X_1 from the full model (23) while $(\gamma\ \gamma_{XF}\ \gamma_{XJ}\ \gamma_{XJF})$ is the vector of coefficients estimates of X_2 from the full model (23). The linear projection of X with respect to X_1 is equal to:

$$(X_1'X_1)^{-1}X_1'X = \begin{bmatrix} / \\ -[(\bar{x}_{Mt} - \bar{x}_{Ft}) - (\bar{x}_{MT} - \bar{x}_{FT})]/2 \\ / \\ / \end{bmatrix}$$

The linear projection of $(F-M)X$ with respect to X_1 is equal to:

$$(X_1^{'}X_1)^{-1}X_1^{'}(F-M)X = \begin{bmatrix} / \\ [(\bar{x}_{Mt}+\bar{x}_{Ft})-(\bar{x}_{MT}+\bar{x}_{FT})]/2 \\ / \\ / \end{bmatrix}$$

The linear projection of JX with respect to X_1 is equal to:

$$(X_1^{'}X_1)^{-1}X_1^{'}JX = \begin{bmatrix} / \\ (\bar{x}_{Ft}-\bar{x}_{Mt})/2 \\ / \\ / \end{bmatrix}$$

The linear projection of $(F-M)JX$ with respect to X_1 is equal to:

$$(X_1^{'}X_1)^{-1}X_1^{'}(F-M)JX = \begin{bmatrix} / \\ (\bar{x}_{Ft}+\bar{x}_{Mt})/2 \\ / \\ / \end{bmatrix}$$

It can be easily shown that:

$$\begin{aligned}\hat{\gamma}_{FJ}^{base} &= \frac{\Big(\bar{y}_{MT}-\bar{y}_{FT}\Big)-\Big(\bar{y}_{Mt}-\bar{y}_{Ft}\Big)}{2} \\ &= \frac{\Delta GPG}{2}\end{aligned}$$

and

$$\hat{\gamma}_{FJ}^{full} = \frac{(\hat{\alpha}_{MT}-\hat{\alpha}_{FT})-(\hat{\alpha}_{Mt}-\hat{\alpha}_{Ft})}{2}$$

where, analogously to Section 3 $\hat{\alpha}_{Mt}, \hat{\alpha}_{MT}, \hat{\alpha}_{Ft}, \hat{\alpha}_{FT}$ are the estimated coefficients of the wage equations for men and women at period t and T, respectively:

$$\begin{aligned} y_{iMt} &= \alpha_{Mt} + X_{Mt}\beta_{Mt} + \epsilon_{iMt} & (24)\\ y_{iFt} &= \alpha_{Ft} + X_{Ft}\beta_{Ft} + \epsilon_{iFt} & (25)\\ y_{iMT} &= \alpha_{MT} + X_{MT}\beta_{MT} + \epsilon_{iMT} & (26)\\ y_{iFT} &= \alpha_{FT} + X_{FT}\beta_{FT} + \epsilon_{iFT} & (27) \end{aligned}$$

Hence, the relationship:

$$\begin{aligned} \hat{\gamma}_{FJ}^{base} &= \hat{\gamma}_{FJ}^{full} + (X_1^{'}X_1)^{-1}X_1^{'}X\hat{\gamma} + (X_1^{'}X_1)^{-1}X_1^{'}X(F-M)\hat{\gamma}_{XF} + \\ &+ \ (X_1^{'}X_1)^{-1}X_1^{'}XJ\hat{\gamma}_{XY} + (X_1^{'}X_1)^{-1}X_1^{'}XJ(F-M)\hat{\gamma}_{XJF} \end{aligned}$$

can be re-written in terms of the ΔGPG as:

$$\begin{aligned} 2\hat{\gamma}_{FJ}^{base} &= \Delta GPG = \\ &= [(\hat{\alpha}_{MT} - \hat{\alpha}_{FT}) - (\hat{\alpha}_{Mt} - \hat{\alpha}_{Ft})] + (\Delta\bar{x}_T - \Delta\bar{x}_t)\hat{\gamma} + \\ &\quad +(\sum \bar{x}_t - \sum \bar{x}_T)\hat{\gamma}_{XF} - \Delta\bar{x}_t\hat{\gamma}_{XY} + \sum \bar{x}_t\hat{\gamma}_{XYF} \end{aligned}$$

where $\Delta\bar{x}_{Year}$ is the difference between the average level of observed characteristics of men and women in a certain year, with $Year = t, T$ and $\sum \bar{x}_{Year}$ represents the sum of observable labor market characteristics present for men and women in year$=Year$. Recall that the model can be re-written in terms of the OVB formula as follows:

$$2\hat{\gamma}_{FJ}^{base} = \hat{\gamma}_{FJ}^{full} + \hat{\delta}_A + \hat{\delta}_B + \hat{\delta}_C + \hat{\delta}_D$$
$$\hat{P} + \hat{Q} = \hat{\delta}_A + \hat{\delta}_B + \hat{\delta}_C + \hat{\delta}_D$$

with P accounting for the price effect and Q for the quantity effect. In particular,

$$\hat{\delta}_A = \underbrace{(\bar{x}_{MT} - \bar{x}_{FT})\hat{\gamma}}_{Q_T} - \underbrace{(\bar{x}_{Mt} - \bar{x}_{Ft})\hat{\gamma}}_{W}$$

$$\hat{\delta}_B = \underbrace{\bar{x}_{MT}(\hat{\beta}_{MT} - \hat{\gamma}) - \bar{x}_{FT}(\hat{\beta}_{FT} - \hat{\gamma})}_{P_T} + \underbrace{(\bar{x}_{Mt} + \bar{x}_{Ft})\hat{\gamma}_{XF}}_{K}$$

$$\hat{\delta}_C = \underbrace{(\bar{x}_{Mt} - \bar{x}_{Ft})(\hat{\gamma} + \hat{\gamma}_{XJF})}_{Q_t} - \underbrace{(\bar{x}_{Mt} - \bar{x}_{Ft})\hat{\gamma}}_{W}$$

$$\hat{\delta}_D = -[\underbrace{\bar{x}_{Mt}(\hat{\beta}_{Mt} - \hat{\gamma} - \hat{\gamma}_{XJ}) - \bar{x}_{Ft}(\hat{\beta}_{Ft} - \hat{\gamma} - \hat{\gamma}_{XY})}_{P_t}] - \underbrace{(\bar{x}_{Mt} + \bar{x}_{Ft})\hat{\gamma}_{XF}}_{K}$$

6 Data and Descriptive Statistics

We use the 2016 and 2005 files of the survey Isfol Plus from the Italian Institute for the Development of Vocational Training for Workers (Isfol). In 2005, Isfol Plus was conducted with 38,940 interviews. In 2016, 54,961 individuals were interviewed. In our analysis, we focus on full-time employees aged between 18 and 64 years. We include only individuals in the sample that work more than 35 hours per week. We exclude autonomous workers from the analysis. The analysis is also constrained to earnings from the main job only, i.e. from the job that yields the highest income. The selection criteria yielded a sample size of 9,718 in 2005 and of 8,871 in 2016. In 2005, there were 4,873 women (50.14%) and 4,845 men in the sample (49.86%). In the 2016-release, 4,026 (45.44%) individuals were female and 4,835 (54.56%) were male. In 2016, 1,912 women (52.69% of total public-sector employment) and 1,717 men (47.31% of total public-sector employment) were occupied in the public sector. Hence, we see in the data that women seem to favor the public sector, what is in line with results in the literature outlined in Section 1 on more egalitarian pay schemes in the public compared to the private sector. Table 1 and 2 report mean and standard deviation for human capital variables included in the analysis for the two cases under consideration, respectively.

For the analysis of the evolution of the gender wage differential over time, we pool together the two cross sections of 2005 and 2016 and drop individuals that were observed only in one year. For the analysis of the PPWG between women and men, we use the latest release, i.e.

the cross section of 2016. Detailed definitions along with the coding of the variables used in the analysis can be found in the Appendix.

Descriptive Statistics Case 1

Table 1: Descriptive Statistics Case 1

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Women 2005		Women 2016		Men 2005		Men 2016	
Variable	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
Lhwage	1.851	0.368	2.072	0.439	2.003	0.403	2.159	0.459
Schooling	12.94	2.693	13.86	2.408	12.22	2.902	13.06	2.642
Exper	17.16	11.77	20.89	12.64	21.12	12.86	24.72	13.13
Tenure	11.82	10.54	15.42	12.04	14.65	11.76	18.11	12.91
Married	0.563	0.496	0.344	0.475	0.592	0.491	0.358	0.479
North	0.523	0.500	0.530	0.499	0.457	0.498	0.470	0.499
Centre	0.208	0.406	0.215	0.411	0.188	0.390	0.203	0.403
Observations	3,983		4,359		5,202		5,789	

Table 1 shows that women have on average higher educational attainment than men and that their human capital characteristics increased from 2005 to 2016. For men, the increase was less significant and even decreasing for the proportion of men that obtained the maximum degree when studying (*Maximum_D_Mark*). Men still outperform women in terms of labor market characteristics (*Exper*, *Tenure* and *Extra_Hours*). However, while those of women increased that of men partly even decreased slightly (*Exper*). The proportion of married women was reduced slightly, while that of men increased over the last decade. In 2016, less individuals have children compared to 2005 (*Kids* and *Kids_3*). Finally, women engaged in the labor market are about two to three years younger than men in 2016 and 2005, respectively.

Descriptive Statistics Case 2

When focusing on differences in characteristics by gender in the public and private sector, we find that the level of educational attainment is on average higher in the public compared to the private sector and that women have on average higher educational attainment than men in both sectors, see Table 2. Hence, women employed in the public sector are even better educated than their colleagues in the private sector. Similarly, men in the public sector exhibit higher

Table 2: Descriptive Statistics Case 2

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Private Sector Men		Private Sector Women		Public Sector Men		Public Sector Women	
Variable	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
Lhwage	2.086	0.471	1.952	0.461	2.295	0.402	2.203	0.372
Schooling	12.70	2.686	13.51	2.553	13.73	2.419	14.25	2.176
Exper	22.42	13.63	16.95	12.13	28.97	10.95	25.19	11.75
Exper2	688.3	615.4	434.6	506.1	959.3	549.1	772.6	560.7
Tenure	15.12	12.68	11.22	10.47	23.64	11.43	20.01	11.98
Married	0.422	0.494	0.454	0.498	0.239	0.426	0.223	0.416
North	0.547	0.498	0.613	0.487	0.327	0.469	0.440	0.497
Centre	0.199	0.400	0.202	0.402	0.211	0.408	0.229	0.421
Observations	3,756		2,275		2,033		2,084	

educational performance (*Schooling* and *Maximum_D_Mark*) compared the men in the private sector. The capability to speak English (*Eng_Skill*) is more pronounced in the private sector for both men and women. Also, men and women are substantially longer employed at the same institution in the public sector (*Tenure*). Men do on average more over-work (*Extra_Hours*), what is more pronounced in the private than in the public sector for both, male and female employees. Next, women are on average younger than men (*Age*). Employees are about ten years older in the public than in the private sector. This relatively huge difference may be due to the stop of recruitment in the public sector in Italy (Mandrone *et al.*, 2012). About the equal amount of male and female employees is married, yet, the proportion of married employees is higher in the public sector. Similarly for the variable *Kids*; employees with children are more often employed in the public sector. The proportion of individuals with children with less than four years is more pronounced for women and for both, men and women, more in the private sector. The latter is related to the fact that younger individuals work on average more often in the private sector. In particular, given the sharp reduction in recruitment in the public sector in Italy.

7 Empirical Results

Empirical Results Case 1: The Gender Pay Gap between 2016 and 2005

Table 3, column (1) shows the base model of case 1, equation (8), and hence the change of the GPG over time. The difference between the GPG in 2016 and 2005 amounts to -0.03 log points and is statistically significant. Given the negative sign, the GPG has decreased over time. The convergence amounts to —0.03— log points, as expected from the standard estimation technique in Section **??**. However, now we can also conclude that this reduction in the GPG is statistically significant. The full model, equation (12), is presented in column (2) of Table 3. We immediately see that the part of the price effect due to differences in the intercepts, $\hat{\alpha}_1^{full}$, is not statistically significant. Similarly, the effect of being a woman or in Year 2005, all else equal, becomes statistically insignificant compared to the base model in column (1). The remaining coefficient estimates show the expected signs. In particular, we observe a positive effect of educational attainment, labor market experience and tenure.

Besides looking at the average change in the GPG over time, we can easily extend our proposed decomposition to test for changes in wages between men and women over time at different points of the unconditional wage distribution. Therefore, we use unconditional quantile regression, i.e. we use Recentered Influence Function (RIF) of the log of hourly wages at specific quantiles as dependent variable. The method was first introduced by **?**. [8]

Table **??** shows the results from our proposed decomposition. The results suggest that the reduction in the GPG from 2005 to 2016 was entirely due to observed wage characteristics. Hence, the closing of the GPG is not explained by anti-discrimination laws, changes in attitudes towards women in the labor market or changes in the family structure and birth control. The latter is catched by $I2$, which accounts for changes in institutional settings, but is not statistically significant. The reduction in the GPG from 2005 to 2016 is explained by women catching up to men in terms of their educational background and their labor market experience. We know from Section 1 that women's human capital (*Schooling*, *Maximum_D_Mark*, *Exper*) is increasing, while that of men is partly even decreasing (*Exper*) or remained lower than that of women (educational

[8] For further details on unconditional quantile regressions see Appendix **??**.

Table 3: Wage Regression Case 1, Base and Full Specification

	(1) Basic Wage Regression Lhwage	(2) Full Wage Regression Lhwage
female	-0.087***	-0.201*
	(0.009)	(0.112)
inter	-0.064***	0.001
	(0.012)	(0.149)
year	-0.156***	-0.116
	(0.008)	(0.080)
Schooling		0.033***
		(0.002)
Exper		0.023***
		(0.002)
Exper2		-0.000***
		(0.000)
Tenure		0.004***
		(0.001)
Married		-0.061***
		(0.010)
North		0.050***
		(0.010)
Centre		0.018
		(0.012)
Constant	2.159***	1.485***
	(0.006)	(0.059)
Industry & Occupation Dummies	No	Yes
Set of Interactions	No	Yes
Observations	19,333	19,333
R-squared	0.064	0.338

Standard errors clustered at the individual level in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

attainment variables). In fact, in educational matters, women have outpaced men (Goldin, 2006). Also, on average, the percentage of women having at least one child (*Kids*) or a child younger than three years (*Kids_3*) declined from 69.80% to 54.80% from 2005 to 2016, respectively. The results from Section **??** suggested that the coefficients part of the GPGs, i.e. the part due to differences in returns on observable wage characteristics, was a main contributor to the GPG in either year. However, by estimating the difference of the GPG over time directly, we have shown that the only factor that contributes statistically significantly to the narrowing of the gap are better observable wage characteristics for women.

Table 4: Decomposition Case 1 at the Mean & Selected Quantiles, Standard Approach

	(1) 2005 Mean	(2) 2016 Mean	(3) 2005 10	(4) 2016 10	(5) 2005 50	(6) 2016 50	(7) 2005 90	(8) 2016 90
Panel A: Decomposition with Male-Reference Category								
GPG	0.152***	0.087***	0.118***	0.029**	0.121***	0.072***	0.252***	0.133***
	(0.008)	(0.009)	(0.013)	(0.013)	(0.009)	(0.007)	(0.017)	(0.013)
Explained	0.006	-0.012*	0.012	-0.009	0.009	-0.015***	-0.008	-0.038***
	(0.006)	(0.007)	(0.007)	(0.009)	(0.006)	(0.005)	(0.015)	(0.010)
Unexplained	0.146***	0.100***	0.106***	0.038***	0.112***	0.086***	0.261***	0.171***
	(0.008)	(0.009)	(0.014)	(0.015)	(0.009)	(0.007)	(0.020)	(0.015)
Observations	9,185	10,148	9,185	10,148	9,185	10,148	9,185	10,148
Panel B: Regression-Compatible Decomposition								
GPG	0.152***	0.087***	0.118***	0.029**	0.121***	0.072***	0.252***	0.133***
	(0.008)	(0.009)	(0.013)	(0.013)	(0.009)	(0.007)	(0.017)	(0.013)
Explained	-0.001	-0.014**	-0.002	-0.015**	0.004	-0.017***	0.006	-0.021***
	(0.006)	(0.006)	(0.007)	(0.007)	(0.006)	(0.004)	(0.011)	(0.008)
Unexplained	0.152***	0.101***	0.120***	0.044***	0.117***	0.089***	0.246***	0.154***
	(0.007)	(0.009)	(0.013)	(0.014)	(0.008)	(0.006)	(0.018)	(0.014)
Observations	9,185	10,148	9,185	10,148	9,185	10,148	9,185	10,148

Robust standard errors in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

Table 5: Detailed Decomposition Case 1 at the Mean and Selected Quantiles, Standard Approach

	(1) 2005 Mean	(2) 2016 Mean	(3) 2005 10	(4) 2016 10	(5) 2005 50	(6) 2016 50	(7) 2005 90	(8) 2016 90
Panel A: Decomposition with Male-Reference Category								
Explained:								
X_HC	-0.023***	-0.023***	-0.023***	-0.023***	-0.023***	-0.023***	-0.023***	-0.023***
	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)
X_LM	0.037***	0.034***	0.037***	0.034***	0.037***	0.034***	0.037***	0.034***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
X_Demo	-0.003**	-0.004***	-0.003**	-0.004***	-0.003**	-0.004***	-0.003**	-0.004***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
X_OccInd	-0.006	-0.019***	-0.005	-0.019***	-0.006	-0.019***	-0.006	-0.019***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)
Total	0.006	-0.012*	0.005	-0.012*	0.006	-0.012*	0.006	-0.012*
	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.007)
Unexplained:								
X_HC	-0.060	-0.132**	-0.060	-0.132**	-0.060	-0.132**	-0.060	-0.132**
	(0.039)	(0.058)	(0.039)	(0.058)	(0.039)	(0.058)	(0.039)	(0.058)
X_LM	0.057***	0.024	0.057***	0.024	0.057***	0.024	0.057***	0.024
	(0.019)	(0.027)	(0.019)	(0.027)	(0.019)	(0.027)	(0.019)	(0.027)
X_Demo	-0.012	-0.016	-0.012	-0.016	-0.012	-0.016	-0.012	-0.016
	(0.014)	(0.015)	(0.014)	(0.015)	(0.014)	(0.015)	(0.014)	(0.015)
X_OccInd	-0.039	0.023	-0.039	0.023	-0.039	0.023	-0.039	0.023
	(0.061)	(0.070)	(0.061)	(0.070)	(0.061)	(0.070)	(0.061)	(0.070)
Total	0.146***	0.100***	0.146***	0.100***	0.146***	0.100***	0.146***	0.100***
	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)	(0.009)
Constant	0.200**	0.200**	0.200**	0.201**	0.200**	0.201**	0.200**	0.201**
	(0.078)	(0.078)	(0.078)	(0.101)	(0.078)	(0.101)	(0.078)	(0.101)
Panel B: Regression-Compatible Decomposition								
Explained:								
X_HC	-0.024***	-0.027***	-0.024***	-0.027***	-0.024***	-0.027***	-0.024***	-0.027***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
X_LM	0.037***	0.033***	0.037***	0.033***	0.037***	0.033***	0.037***	0.033***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
X_Demo	-0.004***	-0.004***	-0.004***	-0.004***	-0.004***	-0.004***	-0.004***	-0.004***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
X_OccInd	-0.009**	-0.017***	-0.009**	-0.017***	-0.009**	-0.017***	-0.009**	-0.017***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Total	-0.001	-0.014**	-0.001	-0.014**	-0.001	-0.014**	-0.001	-0.014**
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
Unexplained:								
X_HC	-0.059	-0.129**	-0.059	-0.129**	-0.059	-0.129**	-0.059	-0.129**
	(0.038)	(0.054)	(0.038)	(0.054)	(0.038)	(0.054)	(0.038)	(0.054)
X_LM	0.057***	0.025	0.057***	0.025	0.057***	0.025	0.057***	0.025
	(0.021)	(0.029)	(0.021)	(0.029)	(0.021)	(0.029)	(0.021)	(0.029)
X_Demo	-0.011	-0.017	-0.011	-0.017	-0.011	-0.017	-0.011	-0.017
	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)	(0.014)
X_OccInd	-0.036	0.021	-0.036	0.021	-0.036	0.021	-0.036	0.021
	(0.080)	(0.079)	(0.080)	(0.079)	(0.080)	(0.079)	(0.080)	(0.079)
Total	0.152***	0.101***	0.152***	0.101***	0.152***	0.101***	0.152***	0.101***
	(0.007)	(0.009)	(0.007)	(0.009)	(0.007)	(0.009)	(0.007)	(0.009)
Constant	0.200**	0.201*	0.200**	0.201*	0.200**	0.201*	0.200**	0.201*
	(0.098)	(0.112)	(0.098)	(0.112)	(0.098)	(0.112)	(0.098)	(0.112)

Observations are 9,185 & 10,148 in 2005 & 2016, respectively. Robust standard errors in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

Table 6: Hypotheses Testing Case 1 at the Mean & Selected Quantiles

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$H0$	χ^2-Statistic	P-value	χ^2-Statistic	P-value	χ^2-Statistic	P-value	χ^2-Statistic	P-value
	Mean		10		50		90	
Panel B: Regression-Compatible Decomposition								
$P_T = P_t$	0.24	0.626	2.24	0.134	0.03	0.854	1.98	0.160
$Q_T = Q_t$	5.34	0.021	1.79	0.185	0.34	0.561	1.13	0.288

Empirical Results Case 2: Public-Private Sector Wage Gap between Women and Men

The regression results of the base model, i.e. equation (10), are outlined in column (1) of Table **??**. The results suggest that there has been a positive and statistically significant difference in the PPWG between women and men, $\tilde{\Delta}^{Female} - \tilde{\Delta}^{Male}$, equal to 0.04 log points. This is in line with the estimation results obtained in Section **??**, Table **??**. The dummy variable for working in the public sector (*pub*) positively and significantly influences wages (0.20). The coefficient on the $female$-dummy shows that being a women has a significant and negative impact on labor income, what is what we expected to find.

Again, we can easily extend the empirical estimation using unconditional quantile regressions.

In the full model, column (2) of Table **??**, the effect of public-sector employment on wages turns negative, but remains highly statistically significant, while the $female$ coefficient becomes statistically insignificant. Yet, the interaction of women and public-sector employment is statistically significant and strongly positive (0.52). This tells us also that $\hat{\alpha}_1^{full}$, i.e. the part of the price or unexplained effect due to differences in the starting points is statistically significant and hence the unexplained component of the PPWG may not only play a role when estimating it separately for male and female subsamples (Section **??**), but also when estimating the difference of the PPWG between women and men directly. Again, the remaining estimation coefficients are in line with the literature, see for example Fortin (2008).

By applying our proposed estimation approach to case 2, we find that the difference in observable wage characteristics across sectors and gender, E, does play a statistically significant role in explaining the gap of the PPWG by gender, see Table **??**). Moreover, also differences in institutional settings across sectors do influence the PPWG differential between women and men ($I2$). Indeed, $I2$ is found to be a main driver of the PPWG between women and men (-0.06). Finally, the part attributed to differences in returns across sectors and gender exhibits a statistically significant negative impact on the difference in the PPWG by gender. As we know from Table **??** that $\hat{\alpha}_1^{full}$ is positive and statistically significant (0.52), we develop a test in order to be able to draw conclusions on the statistical significance of the price effect of the difference in the PPWG between women and men.

All in all, for case 2, the conclusions drawn from the standard estimation are confirmed; both quantity and price effects contribute to the difference in the PPWG between women and men. Nonetheless, we gain the additional insight that the set-up or organization of the sectors does play a role as well in explaining the statistically significant difference in the PPWG by gender in 2016 in Italy.

Table 7: Wage Regression Case 2, Base and Full Specification

	(1) Basic Wage Regression Lhwage	(2) Full Wage Regression Lhwage
Public_Sec	0.252***	0.467**
	(0.013)	(0.203)
inter	-0.042**	-0.358
	(0.017)	(0.236)
year	0.134***	0.243*
	(0.012)	(0.133)
Schooling		0.036***
		(0.003)
Exper		0.020***
		(0.002)
Exper2		-0.000***
		(0.000)
Tenure		0.004***
		(0.001)
Married		-0.042***
		(0.014)
North		0.062***
		(0.015)
Centre		0.031
		(0.019)
Constant	1.952***	1.234***
	(0.010)	(0.110)
Industry & Occupation Dummies	No	Yes
Set of Interactions	No	Yes
Observations	10,148	10,148
R-squared	0.070	0.253

Standard errors clustered at the individual level in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

Table 8: Decomposition Case 2 at the Mean & Selected Quantiles, Standard Approach

	(1) Men Mean	(2) Women Mean	(3) Men 10	(4) Women 10	(5) Men 50	(6) Women 50	(7) Men 90	(8) Women 90
Panel A: Decomposition with Male-Reference Category								
PPWG	-0.210***	-0.252***	-0.214***	-0.228***	-0.168***	-0.221***	-0.179***	-0.188***
	(0.012)	(0.013)	(0.018)	(0.018)	(0.009)	(0.009)	(0.023)	(0.016)
Explained	-0.134***	-0.131***	-0.113**	-0.147***	-0.101***	-0.125***	-0.153***	-0.114***
	(0.024)	(0.022)	(0.046)	(0.035)	(0.017)	(0.015)	(0.036)	(0.020)
Unexplained	-0.076***	-0.121***	-0.101**	-0.081**	-0.068***	-0.097***	-0.026	-0.074***
	(0.026)	(0.024)	(0.048)	(0.038)	(0.018)	(0.017)	(0.041)	(0.025)
Observations	5,789	4,359	5,789	4,359	5,789	4,359	5,789	4,359
Panel B: Regression-Compatible Decompositionn								
PPWG	-0.210***	-0.252***	-0.214***	-0.228***	-0.168***	-0.221***	-0.179***	-0.188***
	(0.012)	(0.013)	(0.017)	(0.018)	(0.009)	(0.009)	(0.023)	(0.016)
Explained	-0.157***	-0.153***	-0.117***	-0.147***	-0.123***	-0.136***	-0.224***	-0.169***
	(0.012)	(0.014)	(0.019)	(0.018)	(0.009)	(0.009)	(0.021)	(0.016)
Unexplained	-0.052***	-0.099***	-0.096***	-0.081***	-0.045***	-0.085***	0.045*	-0.018
	(0.015)	(0.018)	(0.026)	(0.024)	(0.011)	(0.012)	(0.027)	(0.020)
Observations	5,789	4,359	5,789	4,359	5,789	4,359	5,789	4,359

Robust standard errors in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

Table 9: Detailed Decomposition Case 2 at the Mean & Selected Quantiles, Standard Approach

	(1) Men Mean	(3) Women Mean	(9) Men 10	(11) Women 10	(17) Men 50	(19) Women 50	(25) Men 90	(27) Women 90
Panel A: Decomposition with Male-Reference Category								
Explained:								
X_HC	-0.027***	-0.027***	-0.027***	-0.027***	-0.027***	-0.027***	-0.027***	-0.027***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
X_LM	-0.084***	-0.098***	-0.084***	-0.098***	-0.084***	-0.098***	-0.084***	-0.098***
	(0.007)	(0.010)	(0.007)	(0.010)	(0.007)	(0.010)	(0.007)	(0.010)
X_Demo	0.002	0.005	0.002	0.005	0.002	0.005	0.002	0.005
	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)	(0.006)	(0.005)	(0.006)
X_OccInd	-0.026	-0.010	-0.026	-0.010	-0.026	-0.010	-0.026	-0.010
	(0.024)	(0.020)	(0.024)	(0.020)	(0.024)	(0.020)	(0.024)	(0.020)
Total	-0.134***	-0.131***	-0.134***	-0.131***	-0.134***	-0.131***	-0.134***	-0.131***
	(0.024)	(0.022)	(0.024)	(0.022)	(0.024)	(0.022)	(0.024)	(0.022)
Unexplained:								
X_HC	-0.052	0.014	-0.052	0.014	-0.052	0.014	-0.052	0.014
	(0.074)	(0.092)	(0.074)	(0.092)	(0.074)	(0.092)	(0.074)	(0.092)
X_LM	0.003	0.093**	0.003	0.093**	0.003	0.093**	0.003	0.093**
	(0.048)	(0.044)	(0.048)	(0.044)	(0.048)	(0.044)	(0.048)	(0.044)
X_Demo	0.029**	0.031	0.029**	0.031	0.029**	0.031	0.029**	0.031
	(0.014)	(0.020)	(0.014)	(0.020)	(0.014)	(0.020)	(0.014)	(0.020)
X_OccInd	0.055	0.207*	0.055	0.207*	0.055	0.207*	0.055	0.207*
	(0.090)	(0.121)	(0.090)	(0.121)	(0.090)	(0.121)	(0.090)	(0.121)
Total	-0.076***	-0.121***	-0.076***	-0.121***	-0.076***	-0.121***	-0.076***	-0.121***
	(0.026)	(0.024)	(0.026)	(0.024)	(0.026)	(0.024)	(0.026)	(0.024)
Constant	-0.109	-0.467***	-0.109	-0.467***	-0.109	-0.467***	-0.109	-0.467***
	(0.131)	(0.164)	(0.131)	(0.164)	(0.131)	(0.164)	(0.131)	(0.164)
Panel B: Regression-Compatible Decomposition								
Explained:								
X_HC	-0.029***	-0.027***	-0.029***	-0.027***	-0.029***	-0.027***	-0.029***	-0.027***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
X_LM	-0.089***	-0.095***	-0.089***	-0.095***	-0.089***	-0.095***	-0.089***	-0.095***
	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.007)
X_Demo	-0.003	0.001	-0.003	0.001	-0.003	0.001	-0.003	0.001
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
X_OccInd	-0.037***	-0.031***	-0.037***	-0.031***	-0.037***	-0.031***	-0.037***	-0.031***
	(0.010)	(0.012)	(0.010)	(0.012)	(0.010)	(0.012)	(0.010)	(0.012)
Total	-0.157***	-0.153***	-0.157***	-0.153***	-0.157***	-0.153***	-0.157***	-0.153***
	(0.012)	(0.014)	(0.012)	(0.014)	(0.012)	(0.014)	(0.012)	(0.014)
Unexplained:								
X_HC	-0.050	0.015	-0.050	0.015	-0.050	0.015	-0.050	0.015
	(0.070)	(0.082)	(0.070)	(0.082)	(0.070)	(0.082)	(0.070)	(0.082)
X_LM	0.008	0.090**	0.008	0.090**	0.008	0.090**	0.008	0.090**
	(0.049)	(0.043)	(0.049)	(0.043)	(0.049)	(0.043)	(0.049)	(0.043)
X_Demo	0.034**	0.036	0.034**	0.036	0.034**	0.036	0.034**	0.036
	(0.015)	(0.024)	(0.015)	(0.024)	(0.015)	(0.024)	(0.015)	(0.024)
X_OccInd	0.065	0.229	0.065	0.229	0.065	0.229	0.065	0.229
	(0.070)	(0.154)	(0.070)	(0.154)	(0.070)	(0.154)	(0.070)	(0.154)
Total	-0.052***	-0.099***	-0.052***	-0.099***	-0.052***	-0.099***	-0.052***	-0.099***
	(0.015)	(0.018)	(0.015)	(0.018)	(0.015)	(0.018)	(0.015)	(0.018)
Constant	-0.109	-0.467**	-0.109	-0.467**	-0.109	-0.467**	-0.109	-0.467**
	(0.121)	(0.202)	(0.121)	(0.202)	(0.121)	(0.202)	(0.121)	(0.202)

Observations are 5,789 & 4,359 for men and women, respectively.
Robust standard errors in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

Table 10: Hypotheses Testing Case 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$H0$	χ^2-Statistic	P-value	χ^2-Statistic	P-value	χ^2-Statistic	P-value	χ^2-Statistic	P-value
	Mean		10		50		90	
Panel B: Regression-Compatible Decomposition								
$P_T = P_t$	2.14	0.144	0.25	0.614	0.01	0.940	4.67	0.031
$Q_T = Q_t$	0.54	0.462	2.2	0.138	2.73	0.099	0.82	0.366

8 Conclusion

This paper takes the cue from two empirical evidence that have been considered in literature to introduce a new decomposition approach. Adding to the discussion of the convergence of the GPG over time and the persistence of a PPWG between women and men, we propose an alternative decomposition method allowing to draw inference on the difference of two wage gaps. Additionally, the method allows us to catch otherwise unobserved interaction effects across the respective groups of interest.

In fact, the PPWG which is found to differ for females and males is a topic of on-going research (Melly, 2005). Similarly, the observed closing of the GPG over the last decades is heavily discussed in the literature and the determination of the reasons of the narrowing are of huge interest, especially with regard to policy implications (Blau and Kahn, 2006,0; Goldin, 2014).

Applying the decomposition to the case of the GPG over time (Section 7), the convergence of the GPG in the last decade was found to be entirely explained by a reduction in differences in observable labor market characteristics by gender, i.e. by the catching-up of women with respect to men in these characteristics. On the contrary, by estimating the GPG separately for 2005 and 2016, i.e. following the double OB decomposition, the change in the significance of the price component might have led to the conclusion that the implementation of anti-discrimination laws and changing attitudes towards women in the labor market have influenced the narrowing of the pay gap over time as well. Yet, these policies as well as changes in social norms seem to have been less effective than expected a priori. In fact, even if the unexplained part is found to be the main contributor to the GPG in a specific year, it turns insignificant when estimating changes of the GPG over time directly.

The results for the second case examined, i.e. the PPWG between women and men, pointed the attention to differences in the structure of the public and private sector, which are found to be a main driver of the differential. In this case, the results obtained in Section **??** are confirmed. Thanks to the decomposition carried on we can determine whether both parts of the price effect - the difference in the intercepts and the difference in remuneration - drive the change in the pay gap in a statistically significant way or not. Our conclusion is that both parts contribute to the difference in the PPWG between women and men.

All in all, our decomposition method offers a better understanding of what has lead to the narrowing of the GPG in the last ten years and what drives the difference in the PPWG between women and men. Most importantly, we can infer what drives the difference in the respective pay gaps in a statistically significant manner. Thereby, we add to the literature on the convergence of the GPG over time for the case of Italy, the finding that the closing of the pay differential by gender over the last decade was entirely explained by the catching-up of women in terms of observable labor market characteristics. For the second case of interest in this paper, the PPWG between women and men, we can confirm the results obtained with the standard method in the literature for other countries (Mandel and Semyonov, 2014; Melly, 2005).

Despite additional insights on the composition of differences in gaps, the method can be extended to be robust to the choice of the reference category (Reimers, 1983; Cotton, 1988; Neumark, 1988; Oaxaca and Ransom, 1994; Fortin, 2008) as well as to the indeterminacy problem (Lee, 2015). We propose to decompose the GPG following the intercept-shift approach proposed by (Fortin, 2008) and applying the omitted variable bias formula (as proposed by (Gelbach, 2016)). Thereby, when conducting a detailed decomposition, the single parts of the endowments effect can be associated to specific covariates and the invariance problem with respect to categorical variables can be solved (Gardeazabal and Ugidos, 2004; Fortin, 2008).

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Appendices

A Link between the parameters on pag 12.

1. When F=1, i.e. Female, J=1, i.e. year t:

   - $\hat{\alpha}_{Ft} = \hat{\alpha}_0^{full} + \hat{\alpha}_1^{full} + \hat{\alpha}_2^{full} + \hat{\alpha}_3^{full}$
   - $\hat{\beta}_{Ft} = \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \hat{\beta}_4$

2. When F=0, i.e. Male, J=1, i.e. year t, we get:

   - $\hat{\alpha}_{Mt} = \hat{\alpha}_0^{full} + \hat{\alpha}_3^{full}$
   - $\hat{\beta}_{Mt} = \hat{\beta}_1 + \hat{\beta}_3$

3. When F=1, i.e. Female, J=0, i.e. year T, we get:

   - $\hat{\alpha}_{FT} = \hat{\alpha}_0^{full} + \hat{\alpha}_2^{full}$
   - $\hat{\beta}_{FT} = \hat{\beta}_1 + \hat{\beta}_2$

4. When F=0, i.e. Male, J=0, i.e. year T, we get:

   - $\hat{\alpha}_{MT} = \hat{\alpha}_0^{full}$
   - $\hat{\beta}_{MT} = \hat{\beta}_1$

Re-arranging terms slightly, gives us:

$$\begin{aligned}
\hat{\alpha}_0^{full} &= \hat{\alpha}_{MT} \\
\hat{\alpha}_2^{full} &= \hat{\alpha}_{FT} - \hat{\alpha}_{MT} \\
\hat{\alpha}_3^{full} &= \hat{\alpha}_{MT} - \hat{\alpha}_{Mt} \\
\hat{\alpha}_1^{full} &= \hat{\alpha}_{Ft} - \hat{\alpha}_{MT} - \hat{\alpha}_{FT} + \hat{\alpha}_{MT} - \hat{\alpha}_{Mt} + \hat{\alpha}_{MT} \\
&= (\hat{\alpha}_{MT} - \hat{\alpha}_{FT}) - (\hat{\alpha}_{Mt} - \hat{\alpha}_{Ft})
\end{aligned}$$

$$\begin{aligned}
\hat{\beta}_1 &= \hat{\beta}_{MT} \\
\hat{\beta}_2 &= \hat{\beta}_{FT} - \hat{\beta}_{MT} \\
\hat{\beta}_3 &= \hat{\beta}_{Mt} - \hat{\beta}_{MT} \\
\hat{\beta}_4 &= \hat{\beta}_{MT} - \hat{\beta}_{FT} - \hat{\beta}_{Mt} + \hat{\beta}_{Ft} \\
&= (\hat{\beta}_{MT} - \hat{\beta}_{FT}) - (\hat{\beta}_{Mt} - \hat{\beta}_{Ft})
\end{aligned}$$

B Solving the Index Number Problem for the level of the GPG

Section 5 presents the solution to the indeterminacy problem for the variation over time of the GPG. This Appendix shows how to solve the indeterminacy problem for the level of the GPG within the OVB decomposition. Our aim is to have a wage decomposition invariant to the reference category adopted. Following Fortin (2008), we include gender intercept shifts along with an identification restriction, in the regression of females and males pooled together, when considering the standard case of the GPG:

$$y_i = \gamma_0 + \gamma_{0F} F_i + \gamma_{0M} M_i + X_i \gamma + \epsilon_i$$

subject to:

$$\gamma_{0F} + \gamma_{0M} = 0$$

where F_i (M_i) is equal to one if the individual is female (male) and zero otherwise. The identification restriction, $\gamma_{0F}+\gamma_{0M}=0$, imposes that the pooled wage equation truly represents a non-discriminatory wage structure, i.e. a wage structure where the advantage of men is equal to the disadvantage of women:

$$\bar{y}_M - \bar{y}_F = (\bar{X}_M - \bar{X}_F)\hat{\gamma} + (\hat{\gamma}_{0M} - \hat{\gamma}_{0F}) \tag{28}$$

The first component on the RHS, $(\bar{X}_M - \bar{X}_F)\hat{\gamma}$, is the explained part, while $\hat{\gamma}_{0M}$ and $\hat{\gamma}_{0F}$ are the *advantage of men* and the *disadvantage of women*, respectively. In particular:

$$\begin{aligned}\hat{\gamma}_{0M} &= \bar{X}_M(\hat{\beta}_M - \hat{\gamma}) + (\hat{\alpha}_M - \hat{\gamma}_0) \qquad \textit{advantage of men}\\ \hat{\gamma}_{0F} &= \bar{X}_F(\hat{\beta}_F - \hat{\gamma}) + (\hat{\alpha}_F - \hat{\gamma}_0) \qquad \textit{disadvantage of women.}\end{aligned}$$

where $\hat{\alpha}_M, \hat{\alpha}_F, \hat{\beta}_M, \hat{\beta}_F$ are the estimated coefficients of the wage equations for men and women, respectively:

$$y_{iM} = \alpha_M + X_M\beta_M + \epsilon_{iM} \tag{29}$$

$$y_{iF} = \alpha_F + X_F\beta_F + \epsilon_{iF} \tag{30}$$

In order to recast the wage decomposition of the *full model* with the conditional decomposition framework proposed in Section 3 we estimate the following wage equation:

$$y_i = \gamma_0 + \gamma_{0F}F_i + \gamma_{0M}M_i + X_i\gamma + X_iF_i\gamma_{XF} + X_iM_i\gamma_{XM} + \nu_i \tag{31}$$

subject to:

$$\begin{aligned}\gamma_{0F} + \gamma_{0M} &= 0\\ \gamma_{X_kF} + \gamma_{X_kM} &= 0 \text{ for } k = 1\ldots K\end{aligned}$$

where γ_{X_kF} and γ_{X_kM} are the parameters of the interaction term between the *kth* regressor X_k and the dummy F and M, respectively. The error term is represented by v_i. Evaluating equation (31) at the mean yields:

$$
\begin{aligned}
\bar{y}_M &= \hat{\gamma}_0 + \hat{\gamma}_{0M} + \bar{X}_M\hat{\gamma} + \bar{X}_M\hat{\gamma}_{XM} \\
\bar{y}_F &= \hat{\gamma}_0 + \hat{\gamma}_{0F} + \bar{X}_F\hat{\gamma} + \bar{X}_F\hat{\gamma}_{XF}
\end{aligned}
$$

Hence, the GPG is given by:

$$
\begin{aligned}
\bar{y}_M - \bar{y}_F &= (\hat{\gamma}_{0M} - \hat{\gamma}_{0F}) + (\bar{X}_M - \bar{X}_F)\hat{\gamma} + \bar{X}_M\hat{\gamma}_{XM} - \bar{X}_F\hat{\gamma}_{XF} && (32) \\
&= 2\hat{\gamma}_{0M} + (\bar{X}_M - \bar{X}_F)\hat{\gamma} + (\bar{X}_M + \bar{X}_F)\hat{\gamma}_{XM} && (33)
\end{aligned}
$$

First, we observe that there exists the following relationship between the parameter estimates of equations (29)-(30) and (31):

$$
\begin{aligned}
\hat{\gamma} - \hat{\gamma}_{XM} &= \hat{\beta}_F \\
\hat{\gamma}_0 - \hat{\gamma}_{0M} &= \hat{\alpha}_F \\
\hat{\gamma} + \hat{\gamma}_{XM} &= \hat{\beta}_M \\
\hat{\gamma}_0 + \hat{\gamma}_{0M} &= \hat{\alpha}_M
\end{aligned}
$$

Therefore, the GPG of (33) can be re-written in terms of the Fortin-decomposition as:

$$
\begin{aligned}
\bar{y}_M - \bar{y}_F &= (\hat{\alpha}_M - \hat{\gamma}_0) - (\hat{\alpha}_F - \hat{\gamma}_0) + (\bar{X}_M - \bar{X}_F)\hat{\gamma} + \bar{X}_M(\hat{\beta}_M - \hat{\gamma}) - \bar{X}_F(\hat{\beta}_F - \hat{\gamma}) && (34) \\
&= (\bar{X}_M - \bar{X}_F)\hat{\gamma} + \underbrace{[\bar{X}_M(\hat{\beta}_M - \hat{\gamma}) + (\hat{\alpha}_M - \hat{\gamma}_0)]}_{\textit{advantage of men}} - \underbrace{[\bar{X}_F(\hat{\beta}_F - \hat{\gamma}) + (\hat{\alpha}_F - \hat{\gamma}_0)]}_{\textit{disadvantage of women}} && (35)
\end{aligned}
$$

Second, the estimation can be recast in terms of sequential decomposition by considering the following base model:

$$
y_i = \gamma_0^{base} + (M_i - F_i)\gamma_{0M}^{base} + \epsilon_i^{base} \tag{36}
$$

where the set of regressors of the *base* model is given by $X_1 = \Big[1, (M - F)\Big]$, the constant and the difference between the two dummy variables F and M. The full model is defined as follows:

$$y_i = \gamma_0^{full} + (M_i - F_i)\gamma_{0M}^{full} + X_i\gamma + X_i(M_i - F_i)\gamma_{XM} + \epsilon_i^{full} \tag{37}$$

where $X_2 = \Big[X, X(M-F)\Big]$. $X(M-F)$ is the interaction between the matrix of regressors X and the vector that contains the difference between the two dummy variables M and F. By the OVB formula the following relationship holds:

$$\begin{bmatrix} \hat{\gamma}_0^{base} \\ \hat{\gamma}_{0M}^{base} \end{bmatrix} = \begin{bmatrix} \hat{\gamma}_0^{full} \\ \hat{\gamma}_{0M}^{full} \end{bmatrix} + (X_1'X_1)^{-1}X_1'X_2 \begin{bmatrix} \hat{\gamma} \\ \hat{\gamma}_{XM} \end{bmatrix} \tag{38}$$

where $(\hat{\gamma}_0^{base}\ \hat{\gamma}_{0M}^{base})'$ is the vector of coefficient estimates of X_1 from the *base* model (36); $(\hat{\gamma}_0^{full}\ \hat{\gamma}_{0M}^{full})'$ is the vector containing the coefficient estimates of X_1 from the *full* model (37) and $(\hat{\gamma}\ \hat{\gamma}_{XM})'$ is the vector of coefficients estimates of X_2 from the full model (37). Observe that:

$$\begin{bmatrix} \hat{\gamma}_0^{base} \\ \hat{\gamma}_{0M}^{base} \end{bmatrix} = \begin{bmatrix} \frac{\bar{y}_M + \bar{y}_F}{2} \\ \frac{\bar{y}_M - \bar{y}_F}{2} \end{bmatrix} \tag{39}$$

and $\hat{\gamma}_{0M}^{full}$ is equal to $\frac{\hat{\alpha}_M - \hat{\alpha}_F}{2}$.

Given (39), our interest relies on the second row of equation (38), that represents the decomposition of the GPG. We observe that the linear projection of X with respect to X_1 is equal to:

$$(X_1'X_1)^{-1}X_1'X = \begin{bmatrix} / \\ (\bar{X}_M' - \bar{X}_F')/2 \end{bmatrix}$$

The linear projection of $X(M-F)$ with respect to X_1 is equal to:

$$(X_1'X_1)^{-1}X_1'X(F-M) = \begin{bmatrix} / \\ (\bar{X}_M' + \bar{X}_F')/2 \end{bmatrix}$$

Given (38), we observe that:

$$\begin{aligned} 2\hat{\gamma}_{0M}^{base} &= 2(\bar{y}_M - \bar{y}_F) = \Delta(y) = 2\hat{\gamma}_{0M}^{full} + (\bar{X}_M - \bar{X}_F)\hat{\gamma} + (\bar{X}_M + \bar{X}_F)\hat{\gamma}_{XM} \\ &= 2\hat{\gamma}_{0M}^{full} + (\bar{X}_M - \bar{X}_F)\hat{\gamma} + \bar{X}_M(\hat{\beta}_M - \hat{\gamma}) + -\bar{X}_F(\hat{\beta}_F - \hat{\gamma}) \\ &= (\bar{X}_M - \bar{X}_F)\hat{\gamma} + \underbrace{[\bar{X}_M(\hat{\beta}_M - \hat{\gamma}) + (\hat{\alpha}_M - \hat{\gamma}_0)]}_{advantage\ of\ men} - \underbrace{[\bar{X}_F(\hat{\beta}_F - \hat{\gamma}) + (\hat{\alpha}_F - \hat{\gamma}_0)]}_{disadvantage\ of\ women} \end{aligned}$$

that completes the proof of the decomposition equivalence.

B.1 Invariance decomposition with respect to categorical variables

A second type of identification issue arises when dummy variables are considered in the wage decomposition. Oaxaca and Ransom (1999) show that the assignment of the unexplained part of the GPG to specific variables is not invariant to the choice of reference groups. This problem can be easily solved by imposing the following parameters restrictions as proposed by Fortin *et al.* (2011b):

$$\sum_{j=1}^{C_k} \gamma_{jk} = 0, \ k \in C \tag{40}$$

where C denotes the set of categorical variables, and C_k the number of categories for variable k. The *neutral*, i.e. non-sensitive to any left-out category, Oaxaca-Blinder decomposition follows. The zero-sum restriction (40) is applied to the wage equation, when female and male wages are estimated separately as well as to the pooled regression with gender dummies. The latter is additionally estimated with the identification restriction $\gamma_{0M} + \gamma_{0F} = 0$ on the gender parameters. Thereby, the intercepts, α_M, β_{0F} and γ_0, are no longer influenced by the choice of the reference category and the single parts of the endowments effect can be associated to specific covariates (Gardeazabal and Ugidos, 2004; Fortin, 2008). The restriction (40) can also be applied to the method proposed in Section **??** leading to indicator variables that, in case of categorical variables, are invariant to the choice of the left-out category.

C Intercept-shift approach versus pooled-sample approach

Lee (2015) shows that the intercept-shift approach proposed by Fortin (2008) presents two drawbacks. Firstly, the reference parameter for the Oaxaca-Blinder decomposition, i.e. the parameter that would prevail in a 'fair' world under no discrimination, relies on the variance difference among categories. Secondly, the reference intercept is arbitrary: the same Oaxaca-Blinder decomposition holds with vastly different reference intercepts.

However, it can be easily shown that our proposed decomposition does not suffer from any of these aspects. Our decomposition arises from a specification that allows different intercepts and slopes. In addition, the constraints imposed on the parameters that identify the counterfactual reference parameters are the parameters such that the advantage of men is equal to the disadvantage of women.

In fact, in our model the slope that would prevail under *no discrimination*, γ, is the sample average of the group slopes; α_M and α_F:

$$\gamma = 0.5\alpha_M + 0.5\alpha_F$$

i.e. it is equivalent to considering the weights proposed by Reimers (1983).[9]

Moreover, the constraint:

$$\alpha_F - \gamma_{0F} = \alpha_M + \gamma_{0F}$$

prevents the indeterminacy problem shown by Lee (2015) in eq. (6) page 74. It turns out, that in our model, the intercept indeterminacy problem highlighted by Lee (2015) is ruled out by imposing the constraint that the advantage of men should be equal to the disadvantage of women.

D Inference

TO BE ADDED

[9] See also page 72 of Lee (2015).

E Unconditional Quantile Regression

To be precise, the RIF-OLS regression model allows to estimate the effect of explanatory variables, X, on the unconditional quantile, Q_τ, of an outcome variable, Y. The RIF is estimated in quantile regressions by first calculating the sample quantile $\hat{Q}_\tau$ and computing the density at $\hat{Q}_\tau$, that is $f(\hat{Q}_\tau)$ using kernel methods **?**. This approach relies on the indicator function $1\{Y_t \leq Q_\tau\}$ taking value one if the condition in $\{\cdot\}$ is true, zero otherwise. Estimates for each observation i of the RIF, $\widehat{RIF}(Y_{it}; Q_\tau)$, are then obtained by inserting $\hat{Q}_\tau$ and $f(\hat{Q}_\tau)$ in the aggregate RIF-function, defined as:

$$
\begin{aligned}
RIF(Y_t; Q_\tau) &= Q_\tau + IF(Y_t; Q_\tau) \\
&= Q_\tau + \frac{\tau - 1\{Y_t \leq Q_\tau\}}{f_Y(Q_\tau)} \\
&= \frac{1}{f_{Y_t}(Q_\tau)} 1\{Y_t > Q_\tau\} + Q_\tau - \frac{1}{f_{Y_t}(Q_\tau)}(1-\tau)
\end{aligned}
\tag{E.1}
$$

where the RIF is the first order approximation of the quantile Q_τ. $IF(Y_t; Q_\tau)$ represents the influence function for the τth quantile. It measures the (marginal) influence of an observation at Y on the sample quantile. Adding the quantile Q_τ to the influence function yields the RIF. The probability density of Y at time t evaluated at Q_τ is $f_{Y_T}(Q_\tau)$.

The model can then be estimated by OLS using the RIFs as dependent varaibles. ? (?) model the conditional expectation of the RIF-regression function, $E[RIF(Y_t; Q_\tau)|X]$, as a function of explanatory variables, X, in the UQR:

$$
E[RIF(Y_t; Q_\tau)|X] = g_{Q_\tau}(X) \tag{E.2}
$$

where a linear function $X\beta_\tau$ is specified for $g_{Q\tau}(X)$. The explanatory variables, X, contain time-varying controls such as labor market experience and job tenure as well as time constant controls like schooling. The average derivative of the unconditional quantile regression, $E_X\left[\frac{dg_{Q_\tau}(X)}{dX}\right]$, captures the marginal effect of a small location shift in the distribution of covariates on the τth UQ of Y_t keeping everything else constant. Therefore, the coefficients, β_τ, can be unconditionally interpreted, as $E[RIF(Y_t; Q_\tau)] = E_X\left[E\big(RIF(Y_t; Q_\tau)|X\big)\right] = E(X)\beta_\tau$. That is, the unconditional expectations $E[RIF(Y_t; Q_\tau)]$ using the LIE allow for the unconditional mean interpretation. On the contrary, only the conditional mean interpretation is valid in the context of CQRs; $Q_\tau(Y_t|X) = X\beta_\tau^{CQR}$, where β_τ^{CQR} can be interpreted as the effect of X on the τth CQ of Y given X. The LIE does not apply here; $Q_\tau \neq E_X[Q_\tau(Y_t|X)] = E(X)\beta_\tau^{CQR}$, where Q_τ is the UQ. Hence, β_τ^{CQR} cannot be interpreted as the effect of increasing the mean value of X in the UQ Q_τ. In UQR, the coefficients β_τ can be estimated by OLS in the following way:

$$Q_\tau = E[RIF(Y_t; Q_\tau)] = E_X[RIF(Y_t; Q_\tau)|X] = E(X)\beta_\tau \tag{E.3}$$

F Definition of Variables

Variable Name	Definition
	Dependent Variables
Lhwage	Net hourly wages; hourly wages in Euros, net of taxes and social security contributions
	Independent Variable
	Dummy and Interaction Effects
female	One if the respective individual is a woman, zero otherwise
year	One if year is 2005, zero otherwise
pub	One if firm is a publicly owned firm, zero otherwise
femyear	Interactive effect of year and female, i.e. one if employee is observed in 2005 and is female, zero otherwise
fempub	Interactive effect of pub and female, i.e. one if employee is employed in the public sector and is female, zero otherwise
	Labor Market Presence
Exper	Number of years of prior work experience
Exper2	Exper squared
Tenure	Number of years worked for current employer
Extra_Hours	Measures the hours spent working overtime

Educational Attainment

Schooling	Number of years of schooling completed
Maximum_D_Mark	One if maximum degree mark was, i.e. *110 e lode*, attained, zero otherwise
Eng_Skill	One if individual is able to communicate in English, zero otherwise

Demographic Background

Age	Age of individual (in years)
Italian	One if individual is Italian, zero otherwise
Educ_Moth4	One if mother's education is at least equal to *Diploma*, i.e. 13 years of schooling, zero otherwise
Educ_Moth5	One if mother's education is equal to *Laurea*, i.e. mother holds a university degree, zero otherwise
Educ_Fath4	One if father's education is at least equal to *Diploma*, i.e. 13 years of schooling, zero otherwise
Educ_Fath5	One if father's education is equal to *Laurea*, i.e. father holds a university degree, zero otherwise
Homeowner	One if employee owns a house, zero otherwise

Family Background

Kids	One if individual has at least one child, zero otherwise
Kids_3	One if age of youngest child is less or equal to three years, zero otherwise
Married	One if individual is married, zero otherwise

Industry and Occupations

Sec_2	One if individual is engaged in the industrial sector, zero otherwise

Sec_6	One if individual is engaged in tourism, zero otherwise
Sec_7	One if individual is engaged in transport, zero otherwise
Sec_8	One if individual is engaged in communication, zero otherwise
Sec_9	One if individual is engaged in financial sector, zero otherwise
Sec_10	One if individual is engaged in firm services, zero otherwise
Sec_13	One if individual is engaged in health, zero otherwise
Sec_14	One if individual is engaged in science and other professional activities, zero otherwise
Manager	One if individual executes intellectual professions;
	scientific and highly specialized occupations, zero otherwise
Intermediate_Prof	One if individual executes intermediary positions in commercial, technical
	or administrative sectors, health services and technicians, zero otherwise

Geographic Variables

North	One if individual lives and works in the North of Italy, zero otherwise