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The (Ir)relevance of Real Wage Rigidity for Optimal Monetary Policy

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Real wage rigidity is known to create a substantial trade-off between inflation and employment stabilization for monetary policy in New Keynesian models with search frictions on the labor market. This paper shows that, quantitatively, this finding hinges very much on the assumption of constant returns to scale in production. With decreasing returns to scale, monetary policy with a single focus on inflation stabilization is close to optimal. The reason is twofold: Firms cushion the impact of rigid real wages on marginal costs by adjusting the marginal product of labor over the cycle. In addition, given employment fluctuations have a smaller effect on consumption volatility. Decreasing returns to scale thus remove the need for active monetary policy even if wages are rigid. Importantly, this contrasts with the implications of combining real wage rigidity and decreasing returns to scale for other policy instruments.

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1. Introduction

In a seminal paper, Blanchard and Galí (2010) show that real wage rigidity in a model with search frictions creates a significant trade-off between inflation and employment stabilization. While flexible, efficient wages allow for a simultaneous stabilization of inflation and employment (which Blanchard and Galí (2007) call the "divine coincidence"), this is no longer feasible under real wage rigidity and consequently, optimal monetary policy deviates from strict price stability. A different strand of the literature, in turn, has shown that the interaction of real wage rigidity and decreasing returns to scale in production in search and matching models has important positive and normative policy implications. First, fiscal policy is more effective during recessions (Michaillat, 2014). The combination of real wage rigidity and decreasing returns to scale thus provides a theoretical framework for countercyclical fiscal multipliers. Second, optimal unemployment insurance is more generous in times of a slack labor market (Landais et al., 2018). Thus, the interaction of real wage rigidity with decreasing returns to scale matters for fiscal and unemployment insurance policy. This paper shows that this is also true for monetary policy, although in a different and perhaps unexpected way.

The model considered in this paper is a standard New Keynesian model with Rotemberg (1982) adjustment costs for prices and search and matching on the labor market. Blanchard and Galí (2010) show that in a New Keynesian model with search friction real wage rigidity creates a quantitatively important trade-off between inflation and employment stabilization and that the welfare loss of strict inflation targeting is large in this case. I show that this result hinges very much on the assumption of constant returns to scale in production. With decreasing returns to scale, the policy trade-off between inflation and employment stabilization is small. Optimal inflation volatility still deviates from zero. However, the welfare loss of a policy that completely stabilizes inflation is close to zero. Indeed, if the central bank follows a Taylor rule, the (constrained) optimal policy puts zero weight on unemployment deviations under decreasing returns while the weight is negative and quantitatively important under constant returns.

Decreasing returns to scale are a common assumption in New Keynesian models without capital (see textbook example Gali, 2009, Ch. 3) on the basis that capital is fixed or very sluggish to respond in the short run due to investment adjustment costs[1] or time-to-build. Indeed, while the empirical literature on production function estimations finds a range of estimates form decreasing to increasing returns to scale in the long-run, a common identifying assumption is the distinction between variable factors of production

[1] This is especially the case if investment is lumpy.

(i.e. labor) and factors of production that are (quasi-)fix in the short run (i.e. capital).[2] Thus, the assumption of decreasing returns to scale in this paper should be seen as a short-run assumption valid for analyzing monetary policy. In addition, there is a broad literature using decreasing returns to scale in search (and matching) models in order to model firm size dynamics (see e.g. Elsby and Michaels, 2013; Kaas and Kircher, 2015). Real wage rigidity, in turn, is a common assumption in search and matching models as it helps to solve the so-called "Shimer puzzle" – the lack of amplification in the standard search and matching model – as suggested by Shimer (2005) and Hall (2005). Empirical work by Haefke et al. (2013) shows that real wages of newly hired workers in the US are fairly flexible but respond less than one to one to aggregate productivity changes.[3] Gertler et al. (2019) argue that wages of new hires are as rigid as those of existing workers once controlling for composition effects.

In this context, this paper makes two important points: First, even in the presence of real wage rigidity, a "hawkish" monetary policy does not necessarily lead to large welfare losses. Second, in a broader sense, it shows that the interaction of different short-run real rigidities – in wages and in capital formation – matters: Combining them can have non-trivial consequences for policy and the implications can differ dramatically between policy instruments. The combination of real wage rigidity and decreasing returns to scale calls for a passive monetary policy but – as previous research has shown – a more active fiscal and labor policy in response to business cycle fluctuations.

The intuition behind this result is straightforward. With decreasing returns to scale, an adjustment in labor input changes the marginal product of labor. If real wages are rigid, a negative aggregate productivity shock leads to an increase in the real wage relative to productivity and hence raises marginal costs. However, if firms reduce employment at the same time, the resulting increase in the marginal product of labor partly offsets the effect of the rigid real wage. For a given change in employment, marginal costs move by less. This results in a lower inflation volatility in response to aggregate productivity shocks. On the other hand, stabilizing inflation can be achieved with smaller labor market volatility. What is more, any given labor market volatility leads to lower welfare losses under decreasing returns because the changing marginal product of labor reduces the impact on output and consumption volatility. Thus, for a policy maker that aims at maximizing household welfare, stabilizing inflation comes at a much lower cost for two reasons: costly employment volatility is smaller and the impact of employment volatility on consumption volatility is reduced.

[2] See e.g. De Loecker et al. (2018) for a very recent example for the U.S..

[3] The degree of wage rigidity in the quantitative part of the paper is calibrated to the results in Haefke et al. (2013).

This paper adds to the growing literature that features the combination of real wage rigidity and decreasing returns to scale in a search and matching labor market framework. Michaillat (2012) shows that in such an environment there is “rationing unemployment”. Even if recruiting was costless, firms’ optimal decision would not lead to full employment if aggregate productivity is relatively low. The policy implications of this mechanism have been analyzed by Michaillat (2014) for fiscal policy and Landais et al. (2018) for optimal unemployment insurance.[4]

This paper is further related to the literature on optimal monetary policy in the context of a frictional labor market. A number of papers have studied trade-offs arising from nominal (as opposed to real) wage rigidity (Thomas, 2008; Sala et al., 2008) or more general deviations from the efficient wage (Faia, 2009; Ravenna and Walsh, 2011; Faia et al., 2014). In a very recent paper, LePetit (2019) finds significant trade-offs between inflation and unemployment stabilization in search and matching models if there are steady state distortions and the average unemployment rate is much higher than the steady state rate. As common in the search and matching literature and in contrast to the present paper, all these models are based on a constant returns to scale production function.[5] The current paper is further related to Erceg et al. (2000) who investigate monetary policy trade-offs in a model with staggered nominal wages but without a search and matching labor market. Arseneau and Chugh (2008), in turn, study optimal fiscal and monetary policy in a model with costly nominal wage adjustments and a search and matching labor market but flexible product prices. Finally, Krause and Lubik (2007) show that in a New Keynesian model with labor market frictions real wage rigidity is irrelevant for the dynamics of inflation. This paper – in a variation of the former paper’s title – presents another case in which real wage rigidity matters less than previously expected.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 explains the model mechanism in the presence of real wage rigidity and decreasing versus constant returns. Section 4 discusses the calibration and Section 5 presents numerical results. Section 6 briefly concludes.

[4]The model has also served as a theoretical basis for Crépon et al. (2013), who evaluate the displacement effects of a labor market assistance program in France. Finally, Brügemann (2014) provides an extensive discussion of the efficiency properties of the wage rule in this setting.

[5]Sala et al. (2008) study a model with capital where the production function is constant returns with respect to both input factors and a competitive rental market for capital.

2. The Model

The model is a search and matching model embedded in a New Keynesian framework that features both real wage rigidity and decreasing returns to scale in production. It is thus similar to the model presented in Michaillat (2014).

2.1. Household

There is a big representative household that pools income and consumption of its members, called workers. The household derives utility $U(c_t)$ from an aggregate consumption bundle c_t.[6] It receives real wage income, $w_t n_t$, from its employed members n_t and nominal transfers T_t from firms. In addition, the household has access to one-period bonds B_t that pay a gross nominal interest rate R_t in the next period. The future is discounted with discount factor $0 < \beta < 1$.

The household chooses consumption and nominal bond holdings to maximize the stream of discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t\right), \tag{1}$$

subject to the budget constraint

$$c_t + \frac{B_t}{p_t} = n_t w_t + R_{t-1} \frac{B_{t-1}}{p_t} + \frac{T_t}{p_t}, \tag{2}$$

with p_t denoting the aggregate price index. The consumption bundle is given by a standard Dixit-Stiglitz aggregator

$$c_t = \left[\int_0^1 c_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)}, \tag{3}$$

such that demand for variety $c_t(i)$ with price $p_t(i)$ is given by

$$c_t(i) = c_t \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon}, \tag{4}$$

[6] As common in search and matching models, I assume that there is no disutility from work. This also implies that labor only adjusts at the extensive margin. This assumption seems justified given that the empirical literature (see e.g. Hansen (1985) and Merkl and Wesselbaum (2011)) shows that most of the volatility of aggregate hours is driven by the extensive margin.

and the aggregate price level satisfies:

$$p_t = \left(\int_0^1 p_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}. \tag{5}$$

Utility is given by:

$$U(c_t) = \frac{c_t^{1-\nu}}{1-\nu}. \tag{6}$$

Maximization of (1) subject to (2) yields the standard Euler equation:

$$\lambda_t = \beta E_t \left[\frac{R_t}{1+\pi_{t+1}} \lambda_{t+1} \right], \tag{7}$$

where

$$\lambda_t = U'(c_t) = c_t^{-\nu} \tag{8}$$

represents the marginal utility of consumption and $\pi_t = (p_t/p_{t-1} - 1)$ is the net inflation rate.

Workers The household consists of a measure one of workers that supply labor inelastically. The labor market is characterized by search and matching frictions, i.e. only a fraction of workers is employed in every period. Unemployed workers search for jobs. Employed workers earn a real wage w_t and face the risk of loosing their job with an exogenous probability σ in every period. Newly unemployed workers can be immediately rehired. The number of searching workers at the beginning of every period is thus

$$u_t^s = 1 - (1-\sigma) n_{t-1}, \tag{9}$$

with n_t representing the number of employed workers in period t. Searching workers are matched to vacant jobs v_t via a standard Cobb-Douglas matching function

$$m_t = \vartheta v_t^{\xi} (u_t^s)^{1-\xi}, \tag{10}$$

where m_t are the number of new matches at the beginning of period t and $0 < \xi < 1$. New matches become productive immediately. The job-finding probability for a worker is thus

$$f(\theta_t) = \vartheta \theta_t^{\xi}, \tag{11}$$

where $\theta_t = v_t/u_t^s$ denotes market tightness.

2.2. Firms and Production

There are two types of firms. The wholesale sector consists of a continuum of firms on the unit interval. They produce a homogeneous intermediate good using labor. Firms in the retail sector buy the intermediate goods and transform them into differentiated goods that they sell on a monopolistically competitive market subject to price frictions.

2.2.1. Wholesale Firms

Production Firms in the wholesale sector use labor as their sole input factor in production:

$$y_t(j) = a_t n_t(j)^\alpha, \tag{12}$$

where a_t is aggregate productivity and $0 < \alpha \leq 1$. The production technology may thus feature decreasing returns to scale. In contrast to the one-firm one-job model in e.g. Pissarides (2000), firms are large and can have more than one employee.

Search Frictions Firms in the wholesale sector have to post vacancies $v_t(j)$ at period cost κa_t in order to make new hires.[7] Vacancy posting costs are measured in terms of the final output good. Given the matching technology, the vacancy-filling rate is:

$$q(\theta_t) = \frac{m_t}{v_t} = \vartheta \theta_t^{\xi - 1}, \tag{13}$$

where v_t denotes the aggregate number of vacancies in the economy. In every period, a fraction σ of existing employment relations is dissolved. Thus, for a desired number of new hires $h_t(j)$ the firm needs to post $h_t(j)/q(\theta_t)$ vacancies. The total costs for new hires are thus:

$$\kappa a_t v_t(j) = \frac{\kappa a_t}{q(\theta_t)} h_t(j) = \frac{\kappa a_t}{q(\theta_t)} \left[n_t(j) - (1-\sigma) n_{t-1}(j) \right]. \tag{14}$$

Wages For comparability and tractability, I follow Blanchard and Galí (2010) and Michaillat (2012, 2014) in modeling real wage rigidity. The wage is given by the following simple wage rule:

$$w_t = \omega a_t^\gamma, \tag{15}$$

with $0 < \gamma \leq 1$. This formula features real wage rigidity whenever $\gamma < 1$. Hall (2005) shows that in a search and matching labor market even a constant wage could be privately

[7] Recruiting costs are thus increasing in aggregate productivity as e.g. in Pissarides (2000, chapter 1). The main results of this paper are robust to this assumption.

efficient. This is a notable difference to a frictionless labor market, in which a rigid real wage would be subject to the Barro (1977) critique.[8]

Optimization Wholesale sector firms sell on a competitive market to retail firms at relative price Λ_t. A wholesale sector firm thus chooses employment to maximize the following objective function taking into account the household's stochastic discount factor $\beta^t \frac{\lambda_t}{\lambda_0}$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \Lambda_t a_t n_t(j)^{\alpha} - w_t n_t(j) - \frac{\kappa a_t}{q(\theta_t)} \left[n_t(j) - (1-\sigma) n_{t-1}(j) \right] \right\}. \quad (16)$$

Optimization yields the following first-order conditions for employment:

$$\Lambda_t \alpha n_t(j)^{\alpha-1} = \frac{w_t}{a_t} + \frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right]. \quad (17)$$

2.2.2. Retail Firms

Retail firms purchase the intermediate goods at relative price Λ_t and transform them into differentiated goods $y_t(i)$ indexed by i. They face a downward sloping demand curve and price changes are subject to Rotemberg (1982) adjustment costs:

$$\frac{\Phi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 y_t, \quad (18)$$

i.e. firms pay a quadratic cost measured in the final consumption good whenever they change their price level from one period to the next.[9] I assume that production is subsidized at rate τ_s such that retailers' marginal costs, i.e. Λ_t, equal unity in steady state. This way, inefficiencies arising from monopolistic competition are offset in steady state.

[8]Brügemann (2014) demonstrates that private efficiency with decreasing returns to scale can only be guaranteed along the equilibrium path, but not off equilibrium. With decreasing returns, private efficiency could always be violated if firms chose a large enough expansion of employment. However, in equilibrium, such an allocation would never arise. Furthermore, Brügemann (2014) shows that a way around this problem is to assume a wage that follows equation (15) up to a threshold and afterwards adjusts such that firms' profit function is flat. For the sake of simplicity, I keep the simple wage of equation (15) and check numerically that the private efficiency condition is not violated.

[9]As decreasing returns to scale imply that firm level employment is the outcome of a firm's optimal choice, I choose Rotemberg (1982) adjustment costs which have the advantage that firms are confronted with symmetric problems in every period. Note, however, that Rotemberg (1982) costs are equivalent to Calvo (1983) pricing up to a first-order approximation if trend inflation is zero, which is the case in this setup. Equivalence does not hold, however, if trend inflation deviates from zero (Ascari and Rossi, 2012).

Firms in the retail sector set prices in order to maximize the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ (1+\tau_s) \frac{p_t(i)}{p_t} y_t(i) - \Lambda_t y_t(i) - \frac{\Phi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 y_t \right\} \tag{19}$$

subject to

$$y_t(i) = y_t \left(\frac{p_t(i)}{p_t} \right)^{-\epsilon}. \tag{20}$$

This yields the following optimality condition for the individual price $p_t(i)$:

$$\begin{aligned} &(1-\epsilon)(1+\tau_s)\left(\frac{p_t(i)}{p_t}\right)^{-\epsilon} \frac{y_t}{p_t} + \epsilon \Lambda_t \left(\frac{p_t(i)}{p_t}\right)^{-\epsilon-1} \frac{y_t}{p_t} \\ &- \Phi \left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right) \frac{y_t}{p_{t-1}(i)} + \beta \cdot E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \Phi \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 \right) \frac{y_{t+1} p_{t+1}(i)}{p_t(i)^2} \right] = 0. \end{aligned} \tag{21}$$

2.3. Equilibrium

In equilibrium, firms are identical such that $p_t(i) = p_t$, $n_t(j) = n_t$, $y_t(i) = y_t$, and all markets clear. Aggregate labor demand thus becomes:

$$\Lambda_t \cdot \alpha n_t^{\alpha-1} = \frac{w_t}{a_t} + \frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right], \tag{22}$$

with

$$q(\theta_t) = \vartheta \theta_t^{\xi-1}.$$

Note that in contrast to the classical search and matching model with constant returns to scale (see e.g. Pissarides, 2000, chapter 1) there is no free-entry condition for firms. Instead, the number of firms is assumed to be fixed and employment is determined by the optimal choice of labor input.

Equation (21) reduces to the familiar nonlinear Phillips curve:

$$(\pi_t + 1)\pi_t = \frac{1}{\Phi} \cdot [\epsilon \Lambda_t - (\epsilon - 1)(1+\tau_s)] + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} (\pi_{t+1} + 1)\pi_{t+1} \right]. \tag{23}$$

Note that in a first order approximation, the slope of the Philips curve with respect to marginal costs is the same under constant and decreasing returns. Thus any differences in optimal monetary policy between the two assumption stem from differences in marginal costs and not from a different reaction of inflation to marginal costs.[10]

[10] This would be different with Calvo (1983) price staggering and in the presence of strategic complementarities.

Aggregate production is given by:

$$y_t = a_t n_t^{\alpha}. \tag{24}$$

By bond market clearing, $B_t = R_{t-1}B_{t-1}$, and assuming that firms' profits are transferred to the household, the aggregate resource constraint becomes:

$$c_t = y_t - \frac{\Phi}{2}\pi_t^2 y_t - \frac{\kappa a_t}{q(\theta_t)}\left[n_t - (1-\sigma)n_{t-1}\right]. \tag{25}$$

The law of motion for employment is given by:

$$n_t = (1-\sigma)\cdot n_{t-1} + \left[1 - (1-\sigma)\cdot n_{t-1}\right]\cdot f(\theta_t), \tag{26}$$

with

$$f(\theta_t) = \vartheta\theta_t^{\xi}.$$

Finally, the nominal interest rate R_t is set by the central bank in accordance with a Ramsey policy as described in Section 2.4. Alternatively, the optimal Ramsey policy is compared to a policy that completely stabilizes inflation over the business cycle and a policy following an optimized Taylor rule.

Definition 1. *Given initial employment and bond holdings, $\{n_0, b_0\}$, a stochastic path for productivity $\{a_t\}_{t=0}^{\infty}$, and a policy for the nominal interest rate $\{R_t\}_{t=0}^{\infty}$, the equilibrium is a collection of processes $\{c_t, \lambda_t, \pi_t, w_t, \Lambda_t, n_t, \theta_t, y_t\}_{t=0}^{\infty}$ that satisfy equations (7), (8), (15), (22), (23), (24), (25), and (26).*

2.4. The Optimal Policy

Optimal monetary policy in this paper is defined as the policy of a Ramsey planner. The Ramsey approach has become a popular tool for analyzing optimal fiscal and monetary policy (see e.g. Lucas and Stokey, 1983; Chari et al., 1991; Khan et al., 2003; Schmitt-Grohé and Uribe, 2004, 2007; Yun, 2005). The Ramsey planner chooses the interest rate such as to maximize the sum of the household's discounted utility, taking into account the constraints of the competitive economy. Given the utility function, matching function, production function, and the wage rule, the policy maker is subject to the following constraints:

$$c_t^{-\nu} = \beta E_t\left[\frac{R_t}{1+\pi_{t+1}} c_{t+1}^{-\nu}\right], \tag{27}$$

$$\Lambda_t \cdot \alpha n_t^{\alpha-1} = \omega a_t^{\gamma-1} + \frac{\kappa}{\vartheta \theta_t^{\xi-1}} - (1-\sigma)\beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\nu} \frac{a_{t+1}}{a_t} \frac{\kappa}{\vartheta \theta_{t+1}^{\xi-1}} \right], \tag{28}$$

$$\begin{aligned} (\pi_t + 1)\pi_t =& \frac{1}{\Phi} \cdot [\epsilon \Lambda_t - (\epsilon - 1)(1 + \tau_s)] \\ &+ \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\nu} \frac{a_{t+1}}{a_t} \left(\frac{n_{t+1}}{n_t} \right)^{\alpha} (\pi_{t+1} + 1)\pi_{t+1} \right], \end{aligned} \tag{29}$$

$$a_t n_t^{\alpha} = c_t + \frac{\Phi}{2} \pi_t^2 a_t n_t^{\alpha} + \frac{\kappa a_t}{\vartheta \theta_t^{\xi-1}} \left[n_t - (1-\sigma) n_{t-1} \right], \tag{30}$$

$$n_t = (1-\sigma) \cdot n_{t-1} + [1 - (1-\sigma) \cdot n_{t-1}] \, \vartheta \theta_t^{\xi}. \tag{31}$$

Formally, the Ramsey problem is defined as follows:

Definition 2. *Let* $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^{\infty}$ *be the Lagrange multipliers on constraints (27), (28), (29), (30), and (31). For a given path of productivity* $\{a_t\}_{t=0}^{\infty}$*, a first-best constrained allocation is a plan for the control variables* $\Xi_t^n = \{c_t, n_t, \theta_t, \pi_t, \Lambda_t, R_t\}_{t=0}^{\infty}$ *and the co-state variables* $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^{\infty}$ *that solves the following maximization problem:*

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}} Max_{\{\Xi_t^n\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}, \tag{32}$$

subject to (27), (28), (29), (30), and (31).

As constraints (27), (28), and (29) are forward looking, the private agents' decisions are influenced by their expectation about future policy. However, their current choices are also based on past hiring decisions, influenced in turn by past expectations about the evolution of monetary policy. The Ramsey policy thus suffers from a time-inconsistency problem (see e.g. Kydland and Prescott, 1977; Prescott, 1977; Calvo, 1978). As suggested by Kydland and Prescott (1980), this problem can be transformed into a recursive problem by enhancing the state space with additional co-state variables, i.e. the multipliers on the forward looking constraints. Intuitively, these co-state variables represent the costs of the planner of sticking to earlier policy commitments. At time zero, the values of the co-state variables are set to their steady state values which is consistent with a timeless perspective. In other words, it is assumed that the economy has already been evolving around that steady state for some time.

3. Inspecting the Mechanism

3.1. Constrained Efficient Allocation

In order to illustrate potential trade-offs between inflation and employment stabilization under the chosen wage norm, it is useful to first set a benchmark by deriving the constrained efficient allocation of a social planner.

The social planner is constrained by the search technology in the labor market but is not subject to frictions arising from price rigidity or monopolistic competition. The planner chooses consumption, employment, and vacancies to maximize the stream of the household's discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{33}$$

subject to the resource constraint and the employment dynamics equation:

$$a_t n_t^{\alpha} - \kappa a_t v_t - c_t = 0, \tag{34}$$

$$v_t q(\theta_t) + (1-\sigma) n_{t-1} - n_t = 0, \tag{35}$$

with

$$\theta_t = \frac{v_t}{1-(1-\sigma)n_{t-1}}. \tag{36}$$

Let λ_t and τ_t be the multipliers on constraints (34) and (35) respectively. Taking first-order conditions and rearranging gives the following optimality conditions:

$$\tau_t = \lambda_t \alpha a_t n_t^{\alpha-1} + \beta E_t \left[\tau_{t+1} \left((1-\sigma) + v_{t+1} q'(\theta_{t+1}) \frac{\partial \theta_{t+1}}{\partial n_t} \right) \right], \tag{37}$$

$$\lambda_t \kappa a_t = \tau_t \left(q(\theta_t) + v_t q'(\theta_t) \frac{\partial \theta_t}{\partial v_t} \right). \tag{38}$$

Equation (37) describes the value of an additional worker. It is given by the current marginal product weighted by the household's marginal utility and the continuation value of a worker. The latter is composed of two parts: the value of a worker in the next period weighted by the retention probability and a term that takes into account that an employed worker reduces the chances of finding new workers in the next period – a congestion effect. Equation (38) equates the costs of posting a vacancy (the left hand side) with the benefits of a vacancy (right hand side). Again, the social planner takes into account that posting an additional vacancy increases market tightness and thus decreases the chance of filling a vacancy. Combining equations (37) and (38) and using the definition of the vacancy-filling rate (13) and market tightness (36) yields the following optimality condition, which equates the benefits of a new hire to its costs:

$$\begin{aligned}\frac{\kappa}{q(\theta_t)} &= \xi\alpha n_t^{\alpha-1} + (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{a_{t+1}}{a_t}\frac{\kappa}{q(\theta_{t+1})}\right] \\ &\quad - (1-\xi)(1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{a_{t+1}}{a_t}\kappa\theta_{t+1}\right].\end{aligned} \tag{39}$$

If the stochastic discount factor moves one to one with productivity changes,[11] labor market variables are invariant to changes in productivity. Now, let us compare the planner's solution to the optimality condition from the competitive economy (equation (22)):

$$\frac{\kappa}{q(\theta_t)} = \Lambda_t \cdot \alpha n_t^{\alpha-1} + (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{a_{t+1}}{a_t}\frac{\kappa}{q(\theta_{t+1})}\right] - \frac{w_t}{a_t}. \tag{40}$$

The two conditions correspond if the wage takes the following form:[12]

$$w_t^* = (1-\xi)\left(\Lambda_t \alpha a_t n_t^{\alpha-1} + (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\kappa a_{t+1}\theta_{t+1}\right]\right). \tag{41}$$

The optimal wage moves proportionally with the marginal product of labor, with the factor of proportionality equal to the elasticity of matches with respect to unemployment. Under constant returns to scale, this wage corresponds to the familiar Nash bargaining wage under Hosios (1990) rule. It looks the same under decreasing returns to scale adjusted for the marginal product of labor although in this case it does not correspond to the outcome of an intra-firm bargaining game.[13] Plugging the optimal wage back into the competitive economy's optimality condition (40) yields an expression of marginal costs as a function of labor market variables:

$$\begin{aligned}\Lambda_t &= (1-\xi) \\ &+ \frac{1}{\alpha n_t^{\alpha-1}}\left(\frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{a_{t+1}}{a_t}\left(\frac{\kappa}{q(\theta_{t+1})} + (1-\xi)\kappa\theta_{t+1}\right)\right]\right).\end{aligned} \tag{42}$$

The first term on the right hand side is a constant. The second term depends only on labor market variables, productivity, and the stochastic discount factor. Again, if the

[11] This would be the case under log utility.

[12] Remember that marginal costs, Λ, are subsidized to be equal to one in steady state.

[13] Under decreasing returns the nature of the bargaining game changes as firms take into account the impact of an additional hire on the marginal product and wages of the rest of the workforce. For a theory of intra-firm bargaining see Stole and Zwiebel (1996). For an incorporation of the Stole and Zwiebel (1996) bargaining into an equilibrium search and matching model see e.g. Cahuc and Wasmer (2001), Cahuc et al. (2008), and Elsby and Michaels (2013).

stochastic discount factor moves one to one with changes in productivity, labor market tightness and employment are invariant to productivity changes and marginal costs are a constant.[14] Thus, with decreasing returns but a flexible, optimal wage, there is no trade-off between inflation and employment stabilization. The next section explores how the relationship between real marginal costs and employment changes if real wages are rigid.

3.2. The Inflation-Unemployment Trade-off

I follow previous literature (Blanchard and Galí, 2010; Michaillat, 2012, 2014; LePetit, 2019) and model real wage rigidity with the simple wage norm $w_t = \omega a_t^{\gamma}$, where $\gamma < 1$. While this wage is clearly ad hoc, it is convenient to illustrate the central mechanisms. In the quantitative section I provide an alternative specification of real wage rigidity. I start with an intuitive discussion on the trade-off between employment and inflation stabilization generated by real wage rigidity under constant versus decreasing returns and then show more formally how the different model assumptions impact welfare in the next section.

Consider equation (22) with constant returns to scale and the wage norm (15):

$$\Lambda_t = \omega a_t^{\gamma-1} + \left(\frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{a_{t+1}}{a_t}\frac{\kappa}{q(\theta_{t+1})}\right]\right). \tag{43}$$

The first term on the right hand side of equation (43) represents the real wage relative to aggregate productivity. Compare this to equation (42). In (42), the first term on the right hand side is a constant. In contrast, under real wage rigidity, i.e. $\gamma < 1$, the relative real wage fluctuates with aggregate productivity. A drop in aggregate productivity leads to an increase in the relative wage and vice verse. Keeping marginal costs constant would mean that the second term on the right hand side of equation (43) would have to fully compensate for the change in the relative wage. This term represents the value of having a matched worker (i.e. the costs of hiring today minus the saved hiring costs of tomorrow). In particular, stabilizing marginal costs in response to a negative productivity shock would require a sharp drop in today's recruiting costs, i.e. a sharp drop in market tightness. Under complete inflation stabilization the steady state elasticity of market tightness with respect to aggregate productivity is given by the following term:[15]

$$\frac{\partial \log \theta}{\partial \log a} = \frac{1}{1-\xi}(1-\gamma)\frac{\omega a^{\gamma}}{\Lambda a - \omega a^{\gamma}}. \tag{44}$$

[14] Even if households are more risk averse, the effect is quantitatively close to zero. Blanchard and Galí (2010) show that for more general preferences, where households enjoy leisure, inflation and employment can be simultaneously stabilized if income and substitution effects cancel.

[15] I assume that steady state market tightness is equal to one.

If $\gamma = 1$, i.e. if the wage responds one-to-one to aggregate productivity, tightness does not respond to aggregate productivity and hence there is no trade-off between inflation and employment stabilization. This is an extreme form of the Shimer (2005) puzzle. The higher the degree of wage rigidity, i.e. the smaller γ, and the smaller the fundamental surplus $\Lambda a - \omega a^\gamma$, the bigger is the response of tightness and hence employment.[16]

Why is the inflation-employment stabilization trade-off reduced under decreasing returns to scale? Again consider equation (22) with the wage norm (15), i.e. $w_t = \omega a_t^\gamma$, and decreasing returns to scale:

$$\Lambda_t = \frac{\omega a_t^{\gamma-1}}{\alpha n_t^{\alpha-1}} + \frac{1}{\alpha n_t^{\alpha-1}} \left(\frac{\kappa}{q(\theta_t)} - (1-\sigma)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{a_{t+1}}{a_t} \frac{\kappa}{q(\theta_{t+1})} \right] \right). \tag{45}$$

As before, the first term on the right hand side of equation (45) is the ratio between the relative real wage w_t/a_t and the relative marginal product of labor. With real wage rigidity, the relative real wage rises if productivity falls. However, if employment also falls in response, the marginal product of labor rises. This reduces the effect on real marginal costs. In other words, for a given level of employment volatility, the resulting volatility of real marginal costs and hence inflation is lower. As discussed before, with constant returns to scale, stabilizing marginal costs in the presence of rigid real wages means that all the adjustment has to come from the value of the match. Employment has to move by a lot. With decreasing returns to scale, employment also adjusts but the response is smaller because the change in the marginal product of labor works as an offsetting factor to the real wage rigidity. Indeed, under decreasing returns to scale, the elasticity of tightness with respect to productivity under complete inflation stabilization becomes

$$\frac{\partial \log \theta}{\partial \log a} = \frac{1}{1-\xi}(1-\gamma) \frac{\omega a^\gamma}{\underbrace{\Lambda \alpha a n^{\alpha-1} - \omega a^\gamma}_{FS} - \underbrace{\frac{1}{1-\xi}\Lambda \frac{\partial mpl}{\partial n}\frac{\partial n}{\partial \theta}\theta}_{<0}}, \tag{46}$$

where $mpl = \alpha n^{\alpha-1}$ is the marginal product of labor. As the marginal product of labor is decreasing in employment, the elasticity is strictly smaller than under constant returns.[17] Therefore, one reason why inflation stabilization under decreasing returns has low welfare costs is because labor market volatility is much smaller.

However, it is not the only reason. Under constant returns to scale production, the

[16] Indeed, the degree of labor market amplification depends on the combination of real wage rigidity and the fundamental surplus. If the surplus is large, even a high degree of wage rigidity will lead to modest amplification. With a small surplus, a modest degree of wage rigidity suffices for a large labor market response. See LePetit (2019) and Ljungqvist and Sargent (2017) for a detailed discussion on this.

[17] The responsiveness of the elasticity of tightness to the fundamental surplus is also smaller.

response of employment to productivity affects the output elasticity one-to-one. This is not the case under decreasing returns. Even if employment reacted as much to productivity shocks as under constant returns (this could be achieved with a higher degree of wage rigidity, i.e. a lower γ), decreasing returns imply that the impact of the employment response on output volatility, and hence consumption volatility, is reduced. I discuss this argument in more detail in the next section.

3.3. Welfare Analysis under Inflation Stabilization

I now show more formally how decreasing returns affect welfare outcomes under real wage rigidity and complete inflation stabilization. Household welfare is given by:

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\nu}}{1-\nu}. \tag{47}$$

A second-order approximation of the utility function gives

$$U(c_t) \approx U(c) + c^{-\nu}\tilde{c}_t - .5\nu c^{-\nu-1}\tilde{c}_t^2, \tag{48}$$

where $\tilde{c} = c_t - c$ is the level deviation from steady state. Thus,

$$W_0 \approx E_0 \sum_{t=0}^{\infty} \beta^t c^{-\nu}(\tilde{c}_t - .5\nu c^{-1}\tilde{c}_t^2). \tag{49}$$

As is well known, welfare can be decomposed into a level and a volatility term of consumption. Most importantly, with risk averse households, consumption volatility reduces welfare. The level term is obtained by a second-order approximation to the resource constraint under zero inflation volatility and log utility:

$$\tilde{c}_t \approx (n^\alpha - \kappa v) \cdot \tilde{a}_t + mpl \cdot \tilde{n}_t - \kappa a \cdot \tilde{v}_t + mpl \cdot \tilde{a}_t\tilde{n}_t + \frac{1}{2}\frac{\partial mpl}{\partial n} \cdot \tilde{n}_t^2 - \kappa \cdot \tilde{a}_t\tilde{v}_t, \tag{50}$$

where $mpl = \alpha a n^{\alpha-1}$ is the marginal product of labor.

Using the definition of market tightness (36) and ignoring higher-order terms as well as terms independent of policy, the volatility term is governed by the second moments of the labor market:

$$
\begin{aligned}
\tilde{c}_t^2 \approx & mpl^2 \cdot \tilde{n}_t^2 + 2mpl\zeta_2 \cdot \tilde{a}_t\tilde{n}_t + 2mpl\zeta_1 \cdot \tilde{n}_t\tilde{n}_{t-1} + \zeta_3 \cdot \tilde{\theta}_t^2 + \zeta_1^2 \cdot \tilde{n}_{t-1}^2 + \zeta_1\zeta_2 \cdot \tilde{a}_t\tilde{n}_{t-1} \\
& - 2mpl\zeta_3 \cdot \tilde{\theta}_t\tilde{n}_t - 2\zeta_1\zeta_3 \cdot \tilde{\theta}_t\tilde{n}_{t-1} - 2\zeta_3\zeta_1\tilde{\theta}_t\tilde{a}_t
\end{aligned}, \tag{51}
$$

where $\zeta_1 = \kappa a\theta(1-\sigma)$, $\zeta_2 = n^\alpha - \kappa v$, and $\zeta_3 = \kappa a(1-(1-\sigma)n)$.

While welfare losses can also arise from the level term of consumption, the difference between constant and decreasing returns is predominantly driven by the volatility term.[18] The first two terms on the right-hand side of equation (51) capture the direct effect of the labor market response to productivity changes on output and hence consumption volatility. The remaining terms capture effects related to changes in recruiting costs. Clearly, a higher response of employment to productivity shocks (i.e. a high covariance between a_t and n_t and a high variance of n_t) increases consumption volatility and hence reduces welfare. This has already been argued in the last section. From equation (51) it is also clear that the response of consumption volatility to labor market volatility is quite different between constant and decreasing returns. With $mpl < 1$, employment fluctuations in response to productivity shocks have a smaller impact on consumption volatility and the stabilization of employment becomes less of a concern from a welfare perspective.[19] Figure 1 shows a contour plot of the consumption volatility term in equation (51) (relative to the variance of aggregate productivity and net of terms independent of policy) for different combinations of wage rigidity and returns to scale based on the calibration strategy in section 4. The plot shows that moving from constant returns to scale to $\alpha = 0.66$ reduces consumption volatility by a factor of 3. How much of this reduction is due to smaller second moments and how much is due to a reduced impact of labor market volatility on consumption volatility? Figure 2 shows the counterfactual case in which only the coefficients in (51) are allowed to vary with the returns to scale but the moments are fixed at their values under constant returns ($\alpha = 1$). In this case, moving from constant returns to scale to $\alpha = 0.66$ reduces consumption volatility by a factor of 1.4, i.e. both factors - reduced labor market volatility and a reduced impact on consumption volatility - account for around half of of the total reduction.

[18] In a recent paper, LePetit (2019) argues that the combination of steady state distortions and labor market asymmetries generates a trade-off between inflation and unemployment stabilization because it leads to a higher mean unemployment. While there are differences in the level term between constant and decreasing returns in my calibration, these are very small compared to differences in the volatility term. Furthermore, my results are preserved if I use a first-order approximation to the resource constraint, which by construction leads to a mean of consumption equal to its steady state, i.e. $E(\tilde{c}_t) = 0$.

[19] There are offsetting effects through smaller negative coefficients that are related to vacancy posting costs. These are quantitatively small compared to the first two coefficients.

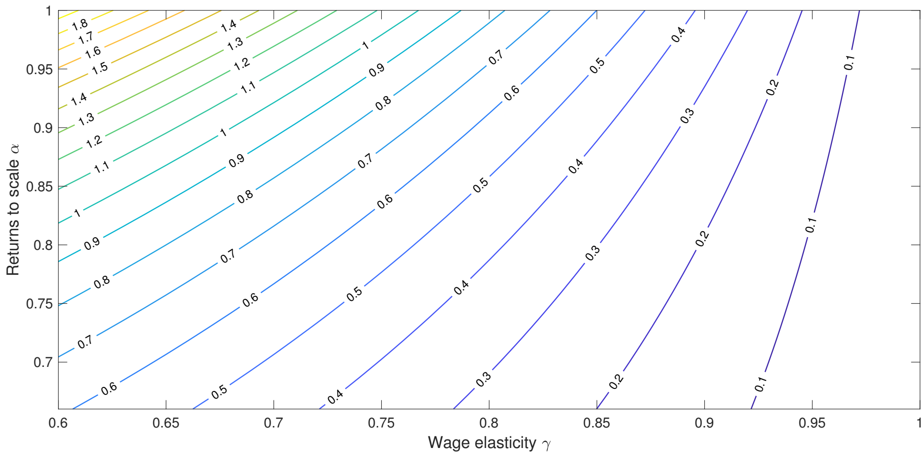

Figure 1: Contour plot of consumption volatility term as in equation (51) for different degrees of wage rigidity and returns to scale. The displayed values are relative to the variance of productivity and exclude terms independent of policy.

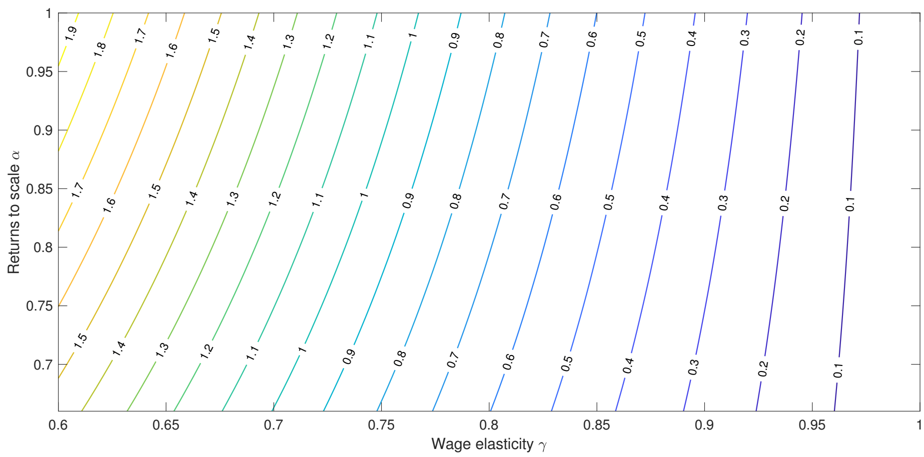

Figure 2: Contour plot of consumption volatility term as in equation (51) for different degrees of wage rigidity and returns to scale with second moments fixed at their constant returns to scale value. The displayed values are relative to the variance of productivity and exclude terms independent of policy.

4. Calibration

The model is calibrated to a quarterly frequency to the U.S. economy. All targets and parameter values are displayed in Table 1 and Table 2.

The household's discount factor is 0.99, consistent with a 4% annual interest rate and the coefficient of relative risk aversion is $\nu = 1$, i.e. utility is log.[20] The parameter determining the elasticity of substitution between different goods is $\epsilon = 11$, which corresponds to a 10% markup in steady state.

In line with the data, the quarterly separation rate is set to $\sigma = 0.1$. I target a steady state unemployment rate of 6%.[21] Given the law of motion for employment, this yields a quarterly job-finding rate of 0.6105. The steady state value of market tightness is normalized to 1. Given the matching function, this pins down the value of matching efficiency at $\vartheta = 0.6105$. I set the weight on vacancies in the matching function to $\xi = 0.3$, which is in line with the survey of matching function estimations by Petrongolo and Pissarides (2001). The coefficient on labor in the production function is set to $\alpha = 0.66$, the value most commonly taken for the labor share.

Haefke et al. (2013) estimate an elasticity of the real (hourly) wage with respect to aggregate productivity for new workers of 0.79. For earnings per person the value is 0.83. I therefore set the elasticity of the real wage with respect to productivity in the model to $\gamma = 0.8$.[22] Following Michaillat (2014), who derives the price adjustment costs from microeconomic evidence presented in Zbaracki et al. (2004), I set $\Phi = 60$. This is an intermediate value between Krause and Lubik (2007), who use a value of 40, and Faia et al. (2014), who use a value of 116.5 for a quarterly calibration.

Note that because of condition (28), choosing recruiting costs simultaneously determines the steady state real wage. Silva and Toledo (2009), using evidence provided by Dolfin (2006), calculate that recruiting costs per hire amount to 4.3% of the quarterly wage of a newly hired. However, these costs only include the manhours spent by the company. Assuming that firms also incur direct financial costs (costs of advertising, travel costs etc.), I set total vacancy posting costs to 10% of the steady state real wage. This simultaneously pins down ω and hence the steady state real wage. This strategy has several advantages. First, it makes sure that the labor market steady state is the

[20]Choosing a higher value of risk aversion does not change the results by much.

[21]Unemployment refers to $u_t = 1 - n_t$, i.e. not the number of searching workers, which, given immediate rehiring, is higher.

[22]Gertler et al. (2019) find that wages of new hires out of unemployment are as sticky as those of existing matches and that the high elasticities from previous studies are driven by the cyclicality of job-to-job transitions. However, as they estimate the semi-elasticity of wages with respect to unemployment (not productivity), the estimates are not directly comparable.

same under constant and decreasing returns to scale. Second, it implies that the fundamental surplus as a fraction of the wage (FS/ω, compare equations (44) and (46)) and the wage relative to its efficient level, i.e. the degree of steady state distortions, are the same under the two specifications.[23]

Finally, aggregate productivity follows an AR(1) process. I estimate the process using data for quarterly total factor productivity (TFP) from 1964 to 2013 from the database constructed by Fernald (2012).[24] This yields an autocorrelation coefficient of 0.96 and a standard deviation for the shock process of 0.00965.

I solve the model using second-order perturbation techniques.[25] For the Ramsey problem, all co-state variables are set to their steady state value in period zero consistent with a timeless perspective.

	Description	Value
Joint targets		
n	Steady state employment	0.94
θ	Steady state market tightness	1
Λ	Steady state marginal costs	1
Joint parameters		
β	Discount factor	0.99
ν	Risk aversion coefficient	1
ϵ	Elasticity of substitution parameter	11
σ	Separation rate	0.1
ξ	Elasticity in matching function	0.3
Φ	Price adjustment costs	60
ϑ	Matching efficiency	0.6105
α	Labor elasticity	0.66
γ	Real wage elasticity	0.8
τ_s	Production subsidy	$1/(\epsilon - 1)$
ρ	AR-coef. productivity	0.96
$sd(a)$	SD productivity	0.00965

Table 1: Joint targets and parameters

[23] My qualitative results are robust to choosing higher or lower recruiting costs. For example, with vacancy posting costs between 7% and 12% of the steady state real wage, the welfare loss in terms of lifetime utility of strict inflation targeting ranges between 0.04 and 0.08 for decreasing returns and 0.77 and 4.36 for constant returns. Thus, while the absolute welfare loss varies, the gap between constant and decreasing returns is always large.

[24] Data at: `www.frbsf.org/economic-research/files/quarterly_tfp.xls`.

[25] I use Dynare 4.5.6 (see Adjemian et al., 2011).

	Description	CRS	DRS
κ	Recruiting costs	$0.1 \cdot \omega$	$0.1 \cdot \omega$
mpl	Marginal product of labor	1	0.674
ω	Steady state wage	0.9825	0.6622
FS/ω	FS as fraction of wage	1.79%	1.79%
$\omega/w^\star$	Wage relative to optimal wage	1.29	1.29

Table 2: Calibration of wage and recruiting costs

5. Results

5.1. Flexible Wage and Decreasing Returns

Before analyzing the interaction between decreasing returns to scale and real wage rigidity, let us first establish a benchmark by looking at both characteristics individually.

Table 3 shows the results of optimal monetary policy when production is characterized by decreasing returns to scale but real wages are flexible. The first line of Table 3 displays the optimal volatility of each variable relative to productivity if the wage follows equation (15) but with γ set to one, i.e. the wage moves one to one with aggregate productivity. As expected, both the optimal inflation volatility and the optimal employment volatility are zero. The Ramsey planner achieves a simultaneous stabilization of both inflation and employment.[26]

Flexible Wage & Decreasing Returns

Policy	π	u	n	θ	c
Ramsey	0.00	0.00	0.00	0.00	1.00

Table 3: Optimal volatility of model variables with flexible wage and decreasing returns. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations of 200 periods with 1000 burn-in periods. Inflation is annualized.

5.2. Real Wage Rigidity and Constant Returns

Next, let us turn to the opposite case with a rigid real wage but constant returns to scale, i.e. $\alpha = 1$, the case considered in Blanchard and Galí (2010). Results are displayed in Table 4. First of all, the real wage rigidity leads to substantial labor market amplification under inflation stabilization as predicted by equation (44). The optimal policy trades

[26] There would be small deviations from complete inflation and employment stabilization for higher degrees of risk aversion (see discussion in Blanchard and Galí, 2010).

off part of the employment volatility for inflation volatility. Under the Ramsey policy, inflation fluctuates nearly as much as productivity, while employment volatility is cut in half compared to the inflation stabilization policy. A simultaneous stabilization of both employment and inflation is not feasible and the Ramsey planner strikes a balance between the two goals. Note that consumption volatility is about 17% higher under inflation stabilization than under the optimal policy.

Rigid Wage & Constant Returns

Scenario	π	u	n	θ	c
Ramsey	0.89	4.00	0.25	5.73	1.14
Infl. stabiliz.	0	7.44	0.51	13.53	1.33
Opt. Taylor Rule	0.74	5.53	0.37	8.28	1.21

Table 4: Volatility of model variables with real wage rigidity and constant returns for Ramsey policy, strict inflation stabilization, and optimized Taylor rule. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations of 200 periods with 1000 burn-in periods. Inflation is annualized.

The discrepancy between the two policies can best be seen by looking at the impulse responses. Figure 3 shows the model responses to a one standard deviation positive productivity shock. While the Ramsey planner immediately reduces the nominal interest rate in response to the shock, strict inflation stabilization requires an initial increase of the nominal interest rate of 40 basis points. As a result, the responses of unemployment, employment and labor market tightness are all more than twice as large compared to the Ramsey policy.

The welfare costs of strict inflation stabilization are quantitatively important. Table 5 shows that (unconditional) expected discounted lifetime utility is 1.4% lower compared to the optimal policy, which corresponds to 0.11% in terms of per-period consumption equivalents.

Welfare loss of strict inflation stabilization

	Lifetime utility	Cons. Equiv.
CRS	1.41%	0.11%
DRS	0.06%	0.003%
$\text{DRS}_{\gamma=.6}$	0.22%	0.012%

Table 5: Welfare loss of strict inflation stabilization policy compared to Ramsey policy for model with constant returns to scale (CRS) and decreasing returns to scale (DRS). The first column reports the welfare loss based on unconditional expected discounted lifetime utility, the second column states the corresponding welfare loss in per-period consumption equivalents.

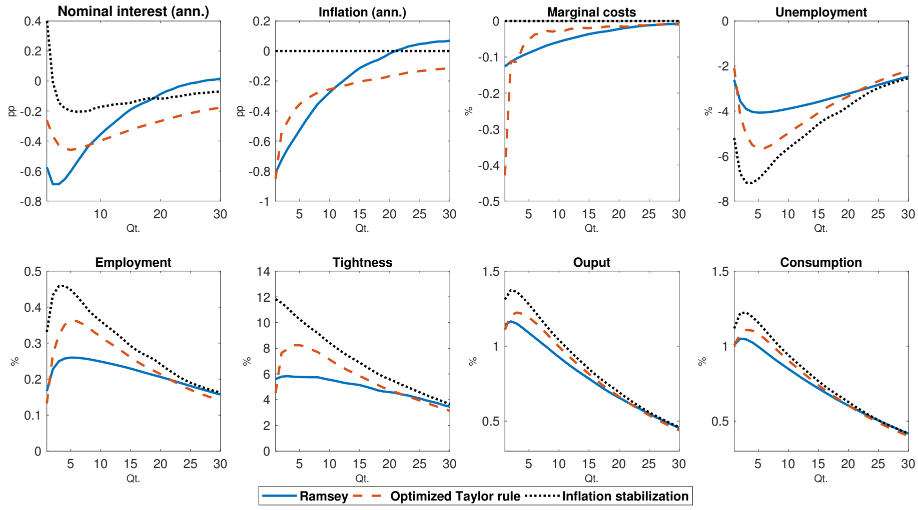

Figure 3: Impulse responses to a one standard deviation positive productivity shock for model with rigid wage and constant returns to scale. The nominal interest rate and inflation are annualized and in percentage point deviation. All other variables are in percent deviation from steady state.

One disadvantage of the Ramsey policy is that it does not provide a simple policy rule. I therefore also provide results for an optimized Taylor rule, i.e. I search for the parameters that maximize welfare under the following simple Taylor rule:[27]

$$R_t = \frac{1}{\beta}\left(\frac{\pi_t + 1}{\bar{\pi} + 1}\right)^{\mu_\pi(1-\mu_R)}\left(\frac{u_t + 1}{\bar{u} + 1}\right)^{\mu_u(1-\mu_R)}(\beta R_{t-1})^{\mu_R}, \tag{52}$$

where $\bar{\pi}$ is the long-run net inflation rate (which is zero in this setting) and $\bar{u}$ is the steady state unemployment rate.[28] I run a grid search for the parameters on the intervals $\mu_\pi \in (1, 5]$, $\mu_u \in [-5, 0]$, and $\mu_R \in [0, 1]$.[29] The optimized Taylor rule in case of rigid real wages and constant returns to scale puts a nontrivial weight on unemployment stabilization (see Table 6). A one percentage point higher unemployment rate leads approximately to a 97 basis points lower nominal interest rate. Note that Table 4 and

[27]Blanchard and Galí (2010) do a similar exercise, although their Taylor rule does not include interest rate smoothing.

[28]The log-linear approximation of this rule reduces to the familiar form $i_t = (1-\mu_R)\bar{i} + \mu_\pi \pi_t - \mu_u \hat{u}_t + \mu_R i_{t-1}$, where i_t is the net nominal interest rate and $\hat{u}_t = u_t - \bar{u}$ is the percentage point deviation of the unemployment rate from steady state. Note that I scale inflation and unemployment in eq. (52) by adding one so that I can interpret the coefficients in terms of percentage point instead of percent deviations.

[29]It is a common phenomenon to obtain corner solutions for the coefficient on inflation. Allowing for even higher values for inflation would result in both higher coefficients on inflation and larger (absolute) values on unemployment without sizable effects on welfare.

Figure 3 show that the optimized rule does not do a perfect job in replicating the Ramsey policy. The welfare loss is still around 0.3% in terms of discounted lifetime utility.

Optimized Taylor rule

	μ_π	μ_u	μ_R
CRS	5.00	-0.97	0.93
DRS	5.00	-0.005	0.91
$\text{DRS}_{\gamma=.6}$	5.00	-0.08	0.95

Table 6: Welfare maximizing coefficients for Taylor rule (equation (52)) for model with constant (CRS) and decreasing (DRS) returns to scale. Based on grid search on the intervals $\mu_\pi \in (1,5]$, $\mu_u \in [-5,0]$, and $\mu_R \in [0,1]$.

5.3. Real Wage Rigidity and Decreasing Returns

How does optimal policy change if the economy is characterized by both real wage rigidity and decreasing returns to scale? The results are displayed in Table 7. The differences to the case with constant returns are striking. Optimal inflation volatility is reduced by a factor of four, while employment volatility is also slightly smaller. Complete stabilization of both employment and inflation is still not feasible and the Ramsey planner allows for some inflation volatility. However, while the volatility of labor market variables is higher under complete inflation stabilization compared to the optimal policy, the increase is relatively modest and consumption volatility is only slightly higher. This is also visible from the impulse responses in Figure 4. The differences between the two policies are quite small compared to the constant returns to scale case in Figure 3.

Rigid Wage & Decreasing Returns

Scenario	π	u	n	θ	c
Ramsey	0.21	3.28	0.21	5.00	1.09
Infl. stabiliz.	0	3.99	0.25	6.25	1.11
Opt. Taylor Rule	0.19	3.49	0.22	5.31	1.09

Table 7: Volatility of model variables with rigid real wage and decreasing returns for Ramsey policy, strict inflation stabilization, and optimized Taylor rule. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600 of 200 periods with 1000 burn-in periods. Results are means over 1000 simulations. Inflation is annualized.

In terms of welfare, the discounted lifetime utility of households under inflation stabilization is only 0.06% lower compared to the Ramsey policy. This corresponds to a loss of just 0.003% in terms of consumption equivalents (see Table 5). This is again in stark contrast to the case with constant returns to scale, for which strict inflation stabilization results in considerable welfare losses.

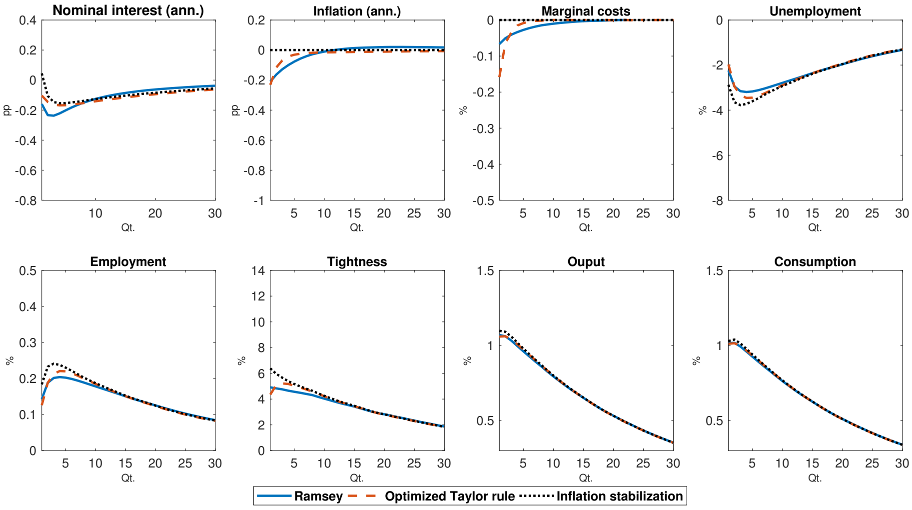

Figure 4: Impulse responses to a one standard deviation positive productivity shock for model with rigid wage and decreasing returns to scale. The nominal interest rate and inflation are annualized and in percentage point deviation. All other variables are in percent deviation from steady state.

The differences also show up when looking at the optimized Taylor rule. Table 6 shows that with decreasing returns, the welfare-maximizing Taylor rule puts close to zero weight on unemployment but a similar weight on inflation and the lagged interest rates compared to the constant returns to scale case. Although employment volatility is a bit higher under this simple rule compared to the Ramsey policy (see Table 7 and Figure 4), the welfare level achieved under this policy is just 0.01% smaller.

How much of these differences is due to a the lower labor market amplification in the model with decreasing returns? In order to answer this, I increase the degree of wage rigidity so that the model with decreasing returns features a similar volatility of employment under inflation stabilization. This is achieved by setting γ equal to 0.6.[30] The results are displayed in Table 8 and Figure 5.

While the Ramsey planner reduces the volatility of employment by half under constant returns to scale (Table 4), optimal employment volatility is only slightly smaller than under inflation stabilization in Table 8. The reason is that the impact of a high labor market volatility on consumption volatility is small and hence there is also less to gain

[30]Note that this makes the wage not just more rigid relative to aggregate productivity but also relative to the marginal product of labor.

Rigid Wage ($\gamma = 0.6$) & Decreasing Returns					
Scenario	π	u	n	θ	c
Ramsey	0.42	6.82	0.42	9.66	1.18
Infl. stabiliz.	0	8.17	0.52	12.33	1.22
Opt. Taylor Rule	0.39	7.03	0.44	10.32	1.18

Table 8: Volatility of model variables with rigid real wage and decreasing returns for Ramsey policy, strict inflation stabilization, and optimized Taylor rule. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations of 200 periods with 1000 burn-in periods. Inflation is annualized.

from reducing labor market volatility further. While the welfare loss of strict inflation stabilization is larger than in the case with lower wage rigidity and lower employment volatility, it is still much lower than under constant returns. In fact, the welfare loss is about 7 times smaller in lifetime utility and about 10 times smaller in consumption equivalents compared to the case with constant returns (see Table 5). The weight on unemployment in the optimized Taylor rule (see Table 6) is still close to zero. Thus, even under a high degree of wage rigidity and an equal amount of labor market amplification, the optimal policy does not stray far from inflation stabilization.

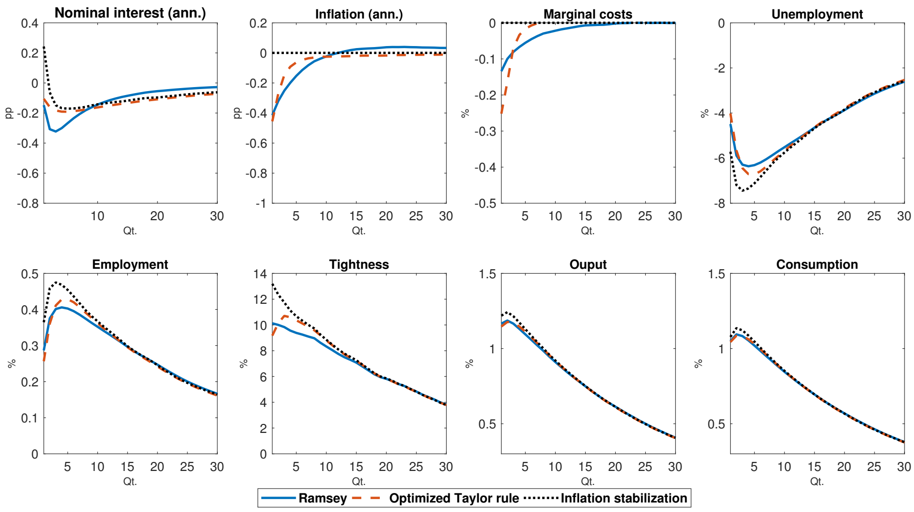

Figure 5: Impulse responses to a one standard deviation positive productivity shock for model with rigid wage ($\gamma = 0.6$) and decreasing returns to scale. The nominal interest rate and inflation are annualized and in percentage point deviation. All other variables are in percent deviation from steady state.

5.4. Alternative wage rule

While the wage norm presented so far offers analytical tractability and direct comparison to the related literature, it is clearly ad hoc. I therefore present an alternative in this section. Following e.g. Krause and Lubik (2007) in modeling wage rigidity, I assume that the wage is a linear combination of the "Nash wage"

$$w_t^N = (1-\eta)\left(\Lambda_t \alpha a_t n_t^{\alpha-1} + (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\kappa a_{t+1}\theta_{t+1}\right]\right), \tag{53}$$

where $1-\eta$ is workers' bargaining power, and its steady state value, such that

$$w_t = \gamma_N w_t^N + (1-\gamma_N)\omega. \tag{54}$$

For comparability, I set workers' bargaining power such that in steady state the wage is equal to the steady state wage from the previous section, i.e. keeping the labor market steady state fixed. The wage rigidity parameter, γ_N, is chosen such that the volatility of the wage relative to aggregate productivity is equal to 0.8 under complete inflation stabilization as in the sections before.[31] With this new wage norm, the degree of wage rigidity is of course no longer independent of the labor market response and - through its dependence on marginal costs - monetary policy. In addition, in case of decreasing returns to scale, intra-firm bargaining would result in a different wage norm (Stole and Zwiebel, 1996). I present results for this version of the wage in the Appendix.

Alternative Rigid Wage & Constant Returns

Scenario	π	u	n	θ	c
Ramsey	1.02	6.16	0.44	9.75	1.33
Infl. stabiliz.	0	7.37	0.51	14.42	1.32

Table 9: Volatility of model variables with rigid real wage as in equation (54) and constant returns for Ramsey policy and strict inflation stabilization. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations of 200 periods with 1000 burn-in periods. Inflation is annualized.

Table 9 shows that under this alternative wage regime the Ramsey planner chooses a lower degree of employment stabilization under constant returns to scale compared to the previous wage norm (see Table 4). However, while the welfare loss of complete inflation stabilization is lower, it is still substantial with a lifetime loss of 0.4% and a

[31]This results in $\gamma_N = 0.4$ in case of the constant returns to scale case and in $\gamma_N = 0.57$ in case of decreasing returns to scale.

loss of 0.03% in terms of consumption equivalents (see Table 10). In contrast, results are quite similar for decreasing returns under the alternative wage norm (see Table 11) and the welfare loss of complete inflation stabilization is again extremely small. Thus, the conclusions from the previous analysis are confirmed under the alternative wage setting: decreasing returns significantly reduce the trade-off between inflation and employment stabilization and the welfare costs of complete inflation stabilization are close to zero in this case. Note that one reason the planner chooses a lower degree of employment stabilization for the case of constant returns and the alternative wage is that marginal costs now feed back into the real wage. If the wage in equation (53) did not respond to marginal costs, the planner would choose to stabilize employment more and the welfare loss of complete inflation stabilization would roughly double.

Welfare loss of strict inflation stabilization

	Lifetime utility	Cons. Equiv.
CRS	0.40%	0.033%
DRS	0.03%	0.001%

Table 10: Welfare loss of strict inflation stabilization policy compared to Ramsey policy for model with constant returns to scale (CRS) and decreasing returns to scale (DRS) and alternative wage rigidity as in equation (54). The first column reports the welfare loss based on unconditional expected discounted lifetime utility, the second column states the corresponding welfare loss in per-period consumption equivalents.

Alternative Rigid Wage & Decreasing Returns

Scenario	π	u	n	θ	c
Ramsey	0.20	3.73	0.25	6.09	1.10
Infl. stabiliz.	0	3.96	0.27	6.76	1.11

Table 11: Volatility of model variables with rigid real wage as in equation (54) and decreasing returns for Ramsey policy and strict inflation stabilization. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations of 200 periods with 1000 burn-in periods. Inflation is annualized.

6. Conclusion

This paper shows that the consequences of real wage rigidity for optimal monetary policy hinge very much on the returns to scale of the production function. In case of constant returns to scale, real wages give rise to a large trade-off between inflation and employment stabilization and optimal policy allows for significant movements in inflation. This case is well established in the literature (Blanchard and Galí, 2010). However, if there are decreasing returns in production – because capital is fixed in the short term – real wage rigidity becomes nearly irrelevant from the point of view of a policy maker. While optimal inflation volatility is still positive, the welfare loss of completely stabilizing inflation is very small. The reason is twofold: First, with decreasing returns firms have an adjustment margin to mitigate the adverse effects of real wage rigidity on marginal costs which implies lower labor market volatility under strict inflation stabilization. Second, any given employment volatility has a smaller effect on consumption volatility under decreasing returns. These results are especially interesting given earlier findings that lend support to a more active fiscal and labor market policy in the presence of real wage rigidity and decreasing returns. The results in this paper thus stress the importance of taking into account the interaction of different short-run real rigidities, which can have very different implications depending on the policy instrument.

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A. Wage Rigidity with Stole and Zwiebel Bargaining

With decreasing returns, the nature of the bargaining game between a firm and a worker changes because the surplus of an individual match depends on the number of employees in the firm. Intuitively, not hiring an additional worker means that the marginal product and hence the wage of the incumbent workers is higher. Stole and Zwiebel (1996) propose a wage bargaining game in this setting in which there is Nash bargaining over the marginal surplus, i.e. the firm takes into account that hiring an additional worker changes the wage bill of the rest of the workforce. This wage norm has been incorporated into a search and matching framework by e.g. Cahuc and Wasmer (2001), Cahuc et al. (2008), and Elsby and Michaels (2013). The resulting wage takes the following form:

$$w_t^{SZ} = (1-\eta)\left(\Lambda_t \alpha a_t n_t^{\alpha-1} - \frac{\partial w_t}{\partial n} n + (1-\sigma)\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\kappa a_{t+1}\theta_{t+1}\right]\right) \tag{55}$$

and the rigid wage based on it is given by

$$w_t = \gamma_N w_t^{SZ} + (1-\gamma_N)\omega. \tag{56}$$

As before I calibrate η such that the $w^{SZ} = \omega$ in steady state and γ_2 such that the volatility of the wage relative to aggregate productivity is around 0.8.[32] The results are very similar to the results in Table 11. Again the welfare loss of strict inflation stabilization compared to the Ramsey policy is close to zero (0.03% for lifetime utility and 0.001% in terms of consumption equivalents).

Stole & Zwiebel Rigid Wage & Decreasing Returns

Scenario	π	u	n	θ	c
Ramsey	0.19	3.86	0.26	6.37	1.11
Infl. stabiliz.	0	4.07	0.27	6.97	1.12

Table A.1: Volatility of model variables with rigid real wage as in equation (56) and decreasing returns for Ramsey policy and strict inflation stabilization. Standard deviations are relative to the standard deviation of productivity. All data is in log-deviations from a Hodrick-Prescott filter with smoothing parameter 1600. Results are means over 1000 simulations of 200 periods with 1000 burn-in periods. Inflation is annualized.

[32] The resulting value is $\gamma_2 = 0.61$.